§9.1-3 Statistical Distributions

$$\boxed{n(\varepsilon) = g(\varepsilon)\cdot f(\varepsilon)}$$

- $n(\varepsilon)$: # of particles (observable)
- $g(\varepsilon)$: # of states (density, degeneracy)
- $f(\varepsilon)$: # particles / state (probability, occupancy, distribution)

example: velocity distribution of ideal gas.

$$g(v)dv = 4\pi v^2 dv \qquad f(v) = e^{-\frac{1}{2}mv^2/kT}$$

$$Z = \int_0^\infty 4\pi v^2 dv \cdot e^{-\beta\cdot\frac{1}{2}mv^2} = \left(\frac{2\pi}{m\beta}\right)^{3/2}$$

$$\langle\varepsilon\rangle = \frac{-\partial Z/\partial\beta}{Z} = \frac{+\frac{3}{2}\left(\frac{2\pi}{m}\right)^{3/2}\beta^{-5/2}}{\left(\frac{2\pi}{m}\right)^{3/2}} = \frac{3}{2\beta}$$

$$= \boxed{\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}kT}$$

see notes, 2009-01-16.

example II: atmospheric pressure.

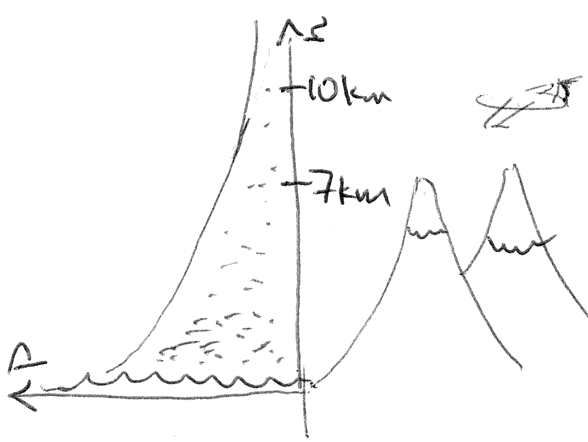
$$PV = NkT \quad \text{or} \quad P = nkT \qquad n = N/V$$

$$dF = dP\cdot A = dN\cdot mg$$

$$dn\,kT = -n\,mg\,dh$$

$$\ln(n) - \ln(n_0) = \frac{-mgh}{kT}$$

$$n = n_0 e^{-\frac{mgh}{kT}} = n_0 e^{-\varepsilon/kT}\ !$$



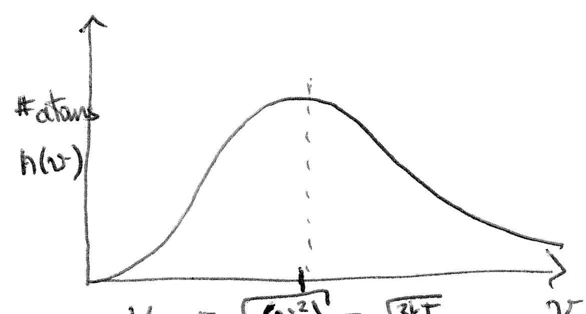
Maxwell-Boltzman distribution

$$\boxed{f_{MB}(\varepsilon) = Ae^{-\varepsilon/kT}} = e^{-(\varepsilon-\mu)/kT} = e^{-(\alpha+\beta\varepsilon)}$$

$$N = \int g(\varepsilon)d\varepsilon\cdot f_{MB}(\varepsilon) = A\underbrace{\int g(\varepsilon)e^{-\beta\varepsilon}d\varepsilon}_{Z(\beta)}$$

$A = N/Z$ "partition fn"

$$\frac{-\frac{\partial Z}{\partial\beta} = \int g(\varepsilon)\cdot\varepsilon e^{-\beta\varepsilon}d\varepsilon}{Z = \int g(\varepsilon)e^{-\beta\varepsilon}d\varepsilon} = \langle\varepsilon\rangle$$



$$v_{rms} = \sqrt{\langle v^2\rangle} = \sqrt{\frac{3kT}{m}}$$

$$Z = \int_0^\infty e^{-\beta mgh}dh = \frac{1}{\beta mg}\int_0^\infty e^{-x}dx = \frac{1}{\beta mg}$$

$$\langle\varepsilon\rangle = \frac{-\partial Z/\partial\beta}{Z} = \frac{+\frac{1}{\beta^2 mg}}{\frac{1}{\beta mg}} = \frac{1}{\beta} = kT$$

$$\langle h\rangle = \frac{\langle\varepsilon\rangle}{mg} = \frac{kT}{mg} = \frac{25\text{ meV}}{28\cdot 1\text{GeV}/c^2\cdot 9.8\text{ m/s}^2}$$

$$\approx \frac{10^{-3}\,(3\times10^8\text{m/s})^2}{10^9\cdot 3^2\text{ m/s}^2} = \boxed{10\text{km}}$$

$$n_0 = \frac{N}{AZ} = \frac{Nmg}{AkT}$$

$$P_0 = n_0 kT = \frac{N}{A}mg.$$

§9.4-6 Quantum Statistics — derivation à là Blackbody radiation

$$u(\nu)\,d\nu = \underbrace{G(\nu)\,d\nu}\cdot\bar{\varepsilon}(\nu)$$

$$G(\nu)\,d\nu = \frac{dN}{V} = \frac{2\cdot 4\pi\nu^2 d\nu}{c^3}$$

§9.5 Rayleigh-Jeans Formula.

Classical Maxwell-Boltzman dist

$$f_{MB}(\varepsilon) = e^{-\varepsilon/kT}$$ ε - continuously varies $0 \to \infty$

$$Z = \int_0^\infty e^{-\beta\varepsilon}d\varepsilon = \frac{1}{\beta}$$

$$\bar{\varepsilon} = \frac{-\partial Z/\partial\beta}{Z} = \frac{+\frac{1}{\beta^2}}{\frac{1}{\beta}} = \frac{1}{\beta} = kT$$

$$u(\nu) = \frac{8\pi\nu^2}{c^3}kT \quad \text{wrong!}$$

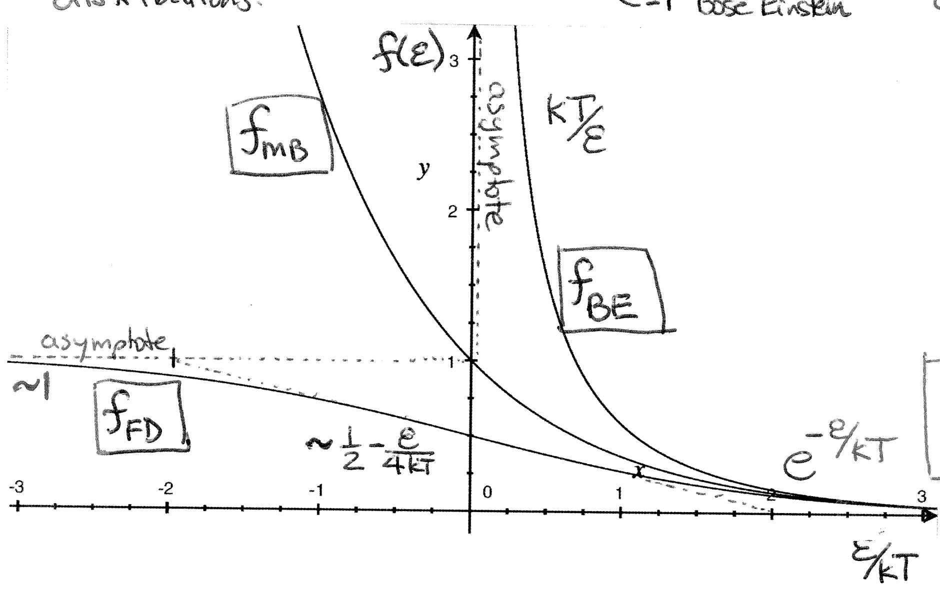
Quantum criterion

$k_n = \frac{\pi n}{L}$ $dp\cdot dx = h\cdot dN$

particle in a box.   or $\boxed{d^3p\cdot d^3x = h^3\,dN}$ $\frac{N}{V} = \frac{d^3p}{h^3}$

summary of all 3 distributions:

$$f(\varepsilon) = \frac{1}{e^{\alpha+\beta\varepsilon} + \begin{cases} +1 & \text{Fermi-dirac} \\ 0 & \text{Maxwell-Boltzman} \\ -1 & \text{Bose Einstein}\end{cases}}$$



§9.6 Planck Radiation Law

Bose-Einstein distribution

$\boxed{\text{photon gas}}$ a) quantization of light
b) B-E statistics.

$\varepsilon_n = nh\nu$ $n = 0,1,2\ldots$ "n-photon state."

$$Z = \sum_{n=0}^\infty e^{-\beta\varepsilon_n} = \sum_{n=0}^\infty\left(e^{-\beta h\nu}\right)^n = \frac{1}{1-e^{-\beta h\nu}}$$

$$\bar{\varepsilon} = \frac{-\partial Z/\partial\beta}{Z} = \frac{+e^{-\beta h\nu}\cdot h\nu}{(1-e^{-\beta h\nu})^2}\Big/\frac{1}{(1-e^{-\beta h\nu})}$$

$$= \frac{e^{-\beta h\nu}\,h\nu}{e^{\beta h\nu}-1} = \bar{n}\cdot h\nu$$

$$u(\nu) = \frac{8\pi\nu^2}{c^3}\cdot\frac{h\nu}{e^{h\nu/kT}-1}$$

$$\boxed{\bar{n} \equiv f_{BE}(\varepsilon) = \frac{1}{e^{\alpha}\cdot e^{\varepsilon/kT}-1}}$$ BOSE-EINSTEIN DISTRIBUTION

spin $S = 0,1,2,3\ldots$

Fermi-Dirac distribution

$\boxed{\text{electron gas}}$ Pauli exclusion principle - 1 particle/state

$$\varepsilon_n = \begin{cases} 0 & n=0 \\ \varepsilon & n=1\end{cases}$$

$$Z = \sum_{n=0}^{1} e^{-\beta\varepsilon_n} = 1 + e^{-\beta\varepsilon}$$

$$\bar{\varepsilon} = \frac{-\partial Z/\partial\beta}{Z} = \frac{+e^{-\beta\varepsilon}\cdot\varepsilon}{1+e^{-\beta\varepsilon}} = \frac{\varepsilon}{e^{\beta\varepsilon}+1}$$

$$= \bar{n}\cdot\varepsilon$$

$$\boxed{\bar{n} \equiv f_{FD}(\varepsilon) = \frac{1}{e^{(\varepsilon-\varepsilon_F)/kT}+1}}$$

spin $S = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$