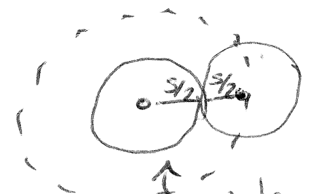
HW #3 solutions

#1 a) by definition of mean free path, one particle is in cylinder λ long

$$n_g = \frac{\#}{Vol} = \frac{1}{\sigma \cdot \lambda}$$

b) the centers of two molecules have to be within a distance "s" for a collision. Thus the second molecule's center can be anywhere within an area of πs^2.



interaction area.

c) The volume of one ~~atom in~~ molecule in a liquid is the volume of the molecule itself: $V_\ell = \frac{4}{3}\pi\left(\frac{s}{2}\right)^3$

The volume of one molecule in a gas is $V_g = \frac{1}{\sigma\lambda} = \pi s^2 \cdot \lambda$

thus $\varepsilon = \frac{n_g}{n_\ell} = \frac{V_\ell}{V_g} = \frac{4/3\,\pi\left(\frac{s}{2}\right)^3}{\pi s^2 \cdot \lambda} = \frac{s}{6\lambda}$.

d) $\varepsilon = \frac{n_g}{n_\ell} = \frac{1/V_m}{\rho_\ell / A} = \frac{A}{\rho_\ell \cdot V_m} = \frac{35.6\,g/mol}{0.870\,g/cm^3 \cdot 24\,L/mol} = 1.70 \times 10^{-3}$

e) $s = 6\varepsilon\lambda = 6 \cdot (1.70\times10^{-3}) \cdot 62\,nm = 0.634\,nm \qquad \sigma = \pi s^2 = 1.26\,nm^2$

f) $n_g = \frac{1}{\sigma \cdot \lambda} = \frac{1}{1.26\,nm^2 \cdot 62\,nm} = 1.28\times10^{19}/cm^3$ compare w/ $2.65\times10^{19}/cm^3$

g) $N_A = n_g \cdot V_m = 1.28\times10^{19}/cm^3 \cdot 24\,L/mol = 3.1\times10^{23}/mol$. [within 2x!]

h) $e = F/N_A = 96485\,C/mol \,/\, 3.1\times10^{23}/mol = 3.15\times10^{-19}\,C$

i) $m_e = e/(e/m) = 3.15\times10^{-19}\,C \,/\, 1.75\times10^{8}\,C/g = 1.8\times10^{-30}\,kg$.

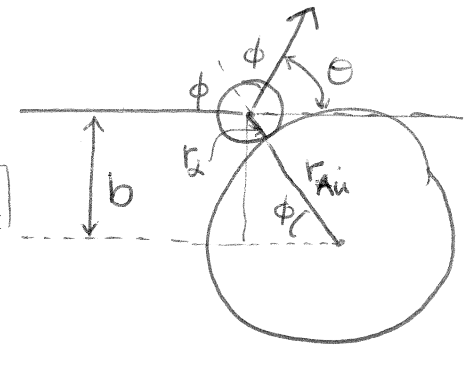
j) $m_p = A/N_A - m_e = 1.008\,g/mol \,/\, 3.1\times10^{23}/mol - 1.8\times10^{-30}\,kg = 3.2\times10^{-27}\,kg$.

Note: this derivation is simplistic and does not include the effect that all molecules are randomly moving at the same time.

25 #2 a) $b = (r_\alpha + r_{Au})\sin\phi$, let $\boxed{R = r_\alpha + r_{Au}}$ $= 9\,fm$

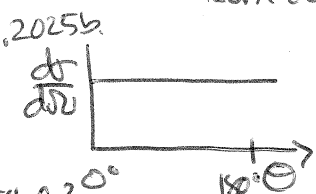
$2\phi + \theta = \pi$

so $b = R\sin\left(\frac{\pi-\theta}{2}\right) = \boxed{R\cos\frac{\theta}{2}}$



b) $\frac{d\sigma}{d\Omega} = \frac{2\pi b}{2\pi\sin\theta}\cdot\frac{db}{d\theta} = \frac{R\cos\theta/2}{\sin\theta}\cdot\frac{1}{2}R\sin\frac{\theta}{2}$ ignore (−) from derivative

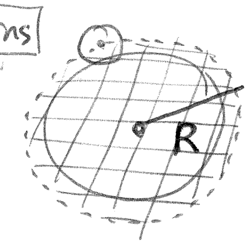
$= \frac{1}{4}R^2\cdot\frac{2\cos\theta/2\,\sin\theta/2}{\sin\theta} = \boxed{\frac{1}{4}R^2} = 0.2025\,b$



c) $\sigma_{tot} = \int_{4\pi}\frac{d\sigma}{d\Omega}d\Omega = 4\pi\cdot\frac{1}{4}R^2 = \boxed{\pi R^2} = 254\,fm^2$

$= \boxed{2.54 \text{ barns}}$ "b"

This is just the area of interaction.
(larger than the physical area of the gold nucleus).



d) $b = R\cos\frac{90°}{2} = (1.7+7.3)fm\cdot 0.707 = \boxed{6.36\,fm}$

$f = \frac{N}{N_i} = \frac{d\sigma}{d\Omega}(\theta)\cdot\underbrace{nt}_{\text{target thickness}}\cdot\underbrace{\frac{A_{det}}{r^2_{det}}}_{\Omega_{det}}$

$n = \frac{\#}{vol} = \frac{\#}{mol}\cdot\frac{mol}{g}\cdot\frac{g}{cm^3} = \frac{N_A\cdot\rho}{A}$

$= \frac{1}{4}R^2\cdot\frac{N_A\cdot\rho}{A}\cdot\frac{t\,A_{det}}{r^2_{det}}$

$n = 5.9\times10^{28}/m^3$ $nt = 2.36\times10^{-6}/b$

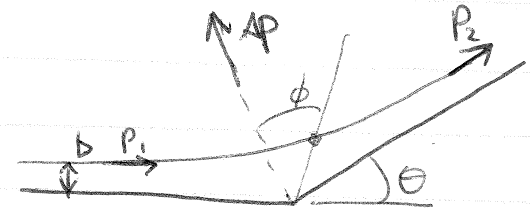
$= \frac{1}{4}(1.7fm + 7.3fm)^2\,\frac{6.02\times10^{23}}{mol}\cdot\left(\frac{19.3g}{cm^3}\right)\cdot\frac{0.00004cm}{197g/mol}\cdot\left(\frac{10cm}{2m}\right)^2$

$= \boxed{1.2\times10^{-9}}$ independent of the angle of the detector.

note: even for a gold foil $t\cdot\sqrt[3]{n} = 1567$ atoms thick (each atom is 2.5 Å wide), the nuclei are extremely far apart!

#3 a) $F=ma \rightarrow \Delta\vec{p} = \int \vec{F}\,dt$

$$2p\sin\frac{\theta}{2} = \int_{\phi_1}^{\phi_2} F\cos\phi \cdot \frac{dt}{d\phi}\,d\phi$$



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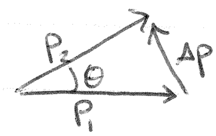
from cons. ang. momentum,

$$L = m\omega r^2 = mr^2\frac{d\phi}{dt} = \text{constant}$$

$L_{initial} = mvb \qquad$ so $\frac{d\phi}{dt} = \frac{r^2}{vb}$



$$\Delta p^2 = p_1^2 + p_2^2 - 2p_1p_2\cos\theta$$
$$= 2p^2(1-\cos\theta) = 4p^2\sin^2\theta/2$$

assume the change in velocity is small.

$$2mv\sin\theta/2 = \int_{-\phi_0}^{\phi_0} F\cos\phi \cdot \frac{r^2}{vb}\,d\phi$$

$$2mv^2 b\sin\theta/2 = \int_{-\phi_0}^{\phi_0} Fr^2\cos\phi\,d\phi$$

The electrostatic force is: $F = \frac{1}{4\pi\epsilon_0}\frac{2Ze^2}{r^2}$

so $\underbrace{2mv^2}_{4E} b\sin\theta/2 = \int_{-\phi_0}^{\phi_0} \frac{2Ze^2}{4\pi\epsilon_0}\cos\phi\,d\phi = \frac{2Ze^2}{4\pi\epsilon_0}\underbrace{\sin\phi\Big|_{-\frac{\pi-\theta}{2}}^{\frac{\pi-\theta}{2}}}_{2\sin(\frac{\pi-\theta}{2}) = 2\cos(\frac{\theta}{2})}$

$$b = \frac{2Ze^2}{4\pi\epsilon_0}\cdot\frac{1}{4E}\cdot\frac{2\cos(\theta/2)}{\sin(\theta/2)}$$

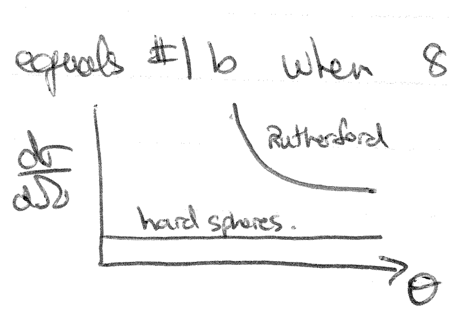
$$\boxed{b = \frac{Ze^2}{4\pi\epsilon_0 E}\cot\left(\frac{\theta}{2}\right)} = \frac{79\cdot(1.44\,\text{eV}\cdot\text{nm})}{2.0\,\text{MeV}}\cot\frac{90^\circ}{2} = \boxed{56.9\,\text{fm}}$$

at $\theta = 90^\circ$

b) $\frac{d\sigma}{d\Omega} = \frac{2\pi(b)}{2\pi(\sin\theta)}\left(\frac{db}{d\theta}\right) = \frac{\left(\frac{Ze^2}{4\pi\epsilon_0 E}\frac{\cos\theta/2}{\sin\theta/2}\right)}{\left(2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}\right)}\left(\frac{Ze^2}{4\pi\epsilon_0 E}\frac{\csc^2\frac{\theta}{2}}{2}\right)$

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{2Ze^2}{4\cdot4\pi\epsilon_0 E}\right)^2\sin^{-4}\theta/2} = \left(\frac{2\cdot79\cdot1.44\,\text{eV}\cdot\text{nm}}{4\cdot2\,\text{MeV}}\right)^2\sin^{-4}\frac{\theta}{2} = 8.09\,\text{b}\cdot\sin^{-4}\frac{\theta}{2}$$

this equals #1 b when $8.09\,\text{b}\sin^{-4}\frac{\theta}{2} = 2.54\,\text{b}$. or $\theta =$ never!



The electric force has a much longer range!

c) $f = \frac{N}{N_i} = f_{(\text{prob } \#1)} \times \frac{8.096 \sin^{-4}(\theta/2)}{0.2026} = 4.81\times10^{-8} / \sin^4\frac{\theta}{2}$

$\theta = 15^\circ \quad \sin^{-4}\frac{\theta}{2} = 3445 \qquad \boxed{f(15) = 1.64\times10^{-4}}$

$\theta = 165 \quad \sin^{-4}\frac{\theta}{2} = 1.034 \qquad \boxed{f(165) = 4.9\times10^{-8}}$

d) The electric force has infinite range. $F \propto \frac{1}{r^2}$
It gets small, but is not 0 at very large distances, so we expect the cross section to be infinite.
However, at $r \gtrsim 1.3\,\text{Å}$ the α^{2+} particle will stay outside the electron cloud of the atom, and will see the neutral atom instead of the (+) nucleus.

80

#11 $\theta_{RMS} = \theta_{single} \cdot \sqrt{n}$

$n = \left(\theta_{RMS}/\theta_{single}\right)^2 = \left(\frac{10^\circ}{0.1^\circ}\right)^2 = \boxed{10^6}$

10 compare: $n = 10^{-6}m / 10^{-10}m = \boxed{10^4}$

the foil is not thick enough to produce 10^6 scatters.

#16 quantization: $L = n\hbar = mv \cdot r = \cancel{2\pi}\, m_\oplus \frac{2\pi r}{1\,\text{year}} \cdot r$ (earth) $m_\oplus = 5.9736 \times 10^{24}$ kg.

$n = \frac{2\pi \cdot mr^2}{\hbar \cdot 1\,\text{year}} = \boxed{2.52 \times 10^{74}}$ (you could say its classical!).

$E_n = -\frac{E_o}{n^2}$ where $E_o = \frac{m_\oplus (\overbrace{G m_\oplus m_\odot}^{ke^2})^2}{2\hbar^2} = 1.689 \times 10^{182}$ J

15 check: $KE = \frac{1}{2} m_\oplus v^2 = \frac{1}{2} m_\oplus \left(\frac{2\pi r}{yr}\right)^2 = 2.65 \times 10^{33}\,J = \frac{E_o}{n^2}$

transition: $\Delta E = E_o\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right) = E_o \frac{\overbrace{n^2 - (n-1)^2}^{n^2-2n+1}}{(n-1)^2 \cdot n^2} \cong E_o \frac{2n-1}{(n-1)^2 \cdot n^2} \approx \frac{2E_o}{n^3}$

$= 2 \cdot (1.689 \times 10^{182} J) / (2.52 \times 10^{74})^3 = \boxed{2.1 \times 10^{-41} J}$

$= 1.3 \times 10^{-22}$ eV

#41 a) $i = q f_{rev} = q \frac{v}{2\pi a_0} = \frac{e\alpha c}{2\pi a_0} = \frac{1.6\times10^{-19}C \cdot \frac{1}{137} \cdot 3\times10^{8} m/s}{2 \cdot 3.14 \cdot 5.29\times10^{-11} m} = \boxed{1.05\, mA}$

10 b) $\mu_B = i a = \frac{e\alpha c}{2\pi a_0} \cdot \pi a_0^2 = \frac{e\alpha c}{2}\left(\frac{\hbar}{mc}\frac{1}{\alpha}\right) = \frac{e\hbar}{2m} = \boxed{9.27\times10^{-24} A\cdot m^2}$ or J/T

"Bohr magneton" see back cover, (error in formula)

#56 $ke^2 \to G m_e m_p$ $\quad \alpha = \frac{ke^2}{\hbar c} \to \frac{G m_e m_p}{\hbar c} = 3.2\times10^{-42}$ ie 10^{40}x weaker!

$E_0 = -\frac{1}{2}mc^2 \cdot \alpha^2$ $\quad E_0^G = E_0 \cdot \left(\frac{\alpha_G}{\alpha}\right)^2 = \boxed{2.64\times10^{-78} eV} = 4.23\times10^{-97} J$

$a_0 = \frac{\hbar}{mc}\frac{1}{\alpha}$ $\quad a_0^G = a_0\left(\frac{\alpha}{\alpha_G}\right) = \boxed{1.2\times10^{29} m}$

10 $\lambda_{H_\alpha} = \frac{hc}{E_0^G}\left(\frac{1}{2^2} - \frac{1}{3^2}\right)^{-1} = \frac{5}{36}\lambda_\infty = \boxed{6.5\times10^{70} m}$ $\quad$ compare: 121.56 nm

$\lambda_\infty = 4\frac{hc}{E_0^G} = \boxed{1.88\times10^{72} m}$ $\quad 6.5\times10^{70} m$ $\quad$ compare: 91.2 nm

$1.88\times10^{72} m$

not going to happen. $\quad$ no comparison!

$\Delta E_{H_\alpha} = 5.85\times10^{-98} J = 3.66\times10^{-79} eV$ $\quad f_{H_\alpha} = 8.28\times10^{-65} Hz$

$\Delta E_{H_\infty} = 6.58\times10^{-79} eV$ $\quad f_{H_\infty} = 1.59\times10^{-64} Hz$