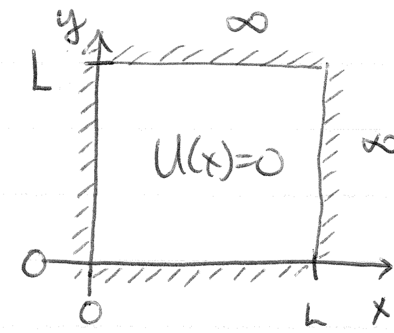
2-D Schrödinger Equation: Separation of Variables

$$\frac{d^2}{dx^2}\Psi(x) \rightarrow \nabla^2\Psi(x,y) = \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Psi(x,y) \qquad \text{ie } p^2 \Rightarrow p_x^2+p_y^2+p_z^2$$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Psi(x,y) = (E-U(x))\Psi(x,y)$$



*eg. Infinite Square Well : $U(x) = \begin{cases} 0 & \text{inside well} \\ \infty & \text{outside} \end{cases}$

let $p^2 = \hbar^2 k^2 = 2mE \qquad \Psi(x,y) = \Psi_x(x)\cdot\Psi_y(y)$

$$\frac{-\frac{\partial^2}{\partial x^2}\Psi_x(x)\cdot\cancel{\Psi_y(y)}}{\underbrace{\Psi_x(x)\cdot\cancel{\Psi_y(y)}}_{k_x^2}} + \frac{-\frac{\partial^2}{\partial y^2}\cancel{\Psi_x(x)}\Psi_y(y)}{\underbrace{\cancel{\Psi_x(x)}\cdot\Psi_y(y)}_{k_y^2}} = \frac{k^2\cancel{\Psi_x(x)}\cancel{\Psi_y(y)}}{\underbrace{\cancel{\Psi_x(x)}\cancel{\Psi_y(y)}}_{k^2}}$$

$$\frac{\partial^2}{\partial x^2}\Psi_x(x) + k_x^2\Psi_x(x) = 0 \qquad \frac{\partial^2}{\partial y^2}\Psi_y(y) + k_y^2\Psi_y(y) = 0$$

$\Psi_x = A_x \sin(\alpha x)$ or $B_x\cos(\beta x)$

$-\alpha^2 + k_x^2 = 0 \rightarrow \alpha=\beta=k_x$

$\int_0^L |\Psi_x|^2 dx = \int_0^L A_x^2 \underbrace{\sin^2(k_x x)}_{\sim 1/2} dx = 1$

$A_x = \sqrt{\frac{2}{L}}$

$\Psi_x(0)=0: \ \cancel{\cos(\beta x)} \qquad \Psi_x(L)=0: \ \sin(k_x L)=0$

$\Psi_x = \sqrt{\frac{2}{L}}\sin(k_x x) \qquad \boxed{k_x L = n_x\pi}$

$\Psi_y = A_y\sin(\alpha y)$

$A_y = \sqrt{\frac{2}{L}} \quad \alpha = k_y$

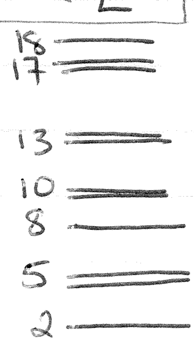
$\Rightarrow \boxed{k_y L = n_y \pi}$

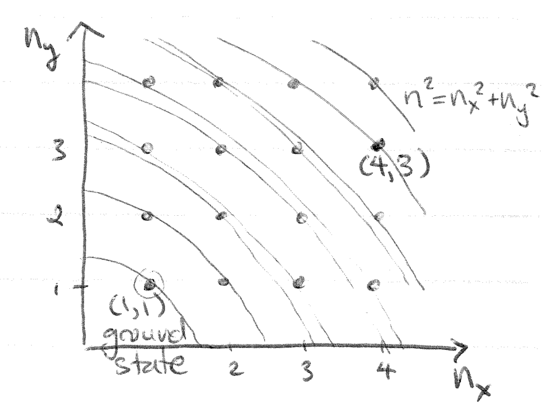
2 independent quantum numbers: (n_x, n_y)

$\Psi_y = \sqrt{\frac{2}{L}}\sin(k_y y)$

$$\boxed{\Psi_{n_x n_y}(x,y) = \frac{2}{L}\sin\left(\frac{n_x\pi x}{L}\right)\cdot\sin\left(\frac{n_y\pi y}{L}\right)}$$

$$\boxed{E_{n_x n_y} = \frac{\hbar^2\pi^2}{2mL^2}(n_x^2+n_y^2)}$$





* which energy states are degenerate ?

* how does this generalize to 3-D ?

* what the node lines for each state ?