University of Kentucky, Physics 361
Problem Set #2, due Monday, Feb 1

1. Comparison of Rayleigh-Jeans and Planck formulas for the black body spectrum.

a) Draw the node lines for the (n_x,n_y) = (1,1); (1,2); (2,1); and (2,2) standing wave modes on a square medium with sides of length L. Show that $n_x \lambda_x = 2L$. Show that the density of modes is $G(f) = 8\pi f^2/c^3$. It is defined as the number of modes per volume per frequency interval, i.e. $G(f)df \equiv d^3n/V$. Remember that $n_{x,y,z} > 0$ and there are two independent polarizations of light.

b) The probability of a mode having energy ϵ is proportional to $e^{-\epsilon/kT}$, the Boltzman distribution. Let $\beta = 1/kT$, and integrate the total probability $Z = \int_0^\infty e^{-\beta\epsilon} d\epsilon$ to obtain the normalization factor. $Z(\beta)$ is also called the partition function. Show that $\langle\epsilon\rangle = -d\ln Z/d\beta = kT$. Show that this leads to the Rayleigh-Jeans formula for the spectral intensity of black body radiation.

c) Assume the energy is not continuous, but quantized to discrete levels $\epsilon = nhf$ for n=0,1,2, ... Repeat the calculation of the normalization factor $Z = \sum_{n=0}^\infty e^{-\beta\epsilon}$. Calculate $\langle\epsilon\rangle$. Show that this leads to Planck's formula.

d) Show that $\langle\epsilon\rangle \approx kT$ in the limit $hf \ll kT$. Show that $\langle\epsilon\rangle \approx 0$ in the limit $hf \gg kT$, thus solving the ultraviolet catastrophe. Qualitatively, this is because the temperature is too low to excite even one photon ($n = 1$).

2. Some of the fundamental constants in SI units are:

$h = 6.62607 \times 10^{-34}$ J·s
$c = 299792458$ m/s
$e = 1.60218 \times 10^{-19}$ C
$k_e = 8.98755 \times 10^9$ N·m^2/C^2 $= \frac{1}{4\pi\epsilon_0}$
$k_B = 1.38065 \times 10^{-23}$ J/K
$m_e = 9.10938 \times 10^{-31}$ kg
$m_p = 1.67262 \times 10^{-27}$ kg
$m_n = 1.67493 \times 10^{-27}$ kg

Note that 1 eV = $e \cdot 1$ V is a compound unit of energy. Calculate the following useful combinations of constants in the units specified: hc [eV·nm], $\hbar c = \frac{h}{2\pi}c$ [MeV·fm], $k_e e^2$ [eV·nm], $\alpha = k_e e^2/\hbar c$ [1], kT [meV] at room temperature $T = 20°$C= 293.15 K, and m_e, m_p, m_n [MeV/c^2]. We will use these combinations in natural units repeatedly throughout the rest of the semester.

Also: Tipler Chapter 3: #24, 26, 31, 40, 47, 49.