# Chapter 9 - Statistical Mechanics

## Multiparticles — separation of variables (all over again!)

schrödinger equation: $\hat{H}\psi = \frac{-\hbar^2}{2m}\nabla_1^2\psi + \frac{-\hbar^2}{2m}\nabla_2^2\psi + \ldots + V_1(\vec{x}_1)\psi + V_2(\vec{x}_2)\psi \ldots = E\psi$

wave function: $\psi(\vec{x}_1, \vec{x}_2, \ldots) = \psi_a(\vec{x}_1)\cdot\psi_b(\vec{x}_2)\cdot \ldots$

independent particle approximation (mean field theory)

each particle has the same separated eq: $H_1\psi_a = \frac{-\hbar^2}{2m}\nabla_1^2\psi_a(\vec{x}_1) + V_1(\vec{x}_1) = E_1\psi_a(\vec{x}_1)$

$a, b, c$ = quantum numbers for individual electron states.

## Symmetry — what if the particles look all the same?

$P(\vec{x}_1, \vec{x}_2) = P(\vec{x}_2, \vec{x}_1)$ $\qquad |\psi(1,2)|^2 = |\psi(2,1)|^2$

$P(\vec{x}_1, \vec{x}_2) = P(1,2) = |\psi(1,2)|^2$ $\qquad \psi(1,2) = \pm\psi(2,1)$

$$\psi_{ab}(1,2) = \frac{1}{\sqrt{2}}\left(\psi_a(1)\psi_b(2) \pm \psi_a(2)\psi_b(1)\right)$$

$\psi_{ab}^S = \frac{1}{\sqrt{2}}(\psi_a\psi_b + \psi_b\psi_a)$ symmetric, $\boxed{\text{bosons}}$ $S = 0, 1, 2, \ldots$

eg. photons, hydrogen atom, 4He, Cooper pairs.

$\psi_{ab}^A = \frac{1}{\sqrt{2}}(\psi_a\psi_b - \psi_b\psi_a)$ antisymmetric, $\boxed{\text{fermions}}$ $S = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \ldots$

eg. electrons, protons, neutrons

$$P(1,2) = P(2,1) = \psi^*\psi = \frac{1}{2}\left(\psi_a^*(1)\psi_b^*(2) \pm \psi_a^*(2)\psi_b^*(1)\right)\left(\psi_a(1)\psi_b(2) \pm \psi_a(2)\psi_b(1)\right)$$

$$= \frac{1}{2}\begin{pmatrix} |\psi_a(1)|^2|\psi_b(2)|^2 & \pm\ \psi_a^*(1)\psi_b(1)\cdot\psi_b^*(2)\psi_a(2) \\ +|\psi_a(2)|^2|\psi_b(1)|^2 & \pm\ \underbrace{\psi_b^*(1)\psi_a(1)}\cdot\underbrace{\psi_a^*(2)\psi_b(2)} \end{pmatrix}$$

these are orthogonal and integrate to 0. if $a \neq b$.

if $a \neq b$ $\quad P = \frac{1}{2} + \frac{1}{2} = 1$

if $a = b$ $\quad P = |\psi_a(1)|^2\cdot|\psi_a(2)|^2 \pm |\psi_a(1)|^2|\psi_a(2)|^2$ $\qquad \iint_{x_1x_2} P\,d\vec{x}_1 d\vec{x}_2 = 1 \pm 1$

fermions: $\psi^A$ total probability $= 1 - 1 = 0$ $\boxed{\text{Pauli Exclusion principle}}$ each electron has unique quantum numbers

bosons: $\psi^S$ $\qquad \iint P\,d\vec{x}_1 d\vec{x}_2 = 1 + 1 = 2$ twice as probable as separate states

"bosons flock together"

# Periodic Table

* spectroscopic notation

$n^{2s+1}L_J \qquad L = S, P, D, F$
$\qquad\qquad\qquad \ell = 0, 1, 2, 3$

* quantum numbers

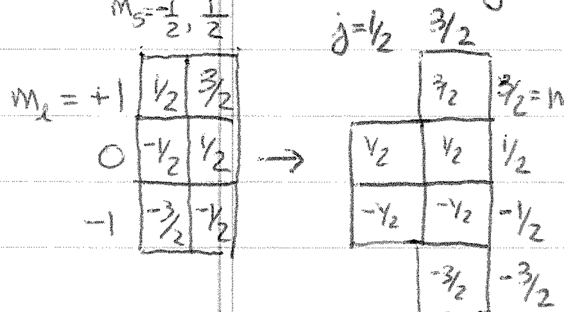
$n = 1, 2, 3, \ldots$

$\ell = 0, 1, \ldots n-1$

$\begin{cases} m_\ell = -\ell, \ldots \ell-1, \ell \\ m_s = -\frac{1}{2}, +\frac{1}{2} \end{cases}$

$\begin{cases} j = \ell-\frac{1}{2}, \ell+\frac{1}{2} \\ m = -j, \ldots j-1, j \end{cases}$

* L·S coupling




$U = -\mu \cdot B \approx \gamma S \cdot \beta L$

$\approx \alpha L \cdot S \approx \frac{1}{2}(J^2 - S^2 - L^2)$

* degeneracy:

$g_s = 2(\frac{1}{2}) + 1 = 2$

$g_\ell = 2\ell + 1$

$g_j = 2j + 1$

so $g_s \cdot g_\ell = g_{\ell-\frac{1}{2}} + g_{\ell+\frac{1}{2}}$

$2(2\ell+1) = 2(\ell-\frac{1}{2})+1 + 2(\ell+\frac{1}{2})+1$