University of Kentucky, Physics 361
EXAM 3, 2008-04-11

Instructions: The exam is closed book and timed (50 minutes). Part III is a take-home portion and is due Monday at 10:00 A.M. sharp. No late exams will be accepted. For the take-home part, you may consult your textbook, but may not discuss the problems with anyone.
I [65 pts] + II [20 pts] + III [45 pts] = [130 pts] total.

$$\begin{array}{lll} L^2 = l(l+1)\hbar^2 & S^2 = l(s+1)\hbar^2 & J^2 = j(j+1)\hbar^2 \\ L_z = m_l\hbar & S_z = m_s\hbar & J_z = m\hbar \\ f_B = e^{-\alpha}e^{-E/kT} & f_{BE} = \frac{1}{e^{\alpha}e^{E/kT}-1} & f_{FD} = \frac{1}{e^{\alpha}e^{E/kT}+1} \end{array}$$

Part I—Short Answer

[3 pts] 1. List similarities and differences in the quantization of angular momentum in the Schrödinger equation and in the Bohr model.

[2 pts] 2. a) Why does the 4s orbital have a lower energy than the 3d orbital?

[2 pts] b) Why is there a big gap above the transition metals in the periodic table?

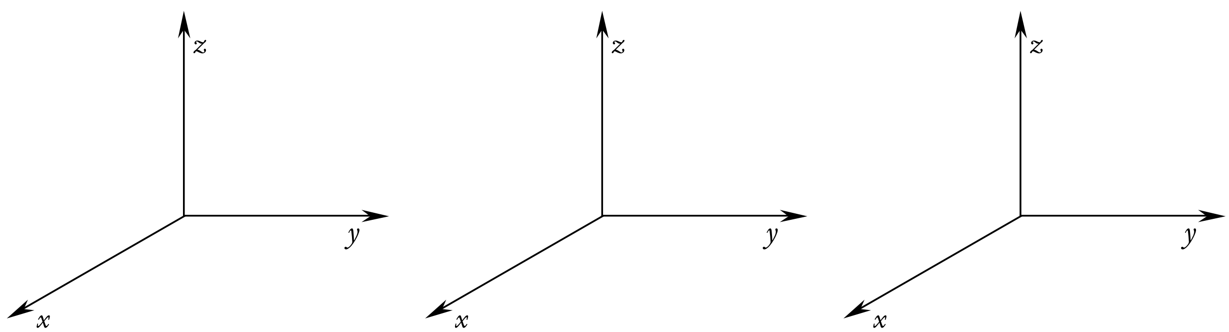
[5 pts] 3. a) For a hydrogen atom, list all of the states in terms of (n, l, m_l, m_s) for $n \leq 2$. How states many are there?

[5 pts] b) List all the states for $n \leq 2$ in terms of (n, l, j, m) where j, m are quantum numbers of $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$. There should be the same number of states.

[2 pts] c) Write down the following states in full spectroscopic notation: $(n{=}1, l{=}0, j{=}\frac{1}{2})$ and $(n{=}3, l{=}2, j{=}\frac{5}{2})$.

[3 pts] d) Write down the electronic configuration of Ge ($Z{=}32$).

[3 pts] 4. Sketch the regions of high probability on the following graph for hydrogen atom electron in the specified states:



2s 1p ($m{=}0$) 1p ($m{=}\pm 1$)

[2 pts] 5. a) Describe the force that causes an atom's spin to precess around a B-field.

[2 pts] b) Describe the force that causes an atom to accelerate in a B-field. What property must the B-field have?

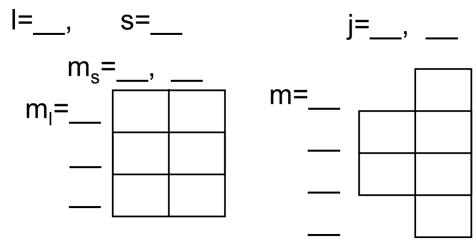
[2 pts] c) Describe the interaction potential energy that causes the Zeeman effect.

[2 pts] d) Describe the potential energy in the spin-orbit interaction ($\boldsymbol{L} \cdot \boldsymbol{S}$).

[6 pts] 6. a) Discuss 3 different types of experimental evidence that electrons have spin, i.e. the additional quantum number m_s.

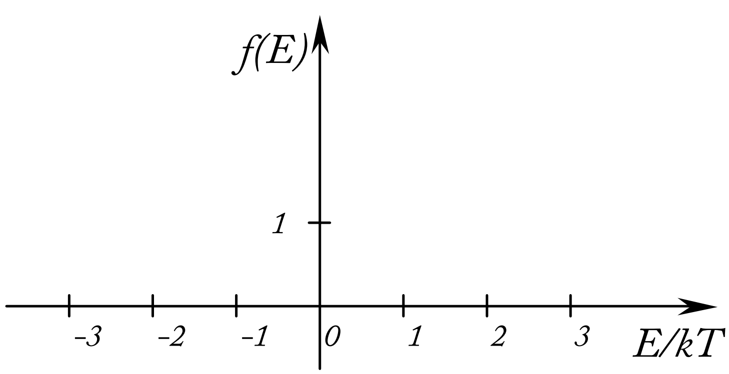
[5 pts] b) List 3 similarities and 2 differences between spin and orbital angular momentum.

[5 pts] 7. Complete the following tables for the addition of angular momenta. Show which groups of states correspond with each other in the two tables.



[3 pts] 8. Describe the meaning of $n(E) = g(E)\, f(E)$.

[6 pts] 9. Sketch a plot of the distributions $f_B(E)$, $f_{BE}(E)$, and $f_{FD}(E)$, where $\alpha = 0$. Label which applies to fermions and which to bosons, and show the features of the graph which are responsible for the Pauli exclusion principle, and for the properties of Bose-Enstein condensates (BEC).



[4 pts] 10. List 4 properties of liquid helium colder than T_c=2.17 K. What is the name of that critical temperature?

[3 pts] 11. What prevents White Dwarf or neutron stars from collapsing?

Part II—Short Calculation

[10 pts] 12. If l=4 and m=2, what is the value of $|L|$ and L_z? What is the angle between $\boldsymbol{L}$ and the z-axis? Sketch the possible positions of the vector $\boldsymbol{L}$ in this state.

[10 pts] 13. Calculate the ratio a hydrogen atoms on the surface of the sun in the first excited state vs. in the ground state. Note that $kT = 0.500$ eV on the surface of the sun and $E_n = -13.6\ \text{eV}/n^2$ for the hydrogen atom.

Part III—Derivation

[5 pts] 14. Show that angular momentum $\boldsymbol{L}$ is conserved by a radial potential $V(r)$.

[15 pts] 15. Show that R_{32} satisfies the radial part of the Schrödinger equation, equation 7-24 in the text, for l=2. Identify the term in this equation containing the effective potential, and show its physical significance. Show that $Y_{21}(\theta, \phi)$ satisfies equations $(L_z)_{op}Y_{lm} = m\hbar Y_{lm}$ and also equation 7-21a in the text. Explain how the above steps show that $\psi_{321}(r, \theta, \phi) = CR_{32}Y_{21}$ is a solution to the Schrödinger equation, and calculate the energy of this state.

[10 pts] 16. Explicitly integrate the probability density over all space to calculate the normalization factor. You may use an integral table or `integrals.wolfram.com`, but show all work.

[15 pts] 17. Use separation of variables to calculate the 2-particle wavefunctions (states) of two particles with position coordinates $x1$ and $x2$ respectively in an infinite square potential where $V(x) = 0$ for $0 \leq x \leq L$ and $V = \infty$ elsewhere. Calculate the total energy of each state. Construct the symmetrized and antisymmetrized wavefunction, and label which type of particles have each type of wavefunction. Show that two fermions can not each occupy the same state.