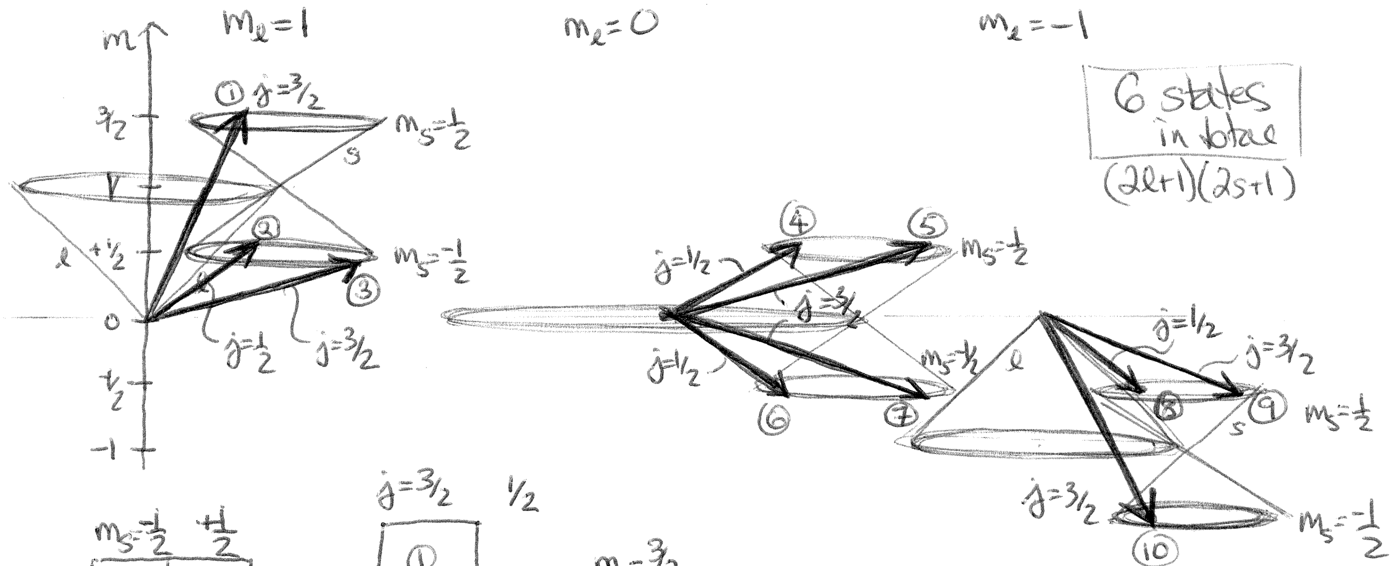
Addition of Angular Momentum (spin-orbit coupling)

$\ell = 1$

$s = \frac{1}{2}$



$\boxed{\text{6 states in total}}$

$(2\ell+1)(2s+1)$

$\ell = 1$

$s = \frac{1}{2}$

	$m_s = -\frac{1}{2}$	$+\frac{1}{2}$
$m_\ell = 1$	②③	①
0	⑥⑦	④⑤
−1	⑩	⑧⑨

	$j = 3/2$	$1/2$
$m = 3/2$	①	
$1/2$	③⑤	②④
$-1/2$	⑦⑨	⑥⑧
$-3/2$	⑩	

$$\boxed{\begin{aligned} j &= |\ell - s|, \ldots \ell + s \\ m_j &= -j, \ldots j-1, j \end{aligned}}$$

triangle inequality

ℓ, s notation:

$p_x^\uparrow \; p_x^\downarrow \; p_y^\uparrow \; p_y^\downarrow \; p_z^\uparrow \; p_z^\downarrow$

Spectroscopic notation: $\;{}^{(2s+1)}L_J$

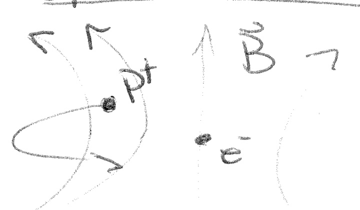
${}^2P_{3/2} \quad {}^2P_{1/2}$

$g_s = 2s+1$

$L = S, P, D, F, G, H, I \ldots$

$J = |L - S|, \ldots L+S$

spin-orbit interaction



$\vec{B} \propto \vec{L}$ due to spinning proton in electron frame.

$$U = -\vec{\mu}_s \cdot \vec{B} = \frac{e}{m_e} S \cdot B$$

$$\propto S \cdot L$$

$$J^2 = (L+S)^2 = L^2 + 2L\cdot S + S^2$$

$$L \cdot S = \frac{1}{2}\left(J^2 - (L^2 + S^2)\right)$$

$$= \frac{\hbar^2}{2}\left(j(j+1) - (\ell(\ell+1) + s(s+1))\right)$$

example: $\ell = 1 \quad s = 1/2$

$\ell(\ell+1) + s(s+1) = 1\cdot 2 + \frac{1}{2}\cdot\frac{3}{2} = 2\,3/4$

$j = 3/2 \quad j(j+1) = \frac{3}{2}\cdot\frac{5}{2} = 3\,3/4$	$j = \frac{1}{2} \quad j(j+1) = 3/4$
$L \cdot S = \frac{1}{2}\hbar^2$	$L \cdot S = -\hbar^2$