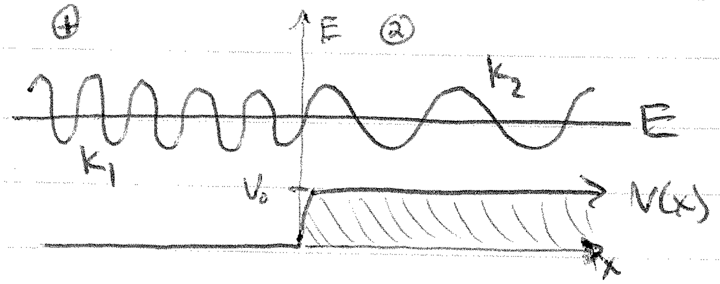
Reflection and Transmission



$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = (E-V)\psi \qquad ①: \ \frac{\hbar^2 k_1^2}{2m} = E \qquad k_1 = \pm\sqrt{2mE}/\hbar$$

$$\psi = e^{ikx} \qquad ②: \ \frac{\hbar^2 k_2^2}{2m} = E - V_0 \qquad k_2 = \pm\sqrt{2m(E-V_0)}/\hbar$$

$$\Psi_1(x,t) = e^{i(\pm k_1 x - \omega t)} \qquad \Psi_2(x,t) = e^{i(\pm k_2 x - \omega t)}$$

$$\psi_1(x) = \underbrace{A e^{ik_1 x}}_{\overrightarrow{\text{incident}}} + \underbrace{B e^{-ik_1 x}}_{\overleftarrow{\text{reflected}}} \qquad \psi_2 = \underbrace{C e^{ik_2 x}}_{\overrightarrow{\text{transmitted}}} + \cancel{D e^{-ik_2 x}} \ \text{comes from nowhere}$$

$$\psi_1(0) = \psi_2(0): \quad A + B = C$$

$$\psi_1'(0) = \psi_2'(0): \quad k_1 A - k_1 B = k_2 C$$

3 unknowns
2 equations
solve for B/A, C/A

transmission

$$\begin{aligned} k_2 A + k_2 B &= k_2 C \\ (-) \quad k_1 A - k_1 B &= k_2 C \\ \hline (k_2 - k_1) A + (k_2 + k_1) B &= 0 \end{aligned}$$

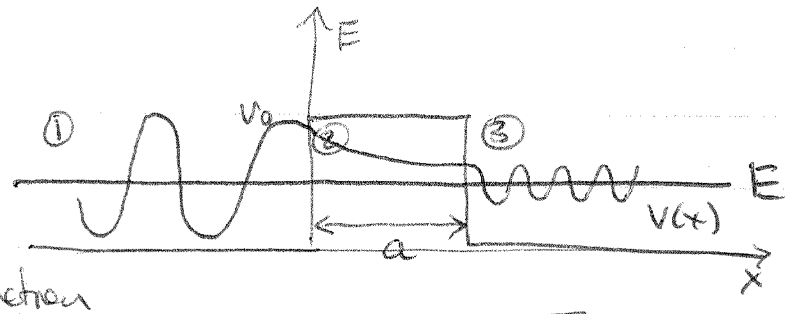
reflection

$$\begin{aligned} k_1 A + k_1 B &= k_1 C \\ (+) \quad k_1 A - k_1 B &= k_2 C \\ \hline 2 k_1 A &= (k_1 + k_2) C \end{aligned}$$

$$R = \left|\frac{B}{A}\right|^2 = \left|\frac{k_1 - k_2}{k_1 + k_2}\right|^2 \qquad T = \underbrace{\frac{k_2}{k_1}}_{v_2/v_1}\left|\frac{C}{A}\right|^2 = \frac{4 k_1 k_2}{|k_1 + k_2|^2}$$

$$R + T = \frac{(k_1 - k_2)^2 + 4 k_1 k_2}{(k_1 + k_2)^2} = \frac{k_1^2 + 2 k_1 k_2 + k_2^2}{(k_1 + k_2)^2} = 1$$

Tunneling



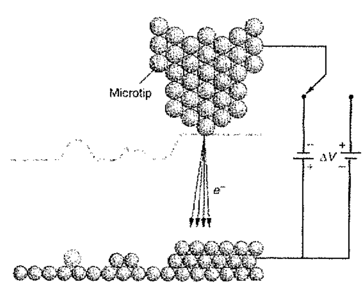
* exponential decay of wave function inside of barrier.
* emerges out the other side

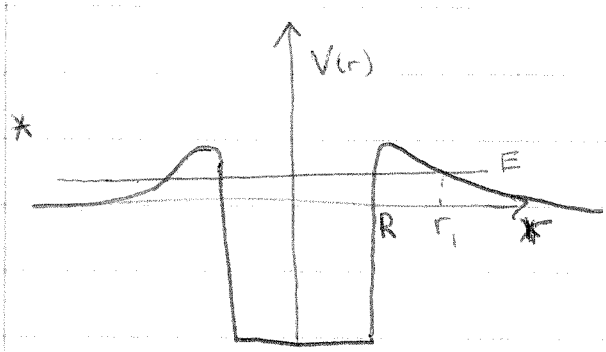
$k_1 = k_3 = \sqrt{2mE}/\hbar$

$ik_2 = \alpha = \sqrt{2m(V-E)}/\hbar$

$$T \approx 16 \frac{E}{V_0}\left(1 - \frac{E}{V_0}\right) \underbrace{e^{-2\alpha a}}_{\text{exponential decay in transmission}}$$

* STM Scanning Tunneling Microscope
  - extremely sensitive to height above sample
  - atomic scale resolution





* Alpha decay

- Coulomb repulsion
- nuclear attraction short range.
- decays had less energy then expected.