University of Kentucky, Physics 361
Problem Set #5, due Friday, Mar 5

1. [15 pts] Exploring Schrödinger's equation. For each of the following potentials, use the applet `http://www.benfold.com/sse/shoot.html` to sketch the potential, determine the first four energy levels E_n, draw lines on plot representing each energy level. Sketch the wave function using the E_n line as the x-axis for each energy level. Which energy spectrum best matches the hydrogen atom? Note this applet uses units where $\hbar^2/2m = 1$.

a) `V=-19.4/(abs(x)+1)`

b) `(x/2)^2` (quadratic), the harmonic oscillator.

c) Three square wells of width 1.0, depth 15.0, and period 2.0.

2. [20 pts] Approximate the hydrogen atom as an electron in an infinite square well.

a) Solve Schrödinger's equation with an infinite square well of width a for the energy levels E_n and normalized wavefunctions.

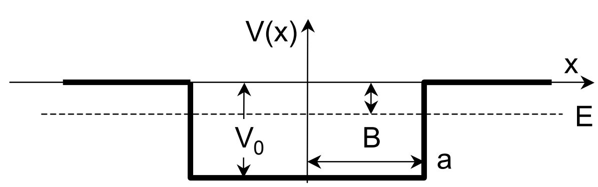
b) Solve for a so that the $n = 2 \to 1$ transition has the same wavelength as hydrogen.

c) Compare the wavelength of the $n = 3 \to 1$ transition with hydrogen.

d) What is the ionization potential in this model?

e) What properties of the infinite square well make it a bad approximation of hydrogen?

3. [30 pts] Consider the nucleus of heavy hydrogen, the deutron, which is a proton and neutron bound by the strong nuclear force. Since the neutron and proton have about the same mass, the reduced mass is $m = m_p m_n/(m_p + m_n) \approx \frac{1}{2} m_p = 469\ \text{MeV}/c^2$. The binding energy $B = 2.225$ MeV has been measured from the energy of the gamma ray produced when a neutron captures on a proton. Approximate this system as a neutron of reduced mass m in a square well of length $2a$, where $a = 2.14$ fm is the radius of the deuteron.



a) Use the wavefunctions $\psi_I(x) = A\cos(kx)$ inside the well, and $\psi_{II}(x) = e^{-\kappa|x|}$ outside. Show that these functions are solutions of the Schrödinger equation, and solve for k, κ as a function of V, B, a, m, and $\hbar$.

b) Solve the two boundary conditions at $x = a$ to come up the with the formula $\tan(ka) = \kappa a/ka$. Using the binding energy, calculate the value of κa and plot the LHS and RHS of the above equation as a function of ka. Circle the solutions for allowed values of ka, where the two curves cross. The lowest value represents the ground state. Using the value of ka from the crossing point, calculate the depth of the potential V and compare it to the binding energy of the hydrogen atom.

c) Are there any excited states of the deutron? Hint: compare the energy of the first excited state to V (no calculation necessary).

4. [25 pts] Consider the step potential

$$V(x) \;=\; V_0 \cdot \theta(x) \;=\; \begin{cases} 0 & \text{in region 1 } (x < 0) \\ V_0 & \text{in region 2 } (x > 0) \end{cases}$$

a) What type of force does this potential describe?

b) Show that $\psi(x) = e^{\pm i k_i x}$ are solutions of the Schrödinger equation for this potential in region 1 ($x < 0$) and region 2 ($x > 0$).

c) Calculate k_i in regions $i = 1, 2$ in terms of the total energy E.

d) To describe the reflection and transmission of a quantum particle, let the total wavefunction be $\psi(x) = Ae^{ik_1x} + Be^{-ik_1x}$ if $x < 0$ and $\psi(x) = Ce^{ik_2x}$ if $x > 0$. Label the incident, transmitted, and reflected wave functions. Why is the term De^{-ik_2x} not included?

e) Apply the boundary conditions at $x = 0$ to obtain formulas for the coefficients of reflection $R \equiv \left(\frac{B}{A}\right)^2$ and transmission $T \equiv \frac{k_2}{k_1}\left(\frac{C}{A}\right)^2$ as a function of E/V_0 (the factor of k_2/k_1 accounts for the difference in velocity).

f) Show that $R + T = 1$, ie. the particle is either reflected or transmitted.

Also: Tipler Chapter 6: #3, 4, 10.