University of Kentucky, Physics 361
EXAM 2, 2008-03-18 18:00–20:00

Instructions: The exam is closed book and timed (120 minutes), so pace yourself. In particular, don't waste your time on the question worth [0 pts]. There are [100 pts] in total. Problems will be graded on both technique and answer, so show your work. Here is a compilation of formulas:

$$\begin{aligned} E &= \hbar\omega & \Delta E\ \Delta t &\ge \tfrac{\hbar}{2} & \hat{E} &= i\hbar\tfrac{\partial}{\partial t} \\ p &= \hbar k & \Delta p\ \Delta x &\ge \tfrac{\hbar}{2} & \hat{p} &= -i\hbar\tfrac{\partial}{\partial x} \\ L_z &= \hbar m & \Delta L_z\ \Delta\phi &\ge \tfrac{\hbar}{2} & \hat{L}_z &= -i\hbar\tfrac{\partial}{\partial\phi} \end{aligned}$$

$$\text{TDSE:}\quad \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\,\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$

$$\text{TISE:}\quad \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\,\psi(x) = E\,\psi(x)$$

$$\langle M\rangle = \int_{-\infty}^{\infty}\psi^*\hat{M}\psi\,dx \qquad v_p = \frac{\omega}{k} \qquad v_g = \frac{d\omega}{dk}$$

Part I—Short Answer

[5 pts] 1. Name five requirements for $\Psi(x,t)$ to represent a quantum mechanical wave function.

[3 pts] 2. What is the relation between de Broglie matter waves and the Schrödinger equation?

[3 pts] 3. Why must the Schrödinger equation be linear?

[2 pts] 4. Why must $\mathcal{E}$ be real, while Ψ may be real or imaginary?

[2 pts] 5. Why is the probability density written $\Psi^*\Psi$ instead of Ψ^2?

[6 pts] 6. When the two pure waves $\Psi_1 = \cos(k_1x - w_1t)$ and $\Psi_2 = \cos(k_2x - w_2t)$ are superimposed, they add up to the function $\Psi = 2\cos(\frac{1}{2}\Delta k\,x - \frac{1}{2}\Delta w\,t)\cos(\bar{k}\,x - \bar{w}\,t)$. What is the group velocity of the train of packets? What is the phase velocity? What is the condition for no dispersion in terms of $k_{1,2}$ and $\omega_{1,2}$?

[0 pts] 7. a) What is the average wingspan of an African swallow?

[3 pts] b) Why are African swallows almost always unsuccessful in their attempts to tunnel through brick walls?

[3 pts] c) List 3 more relevant applications or examples of quantum mechanical tunnelling.

[2 pts] 8. What is Δx and Δk for a pure harmonic wave of a single frequency and wavelength?

[4 pts] 9. How is $\Psi(x,t)$ different than a classical wave? How is it similar to a classical particle?

[3 pts] 10. a) What requirements does the angular wavefunction $\Phi(\phi) = e^{i3.5\phi}$ violate?

[2 pts] b) What is the angular momentum of the wavefunction $\Phi(\phi) = e^{i7\phi}$?

[3 pts] c) Why is angular momentum always quantized for a particle in a central potential regardless of whether or not it is bound?

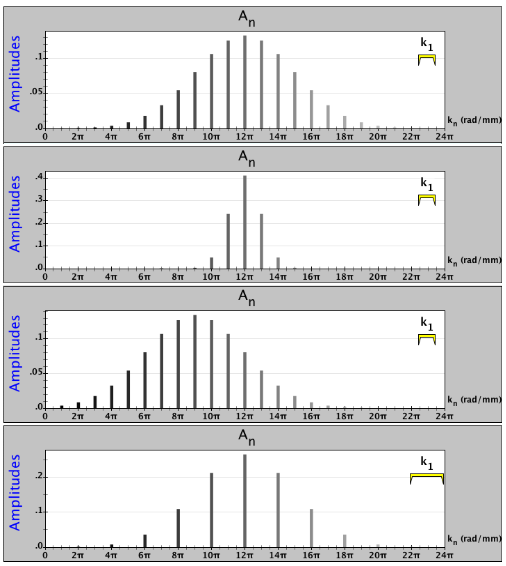
[2 pts] 11. Name two physical differences between the Schrödinger equation and the classical wave equation.

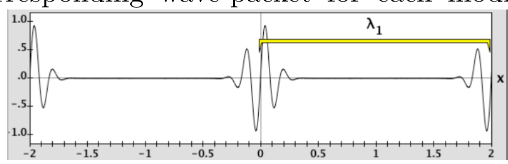
[2 pts] 12. What happens to the zero-point energy of a bound particle as the width of the box increases $L \to \infty$?

[4 pts] 13. Why do electrons shot one at a time through a slit still produce an interference pattern? Why don't they once they are observed behind one slit?

[6 pts] 14. Draw the node lines of the 6 lowest states for infinite square wells over rectangular and circular domains. Show which states are degenerate.

[9 pts] 15. Drawn below is a wave packet $\psi(x)$ composed from the specified frequency component amplitudes A_k. Draw the corresponding wave-packet for each modified A_k spectrum.





Part II—Calculation

[12 pts] 16. Show that $\Psi(x) = A_0 e^{-\frac{1}{2}(x/x_0)^2}$ is a wavefunction for the harmonic oscillator potential $V = \frac{1}{2}m\omega x^2$, and determine the value of x_0. Draw the fifth energy state wavefunction for a simple harmonic oscillator potential, paying special attention to curvature.

[12 pts] 17. Starting from the TISE, solve for the energy states E_n and the wavefunctions $\psi_n(x)$ of the infinite square well in one dimension. Normalize the wavefunctions. What is the full wavefunction $\Psi_n(x,t)$? What is the probability that a particle in the third energy state lies in the middle third of the box, i.e. $\frac{1}{3}<x<\frac{2}{3}$? Showing all steps except for evaluation of the integral, write down the expression for the expected value of momentum.

Part III—Derivation

[12 pts] 18. Starting from the time-dependent Schrödinger equation, perform separation of variables to derive the time-independent Schrödinger equation. What is the time dependence of the wave function? Hint: use $\Psi(x,t) = \psi(x) \cdot \phi(t)$. Obtain separate equations for $\psi(x)$ [TISE] and $\phi(t)$, and solve the equation for the time dependence $\phi(t)$.