University of Kentucky, Physics 361
Problem Set #1, due Friday, Jan 23

1. The normal distribution (Gaussian distribution or bell curve) has the form

$$f(x) = Ce^{-\frac{1}{2}\left(\frac{(x-\mu)}{\sigma}\right)^2}$$

a) Calculate the normalization factor C by requiring the distribution to be normalized

$$\int_{x=-\infty}^{\infty} f(x)dx = 1.$$

b) Calculate $\langle x \rangle$, the expected value of x, defined by

$$\langle u \rangle \equiv \frac{\int_{-\infty}^{\infty} u\, f(x)dx}{\int_{-\infty}^{\infty} f(x)dx}.$$

What physical interpretation does it have?

c) Calculate $\langle x^2 \rangle$. Why is this different than $\langle x \rangle^2$? Express σ in terms of $\langle x \rangle$ and $\langle x^2 \rangle$. What physical interpretation does it have? Show that

$$\langle (x - \langle x \rangle)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

d) Given $\mu = 3$ and $\sigma = 2$, calculate the probability that $x < 0$. Note: there is no analytic formula for the result, so you will have to calculate the integral numerically or look it up in a table.

2. Calculate $v_{RMS} \equiv \sqrt{\langle v^2 \rangle}$ for H_2 molecules at $T = 300$ K. Using potential energy, show that the escape velocity of the earth's gravitational field is $v_{escape} = \sqrt{2GM/R}$. Compare this value with v_{RMS}. The earth's atmosphere contains very little H_2. How is it possible that the H_2 the molecules escape with a relatively small v_{RMS}? Why doesn't N_2 escape? [Tipler & Llewellyn, p 8]

3. Compare v_{RMS} and the average kinetic energy per atom/molecule for H_2, He, O_2, and N_2 gas under standard temperature and pressure.

4. Show that the following blackbody distributions written in terms of wavelength or frequency are equivalent:

$$\tilde{u}(f)df = \frac{8\pi f^2}{c^3}\frac{hf}{e^{hf/kT}-1}|df| \qquad \text{equals} \qquad u(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}|d\lambda|$$

5. Integrate Planck's law over all wavelengths to derive Steffan's law: that the total power radiated by a black body is

$$R = \sigma T^4, \quad \text{where} \quad \sigma = \frac{2\pi^5 k^4}{15 h^3 c^2}, \qquad \text{using} \quad \int_0^\infty \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15}.$$

6. Set the derivative of Planck's law equal to zero to show Wien's displacement law: that

$$\lambda_m T = \text{const} = 2.989 \times 10^{-3} \text{ m} \cdot \text{K},$$

where λ_m is the wavelength of maximum intensity radiation from a blackbody. The Sun's surface temperature is 5800 K. Calculate λ_m. What color is the sun?