Wave Mechanics – DUALITY!

WAVES – collective

$\mathcal{E}(\vec{x},t)$ wave function

$\frac{\partial^2}{\partial x^2}\mathcal{E} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathcal{E}$ wave equation

CLASSICAL

* EVERYTHING propagates as a wave!
- interference & dispersion.

* §5-3 dispersion – "mass"
- $v(\lambda)$ or $\omega(k)$ or $E(p)$
- e.g. $c = \lambda f$ or $E = p^2/2m$
- phase velocity "pure wave" $v_p = \omega/k$
- group velocity "wave packet" $v_g = d\omega/dk$
- example: beating

* §6-1 Schrödinger Equation
- time dependent S.E.

$$\hat{H}\Psi(x,t) = \hat{E}\Psi(x,t)$$

$$\boxed{\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi + V(x)\cdot\Psi = i\hbar\frac{\partial}{\partial t}\Psi}$$

- time-independent S.E.

seperation of variables: $\Psi(x,t) = \psi(x)\phi(t)$

$$\boxed{\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V(x)\psi = E\psi} \qquad = \psi(x)e^{-i\omega t}$$

- boundary conditions
  - $\psi(x), \psi'(x)$ continuous, finite, single valued
  - yield quantized energy eigenvalues (states) for bound potentials.
- strategy: §6-3,4
  - start with $E = T + V$
  - solve TISE (wiggles & boundary) for $\psi(x)$
  - calculate probabilities, expectations.

---

§3 Planck-Einstein quantization — $E = \hbar\omega$

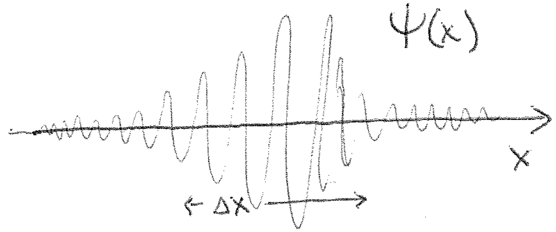
§5-1,2 deBroglie waves — $p = \hbar k$

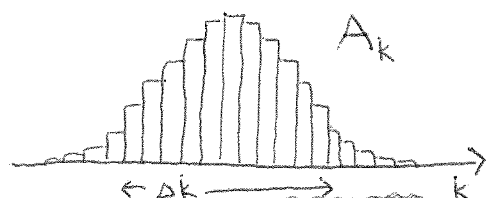
§4 Bohr atom — $L_z = \hbar m$

§5-7 Quantum Wave function $\Psi(x,t)$

§5-3 Wave packets

wave-particle duality





position-momentum duality

§5-5,6 * Heisenberg Uncertainty Principle

$\Delta\omega\,\Delta t \geq \frac{1}{2}$	$\Delta E\,\Delta t \geq \hbar/2$
$\Delta k\,\Delta x \geq \frac{1}{2}$	$\Delta p\,\Delta x \geq \hbar/2$
$\Delta m\,\Delta\phi \geq \frac{1}{2}$	$\Delta L_z\,\Delta\phi \geq \hbar/2$

* applications
- particle in a box
- spectral line widths
- size of H-atom

PARTICLES – individual

$\vec{x}(t)$ trajectory (kinematics)

$\vec{F} = m\vec{a}$ dynamics

$p = mv$ $T = \frac{1}{2}mv^2$ $V = -\int F\,dx$ conservation principles

CLASSICAL

* EVERYTHING interacts as a particle!
- localization & detection.

* §5-4 Born (Copenhagen) Interpretation
- probability amplitude $\psi(x)$
- probability density $|\psi|^2$
- probability normalization $\int_{-\infty}^{\infty}|\psi|^2 dx = 1$

* §6-4 Expectation Values
- OPERATORS "act on functions"

$$\underbrace{-i\hbar\frac{\partial}{\partial x}}_{p_{op}}\underbrace{e^{ikx}}_{\psi} = \hbar k\cdot\underbrace{e^{ikx}}_{\psi}$$

$$p_{op}\,\psi = p\,\psi$$

quantization $\hbar$

particle	wave	i operator-wavefn		conjugate co-ordinate
E	ω	$i\hbar\frac{\partial}{\partial t}$	$e^{-i\omega t}$	t
p	k	$-i\hbar\frac{\partial}{\partial x}$	e^{ikx}	x
L_z	m	$-i\hbar\frac{\partial}{\partial\phi}$	$e^{im\phi}$	ϕ

dispersion "mass"

separation of variables

- all other operators can be constructed from $\hat{E}, \hat{t}, \hat{p}, \hat{x}, \hat{L}_z, \hat{\phi}$ by substitution

$$\hat{H} = \hat{T} + \hat{V} \qquad \hat{H}\psi = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\cdot\psi$$

$$\hat{T} = \frac{\hat{p}^2}{2m} = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$$

$$\hat{V} = V(x)$$

Key Concepts for Exam II

2010-03-10

* wave-particle duality

$\psi(x)$ = prob. amplitude.

det. as particle (interact), propagate as wave (interfere)

* complex numbers

$z = x + iy = re^{i\theta}$ $|z|^2 = z^* z$

pure waves $N e^{ikx - \omega t}$

$f(x - vt)$ → transforms $\left(\frac{y - y_0}{\Delta y}\right) = f\left(\frac{x - x_0}{\Delta x}\right)$

* wave packets (frequency components)

standing waves, beats,

wave packets

Heisenberg Uncertainty.

* dispersion relations.

$E(p)$ $\omega(k)$

v_ϕ, v_g.

matter ~~E=~~ $T = \frac{p^2}{2m}$ $\omega = ck$ (light)

* functions and operators

conjugate variables.

eigenvalues

expected value.

* potentials - classical & quantum

± T conserved E (not p!) forces real imaginary zombie

turning pts, tunnelling, $e^{\pm ikx}$ $e^{\pm \kappa x}$ oscillating or decaying

* boundary value problems ψ, ψ' match.

2nd order ODE 2 B.C.'s. 2 const. of integration.

or separate boundaries → E spectrum, normalization condition

* separation of variables

TDSE → TISE x, y. square.