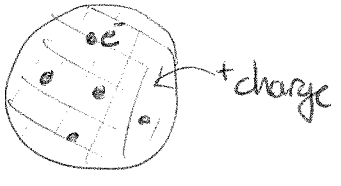
Rutherford's Nuclear Model

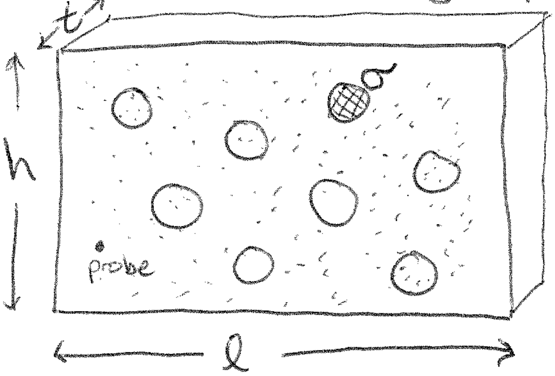
- before we discuss energy levels of atom ⇒ line spectra we need to have a clear picture of the atom.
  - J.J Thompson's "plumb padding" model
    - modes of vibration → atomic spectra? (like a vibrating drumhead). frequencies?
    - NO!



  - Ernest Rutherford - nuclear model.
    - discovered "α", "β" radiation from uranium
      He^{2+} e^- also "γ" = photon @ high energy
    - $\frac{q}{m}$ = half of the proton ⇒ showed $\alpha = He^{2+}$
    - brilliant idea to use 'α' as a probe
      "bread & butter" of nuclear physics.
  - size of atom: $N_A = 6.02 \times 10^{23}/mol$ $\rho \approx 1\ g/cm^3$ $A \geq 1\ g/mol$

$n = \frac{\#}{mol} \cdot \frac{mol}{g} \cdot \frac{g}{cm^3}$
$= N_A \cdot \frac{1}{A} \cdot \rho$
$l \approx 1.2 \times 10^{-10}\ m$
$\approx 1 Å$

- example scattering "particle" experiment.



σ ≡ "cross-sectional area of a single target.
- not physical area, but area of interaction.
- can't be observed directly

- what do we know?
  - density $nt = \frac{\# \cdot t}{l \cdot h \cdot t} = \frac{\#}{A} = \boxed{\qquad cm^{-2}}$
  - beam current $I_o = \frac{\#}{time} = \boxed{\qquad / \quad s}$
  - detector rate $N = \frac{\#}{time} = \boxed{\qquad \pm \qquad}$

- "Monte Carlo"
- counting statistics
  $N \pm \sqrt{N}$

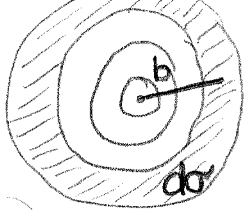
- total (absorbtion) cross-section

$$\sigma = \frac{\text{detector rate}}{\text{luminosity}} = \frac{N}{I_o\, nt}$$

i.e. $\frac{\# \cdot \sigma}{A} = \frac{\#\,hit}{\#\,thrown}$

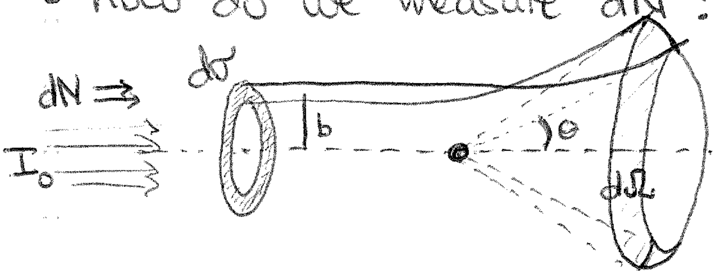
Differential Cross section

- can we do better?
  - yes, we can measure the "Force law" by scattering



$$dN = I_0 \overset{nt}{\frac{\#}{A}} \cdot d\sigma$$

- how do we measure dN?



dσ = differential cross section $\left(\frac{d\sigma}{d\Omega}\right)$

dΩ = solid angle

$= A_{sc}/r^2$

b = impact parameter

θ = scattering angle.

  - what shape of detector do you need?
  - can't "aim" at target, cant measure 'b' directly.

$$\frac{d\sigma}{d\Omega} = \frac{\overset{\text{count rate}}{dN}}{\underbrace{I_0\, nt}_{\text{luminosity}}}\underbrace{\frac{r^2}{A_{sc}}}_{\text{detection solid angle.}}$$

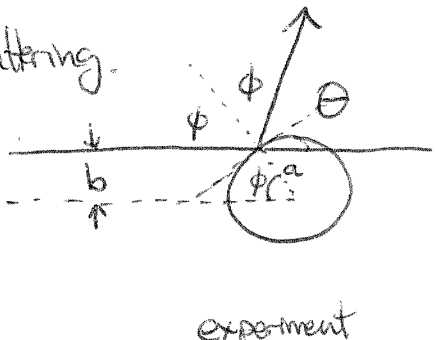
- how does dN or $\frac{d\sigma}{d\Omega}$ relate to the force law?
  - look at trajectory to determine function b(θ).

$$\frac{d\sigma}{d\Omega} = \frac{2\pi b\, db}{2\pi \sin\theta\, d\theta} = \frac{b}{\sin\theta}\frac{db}{d\theta}$$

- example - hard sphere scattering. radius "a"

$2\phi + \theta = 180°$

$b = a \sin\phi$



- Rutherford cross section

$$b = \frac{k_e q_\alpha Q}{m_\alpha v^2} \cot\frac{\theta}{2}$$

$$\overset{\text{"meeting point"}}{\boxed{\frac{d\sigma}{d\Omega} \equiv \overset{\text{experiment}}{\frac{\Delta N}{I_0\, nt}\frac{r^2}{A_{sc}}} = \overset{\text{theory}}{\frac{k_e e^2 \cdot Z}{2E_k}\frac{1}{\sin^4(\frac{\theta}{2})}}}}$$

- verified by Geiger & Marsden