University of Kentucky, Physics 361
EXAM 2, 2009-03-13 11:00–11:50

Instructions: The exam is closed book and timed (50 minutes).

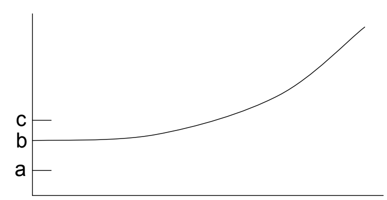
$$E = \hbar\omega \qquad \Delta E\ \Delta t \geq \frac{\hbar}{2} \qquad \hat{E} = i\hbar\frac{\partial}{\partial t}$$
$$p = \hbar k \qquad \Delta p\ \Delta x \geq \frac{\hbar}{2} \qquad \hat{p} = -i\hbar\frac{\partial}{\partial x}$$
$$L_z = \hbar m \qquad \Delta L_z\ \Delta\phi \geq \frac{\hbar}{2} \qquad \hat{L}_z = -i\hbar\frac{\partial}{\partial \phi}$$

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\,\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$
$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\,\psi(x) = E\,\psi(x)$$

$$\hbar c = 197\text{ eV nm} \qquad k_e e^2 = e^2/4\pi\epsilon_0 = 1.44\text{ eV nm}$$
$$m_e c^2 = 0.511\text{ MeV}, \qquad m_p c^2 = 938\text{ MeV}$$
$$E_n = -Z^2E_0/n^2 \qquad E_0 = m_e k_e^2 e^2/2\hbar^2 = 13.6\text{ eV}$$
$$r_n = n^2 a_0/Z \qquad a_0 = \hbar^2/m_e k_e e^2 = 0.529\ (\text{Å} = 0.1\text{ nm})$$
$$\langle G\rangle = \int_{-\infty}^{\infty}\psi^*\hat{G}\psi\,dx \qquad \mu = \frac{mM}{m+M}$$

Part I—Short Answer

[6 pts] 1. In the following diagram, label the impact parameter and scattering angle. Draw the scattering trajectories starting at a) and c). Show the location of nucleus with an '×'. Draw a detector and label the corresponding differential cross section $d\sigma$ being measured. Rank the three trajectories as having the smallest (1) to largest (3) differential cross section.

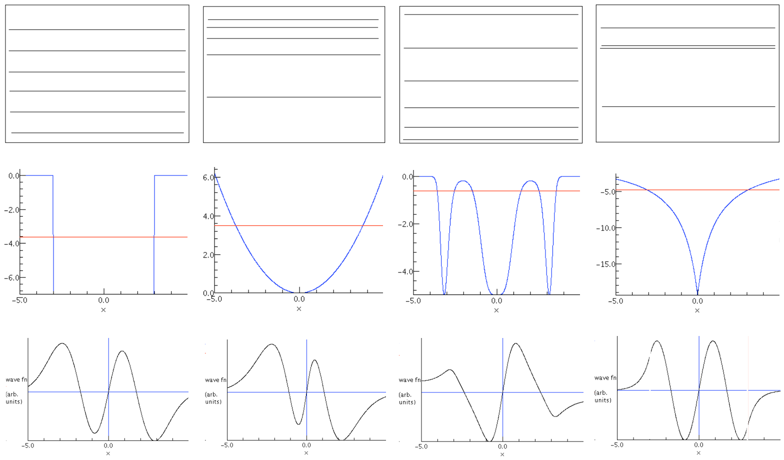


[3 pts] 2. Using a picture and brief labels, show the analogy between tides on the Earth and hydrogen and deuterium atoms. Which atom has lower ground state energy?

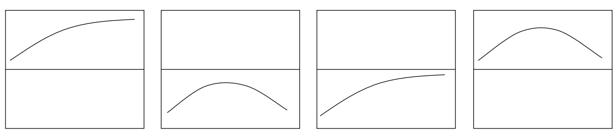
[3 pts] 3. Why does the ideal harmonic oscillator have an infinite number of energy levels, while the finite square well has only a finite number of energy levels?

[2 pts] 4. What two energies are quantized in the Bohr model of atomic spectra?

[8 pts] 5. Match each of the following potential functions (middle) to the corresponding energy level (top), and to the corresponding 4th state wave function (bottom).



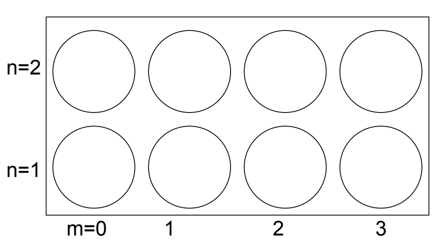
[4 pts] 6. Circle the wave function(s) which represent tunneling. What happens classically at the interface between positive and negative kinetic energy?



[5 pts] 7. a) Draw the wave function for the 10th excited state of the harmonic oscillator potential. b) Show two features of the wave function which illustrate the correspondence principle.

[4 pts] 8. How are momentum and position represented quantum mechanically?

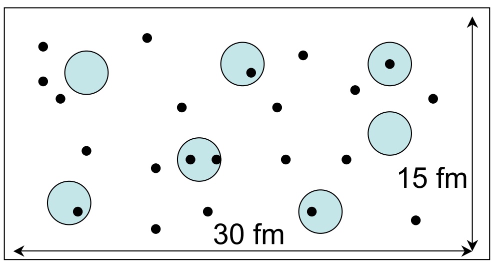
[5 pts] 9. a) Draw the node lines of the first few wave functions of the infinite square well on a disk. b) Indicate the degeneracy of each state.



Part II—Short Calculation

[5 pts] 10. Derive the dispersion relation between w and k starting from $E = p^2/2m$. Compare this with terms in the time-dependent Schrödinger equation (circle the parts that are the same).

[10 pts] 11. Suppose Rutherford were able to peek at the alpha bullet-holes in his gold sample, and saw the result below. a) what would he measure for the nuclear cross section σ? b) List two errors in this supposition.



[10 pts] 12. a) Which of the following are eigenfunctions of the operator $\frac{\partial^2}{\partial x^2}$? b) Calculate the eigenvalue for each eigenfunction.

i) $e^{2i\,x}$ ii) e^{-x} iii) $\cos(3x+4)$ iv) $3x+4$ v) x^2

Part III—Full Problems

[15 pts] 13. Show that $\psi(x) = A\sin(k_n x)$ is a solution of the time-independent Schrödinger equation (TISE) for a particle in an infinite square well, with $U(x) = 0$ for $0<x<L$ and $U(x) = \infty$ for $x<0$ or $x>L$. Show that the boundary conditions are satisfied on both sides of the well, and determine the values of k_n and therefore E_n. Normalize the wave functions.

[15 pts] 14. Use the solutions above to construct wave functions $\psi_{nm}(x, y)$ of a particle in a 2-D infinite well with a square boundary, ie. $U(x, y) = 0$ for $0<x<L, \quad 0<y<L$, and $U(x, y) = \infty$ elsewhere, and calculate the energy values E_{nm}. Draw the node lines for the six lowest energy states, and show which ones are degenerate. No need to do a full separation of variables.