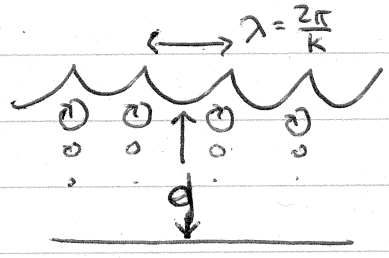
2010-02-19

Water Waves - example of dispersion

Gravity waves

1) $v_\phi = \sqrt{\frac{g\lambda}{2\pi}\tanh\left(\frac{2\pi d}{\lambda}\right)} = \sqrt{\frac{g}{k}\tanh(d\cdot k)}$

$\omega(k) = v_\phi \cdot k = \sqrt{gk \cdot \tanh(d\cdot k)}$



note: gravity waves are independent of the mass of the fluid.

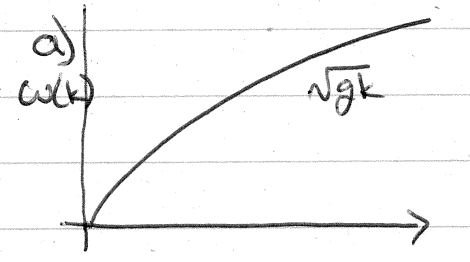
a) deep water: $dk \gg 1$ $\quad \boxed{\omega \approx \sqrt{gk}}$

$v_\phi = \frac{\omega}{k} = \sqrt{\frac{g}{k}} \qquad v_g = \frac{d\omega}{dk} = \frac{1}{2}\sqrt{\frac{g}{k}} = \frac{1}{2}v_\phi$

ex. $d = 1\,km$, $k \approx 1/km$ (longest deep water waves).

$v_\phi = 100\,m/s \qquad v_g = 50\,m/s$ fast!

$\omega = \sqrt{gk} = 1/10\,s$ (tsunami's)



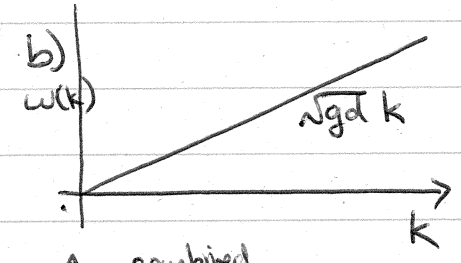
b) shallow water: $dk \ll 1$ $\quad \boxed{\omega \approx \sqrt{gd}\,k}$

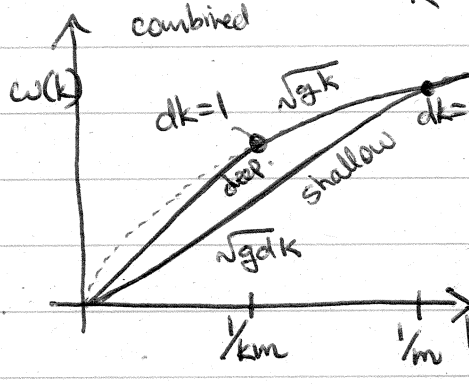
$v_\phi = v_g = \sqrt{gd}$

ex. $d = 1m \Rightarrow v \approx 3m/s$ shore waves.

$d = 1km \Rightarrow v = 100m/s$

wave "scapes" the bottom





2) Surface waves - surface tension

$v_\phi = \sqrt{\frac{2\pi S}{\lambda\rho}} = \sqrt{\frac{kS}{\rho}}$ $\quad S = 73\,mN/m$ surface tension

$\rho = 1\,g/cm^3$ density.

$\boxed{\omega = \sqrt{\frac{k^3 S}{\rho}}}$

$v_g = \frac{3}{2}\sqrt{\frac{kS}{\rho}} = \frac{3}{2}v_\phi$

estimate surface tension by: $F = mg$.

$S \cdot r = \frac{4}{3}\pi r^3 \cdot \rho \cdot g$ for drop $r \approx 1mm$ natural size.

compare $\sqrt{\frac{kS}{\rho}} = v_\phi = \sqrt{\frac{g}{k}}$ at $k \approx 1/mm$.