University of Kentucky, Physics 361
EXAM 2, 2009-03-11 18:00–20:00

Instructions: The exam is closed book. Show your work as problems will be graded on both technique and final answer. Formulas:

$$E = \hbar\omega \qquad \Delta E\ \Delta t \geq \hbar/2 \qquad \hat{E} = i\hbar\,\partial/\partial t$$
$$p = \hbar k \qquad \Delta p\ \Delta x \geq \hbar/2 \qquad \hat{p} = -i\hbar\,\partial/\partial x$$

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\,\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$
$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\,\psi(x) = E\,\psi(x)$$

$$\hbar c = 197\text{ eV nm} \qquad k_e e^2 = e^2/4\pi\epsilon_0 = 1.44\text{ eV nm}$$
$$m_e c^2 = 0.511\text{ MeV}, \qquad m_p c^2 = 938\text{ MeV}$$
$$\langle G\rangle = \int_{-\infty}^{\infty}\psi^*\hat{G}\psi\,dx \qquad E_0 = m_e k_e^2 e^4/2\hbar^2 = 13.6\text{ eV}$$
$$\mu = \frac{mM}{m+M} \qquad a_0 = \hbar^2/m_e k_e e^2 = 0.529\text{ Å}$$

Part I—Short Answer

[2 pts] 1. How does the de Broglie wavelength appear in the Schrödinger equation?

[2 pts] 2. What is the effect of a force on a particle's wave function?

[4 pts] 3. Compare the i) time-independent and ii) time-dependent Schrödinger equations:

a) What information is needed to solve each?

b) What must be determined in the solution of each?

[5 pts] 4. a) Express $\psi = 2 + 2i$ in the form $re^{i\theta}$.

b) Evaluate ψ^* in both forms.

c) Evaluate ψ^2.

d) Evaluate $|\psi|^2$.

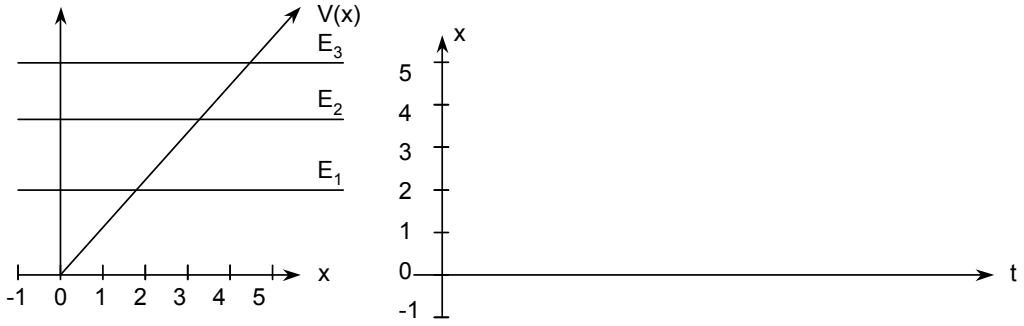
[4 pts] 5. a) Write the formula for A in $\psi(x) = Ax^2e^{-x/a}$. (Don't evaluate any integral!)

b) Write a formula for the expected value of the kinetic energy T of the particle.

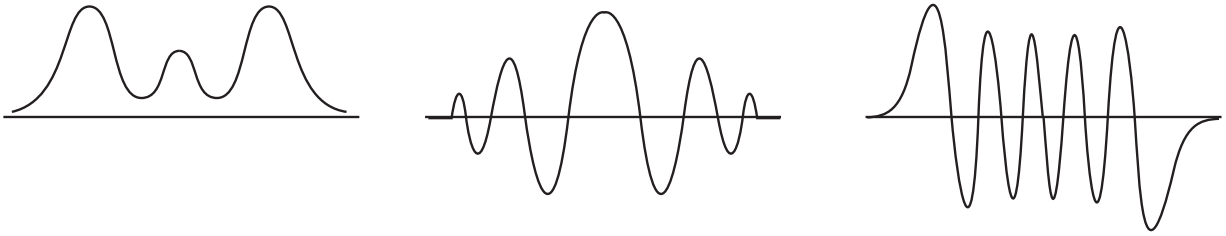
[6 pts] 6. a) Draw the wave functions $\psi_i(x)$ of the following potential.

b) Graph the position vs. time of a classical particle with total energy E_3.

c) Describe the force represented by this potential.



[9 pts] 7. For each of the following three wave functions, sketch the corresponding potential, using the x-axis as the energy level of this state. What is the value of n for each state (starting at $n = 1$)?



[3 pts] 8. What is dispersion of waves and what causes it? Give a physical example.

[5 pts] 9. The phase velocity of surface waves is $\sqrt{kS/\rho}$ where S is the surface tension and ρ the density. What is the group velocity in terms of k, S, and ρ?

[5 pts] 10. What is the decay lifetime of a particle with mass energy $mc^2 = (1232 \pm 120)$ MeV?

Part II—Long Answer

[15 pts] 11. Show that $\psi_1(x) = xe^{-x/b}$ is a solution to the one-dimensional TISE with the potential $V(x) = -k_e e^2/x$ for $x > 0$. Calculate the value of b [nm], and the energy level [eV].

[10 pts] 12. Starting from the time-dependent Schrödinger equation, perform separation of variables to derive the time-independent Schrödinger equation, using $\Psi(x,t) = \psi(x) \cdot \phi(t)$. Obtain separate equations for $\psi(x)$ [TISE] and $\phi(t)$, and solve the equation for the time dependence $\phi(t)$.

[15 pts] 13. a) Show that $\psi(x) = A\sin(k_n x)$ is a solution of the time-independent Schrödinger equation (TISE) for an electron in an infinite square well, with $V(x) = 0$ for $0<x<a_0$ and $V(x) = \infty$ for $x<0$ or $x>a_0$.
b) Show that the boundary conditions are satisfied on both sides of the well, and determine the values of k_n and therefore E_n.
c) Draw the energy levels and wave functions on a graph of the potential.
d) Derive the wavelength of the $n = 2$ to $n = 1$ transition.