University of Kentucky, Physics 361
EXAM 1, 2009-02-12 11:00–11:50

Instructions: The exam is closed book and timed (50 minutes), so pace yourself. Problems will be graded on both technique and answer, so show your work. Credit will be awarded for later parts of a question even if the first part was answered incorrect. A formula sheet is on the last page.

Part I—Short Answer

[5 pts] 1. a) Describe one experiment that demonstrated the particle-like nature of a classical wave. Justify the interpretation of this experiment.

[5 pts] b) Describe one experiment that demonstrated the wave-like nature of a classical particle. Again, justify the interpretation.

[3 pts] 2. a) How can identifying an electron as a wave result in quantization of energy?

[3 pts] b) Why do classical electromagnetic waves not have quantized energy levels?

[4 pts] 3. List two principles which regard energy and time as conjugate variables. What is another conjugate pair? [bonus: What is a third?]

[4 pts] 4. In what aspects do both light and matter behave as particles, and in what aspects do they behave as waves?

[4 pts] 5. List two aspects of the wavefunction $\Psi(x,t)$ which embody its particle-like nature? How does this relate to the Heisenberg uncertainty principle?

[4 pts] 6. Compare and contrast the cathode ray tube, the photoelectric effect, and X-ray tube.

[8 pts] 7. Separate Plank's law into two parts $u(\nu)d\nu = g(\nu)d\nu \cdot \bar{\epsilon}$, and explain the general properties of each factor. What was the classical formula? In what limit are they equal? Explain in terms of energy levels how Plank's new assumption solved the ultraviolet catastrophe.

Part II—Calculation

[10 pts] 8. The luminosity (total power radiated) of the sun is 3.85×10^{26} W. What is the surface temperature of the sun? ($r = 6.955 \times 10^8$ m) What is the most intense wavelength of sunlight?

[15 pts] 9. Given $E = p^2/2m$, derive the dispersion relationship for $w(k)$ [w as a function of k]. Calculate the group and phase velocities for such a particle as a function of w and/or k.

[15 pts] 10. Calculate the maximum shift in the wavelength of a 40 keV X-ray scattered from a salt crystal with lattice spacing d=0.219 nm. What scattering angle would this correspond to? How much energy (eV) would an electron have if its de Broglie wavelength was the same as the wavelength of the incident X-ray?

[20 pts] 11. The wavefunction $\psi(x) = A\sin(3\pi x/2L)$ is for a non-relativistic particle trapped between $0 < x < L$. Plot this function, and determine the normalization constant A. Calculate the probability of observing the particle in the region $L/3 < x < 2L/3$. What is the momentum? What is the energy? How many lower energy states are there?

Physical Constants and Useful Combinations:

Speed of light	c	3.00×10^8 m/s
Planck's constant	h	6.63×10^{-34} J s; $hc = 1240$ eV nm
	$\hbar = h/2\pi$	1.05×10^{-34} J s; $\hbar c = 197$ eV nm
Coulomb force constant	$k_e = 1/4\pi\epsilon_0$	8.99×10^9 Nm^2/C; $k_e e^2 = 1.44$ eVnm
Elementary charge	e	1.602×10^{-19} C; 1 eV $= 1.602 \times 10^{-19}$ J
Fine structure constant	$\alpha = k_e e^2/\hbar c$	$0.0730 \approx 1/137$
Permeability of vacuum	μ_0	$4\pi \times 10^7$ N/A^2 $= 4\pi$ mm G/A; 1 T $= 10^4$ G
Gravitational constant	G	6.67×10^{-11} N m^2/kg^2
Avogadro's number	N_A	6.02×10^{23}/mol
Boltzman's constant	k_B	1.38×10^{-23} J/K $= 25$ meV/293 K
Gas constant	$R = N_A k_B$	8.31 J/mol K
Compton wavelength	$\lambda_c = h/m_e c$	0.00243 nm
Bohr radius	$a_0 = \hbar^2/m_e k_e e^2$	0.529 Å
Ionization energy of H	$E_0 = m_e k_e^2 e^2/2\hbar^2$	13.6 eV
Bohr magneton	$\mu_B = e\hbar/2m_e$	9.27×10^{-24} J/T
Unified mass unit	u	1.66×10^{-27} kg $=$ 931 MeV/c^2
Mass of electron	m_e	9.11×10^{-31} kg $=$ 0.511 MeV/c^2
proton	m_p	1.67×10^{-27} kg $=$ 938 MeV/c^2
α-particle	m_α	6.64×10^{-27} kg $=$ 3727 MeV/c^2

Formulas:

Stefan-Boltzmann law	$R = \sigma T^4 \qquad \sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4
Wein's displacement law	$\lambda_m T = 2.898 \times 10^{-3}$ m K
Rayleigh-Jeans formula	$u(\lambda) = 8\pi k_B T \lambda^{-4}$
Planck's radiation law	$u(\nu) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/k_BT}-1}; \qquad E_n = n\,h\nu = n\,hc/\lambda$
Photoelectric effect	$eV_0 = hf - \phi$
Bragg diffraction	$n\lambda = 2d\sin\theta$
Compton effect	$\lambda_2 - \lambda_1 = \frac{h}{m_e c}(1-\cos\theta)$
Rydberg-Ritz formula	$\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right), n > m$
Impact parameter	$b = \frac{k_e q_\alpha Q}{m_\alpha v^2}\cot\frac{\theta}{2}$
Scattered fraction f	$f = \pi b^2 n t$
Number of scattered4s6s2s α's observed	$\Delta N = \left(\frac{I_0 A_{sc} n t}{r^2}\right)\left(\frac{Z k_e e^2}{2E_k}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}$
Size of nucleus	$r_d = \frac{k_e q_\alpha Q}{\frac{1}{2}m_\alpha v^2}$
Bohr's postulates	$L = n\hbar$ for integer n; $hf = E_n - E_m$
atomic energy levels	$E_n = -\frac{Z^2 E_0}{n^2}$ where $E_0 = \frac{m_e k_e^2 e^2}{2\hbar^2} = 13.6$ eV
atomic orbital radii	$r_n = \frac{n^2 a_0}{Z}$ where $a_0 = \frac{\hbar^2}{m_e k_e e^2} = 0.529$ Å
reduced mass	$\mu = \frac{mM}{m+M}$
Moseley equation	$f^{1/2} = A_n(Z-b)$
De Broglie relations	$\nu = E/h$ and $\lambda = h/p$
Davisson and Germer diffraction	$n\lambda = D\sin\phi$