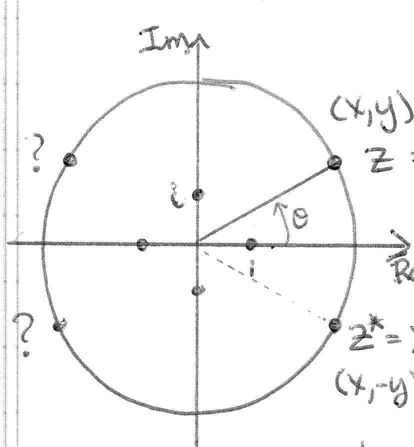
2009-02-06
2010-02-12

Complex Numbers



$(x,y) = (r\cos\theta, r\sin\theta)$

$z = x+iy = re^{i\theta}$

$z^* = x-iy = re^{-i\theta}$

$(x,-y)$

complex conjugation $*: i \leftrightarrow -i$

$$r^2 = x^2+y^2 = z^* z = |z|^2$$

solutions to: $x^2+1=0$
new number $i = \sqrt{-1}$
only new number needed (FTA).

Euler's Theorem

$$e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2!} - i\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + i\frac{\theta^5}{5!} + \dots$$

$$= \cos\theta + i\sin\theta$$

$$\frac{de^{i\theta}}{d\theta} = ie^{i\theta} = -\sin\theta + i\cos\theta$$

$$e^{i(\theta_1+\theta_2)} = e^{i\theta_1} \cdot e^{i\theta_2}$$

$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2} \qquad \sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$$

Plane Wave function

$$\Psi(x,t) = N e^{i(kx-\omega t)} = N e^{i/\hbar (px - Et)}$$

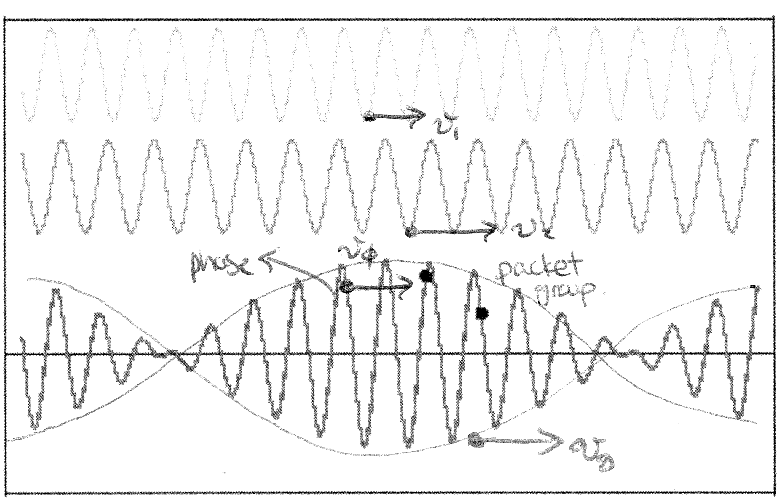
$$\boxed{\begin{matrix} E = \hbar\omega \\ p = \hbar k \end{matrix}}$$

pure energy & momentum.

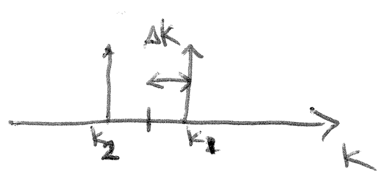
Beating

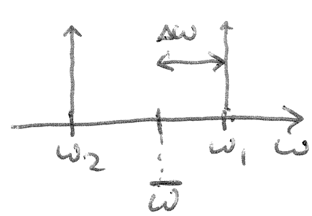
$e^{ik_1x-\omega_1t}$
$+$
$e^{ik_2x-\omega_2t}$

interference.



$$e^{i(k_1x-\omega_1t)} + e^{i(k_2x-\omega_2t)}$$
$$= e^{i\theta_1} + e^{i\theta_2}$$
$$= e^{i\bar\theta+\Delta\theta} + e^{i\bar\theta-\Delta\theta}$$
$$= e^{i\bar\theta}(e^{i\Delta\theta} + e^{-i\Delta\theta})$$
$$= \underbrace{e^{i\bar\theta}}_{\text{phase}} \cdot \underbrace{2\cos(\Delta\theta)}_{\text{group.}}$$





$k_1 = \bar k + \Delta k \qquad \omega_1 = \bar\omega + \Delta\omega$

$k_2 = \bar k - \Delta k \qquad \omega_2 = \bar\omega - \Delta\omega$

$$v_\phi = \frac{\bar\omega}{\bar k} \rightarrow \frac{\omega}{k}$$

$$v_g = \frac{\Delta\omega}{\Delta k} \rightarrow \frac{d\omega}{dk}.$$