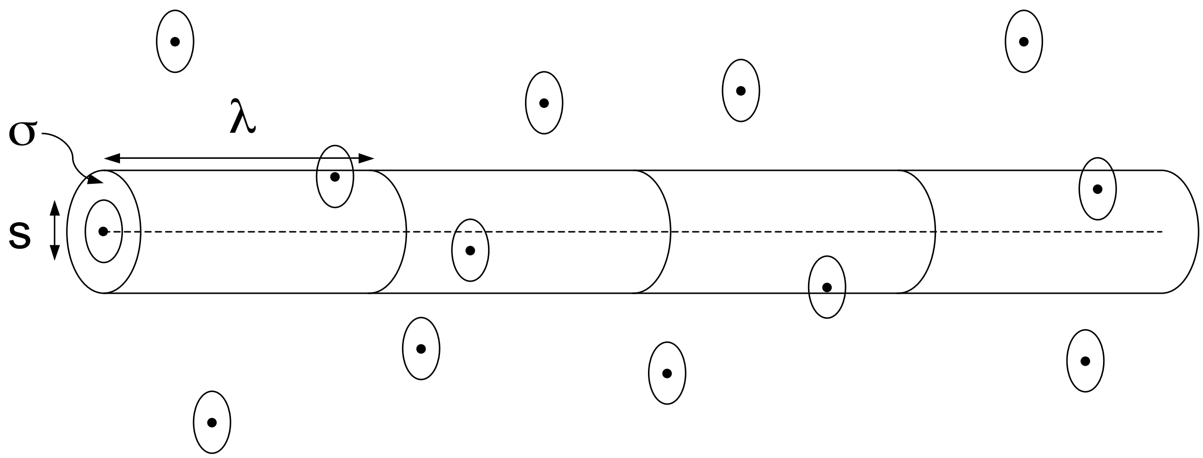
University of Kentucky, Physics 361
Problem Set #3, due Friday, Feb 5

1. [30 pts] In 1866 when atoms and molecules were still quite hypothetical, Joseph Loschmidt used kinetic theory to get the first reasonable estimate of molecular size. He used the liquid to gas expansion ratio of air $\epsilon = n_g/n_l$ (where $n = N/V$) together with the mean free path λ between collisions to calculate the average diameter s of air molecules. In the following diagram, σ is the cross sectional area of interaction between two air molecules (cross section). The purpose of this problem is to reproduce his results and show the profound consequences in atomic physics.



a) Justify that the number density of air [molecules/cm^3] is $n_g = 1/\lambda\sigma$.

b) Justify that $\sigma = \pi s^2$, not, for example, $\pi(s/2)^2$.

c) Show that $\epsilon = s/6\lambda$, assuming that the molecules are tightly packed in a liquid.

d) Given the density of liquid air ρ_l=0.870 g/cm^3, the molar volume of an ideal gas $V_m = 24$ L/mol, the atomic weight of air $A = 35.6$ g/mol, calculate the expansion ratio ϵ from liquid to air at STP (1 atm, 20°C). Loschmidt had to estimate ρ_l, since air had not been liquefied by 1866.

e) Maxwell determined $\lambda = 62$ nm from viscosity measurements of air. A later extraction $\lambda =$ 140 nm by Oskar Emil Meyer (don't laugh!) was less accurate. Use the first value to calculate s and σ.

f) Using σ, calculate the density of air n_g [cm^{-3}] at STP (the Loschmidt number or 1 Amagat).

g) Using n_g and V_m calculate Avogadro's number N_A (also called the Loschmidt number).

h) Faraday's constant $F = N_A e = 96485$ C/mol was determined by weighing the amount of silver deposited during electrolysis of silver ions. Use this to calculate the elementary charge e.

i) Using the cathode ray tube, Thompson discovered electrons, negatively charged particles with charge-to-mass ratio $e/m = 1.75 \times 10^8$ C/g. Calculate the mass of an electron.

j) Using the atomic weight $A = 1.008$ g/mol of hydrogen, calculate the mass of a proton.

2. [20 pts] Consider the scattering of hard spheres as discussed in class. It is assumed the target is infinitely massive so it does not recoil, and the the probe scatters by reflection, ie. the angle of reflection equals the angle of incidence. The radius of the probe and target are a_b and a_t respectively.

a) Determine the relation between the impact parameter b and the scattering angle θ.

b) Using part (a), calculate the cross section $d\sigma/d\Omega$. Plot it as a function of θ.

c) Integrate part (b) to calculate the total cross section

$$\sigma_{tot} = \int d\sigma = \int_{4\pi} \frac{d\sigma}{d\Omega} d\Omega = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{d\sigma}{d\Omega} \sin\theta d\theta d\phi$$

What is an intuitive interpretation of the result?

d) In a model of alpha particle scattering from gold nuclei, let $a_b = r_\alpha = 1.7$ fm and $a_t = r_{\mathrm{Au}} =$ 7.3 fm. What value of b leads to scattering at $\theta = 90°$? 10×10 cm^2 detectors are placed 2 m away from the target, which is a 0.00004 cm thick gold foil, Z=96, A=197 g/mol, ρ=19.3 g/cm^3. What percentage of the incident α particles will scatter into detectors at $\theta = 15°$ and $\theta = 165°$?

3. [20 pts] We will now compare the above with Rutherford scattering (see Beiser, Chapter 4).

a) It can be shown (extra credit!) that the electric force between an α^{2+} particle and nucleus of atomic number Z,

$$F = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{r^2}$$

produces the scattering relation

$$\cot\frac{\theta}{2} = \frac{4\pi\epsilon_0 E}{Ze^2} b.$$

where E is the kinetic energy of the α-particle. What value of b leads to scattering at $\theta = 90°$?

b) Calculate $d\sigma/d\Omega$ and plot it as a function of θ on the same graph as problem 2.

c) In the same conditions as in problem 1, for α-particles with energy E=7.7 MeV, what percentage of the incident α particles will scatter into the detector at i) $\theta = 15°$, ii) $\theta = 165°$?

d) Explain why the total cross section is actually infinite. Up until now, we have ignored the electrons of the gold atom. What is the effect on small angle (large b) scattering from the entire atom? (hint: what is the total charge?)

Also: Tipler Chapter 4: #11, 16, 41, 56.