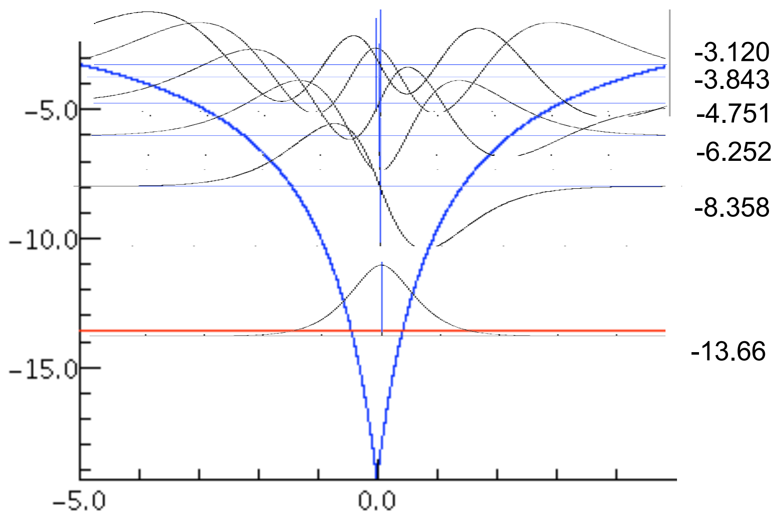
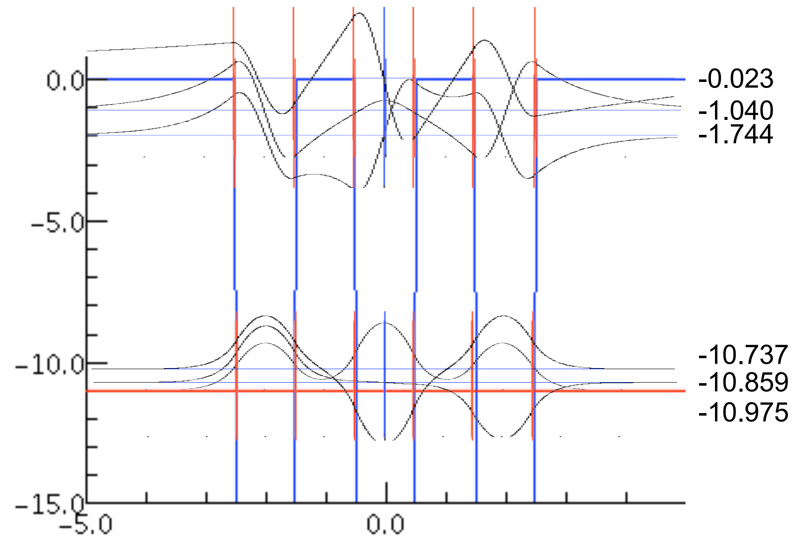
120 pts total

#1 a) and c)





b) E6= -0.02326
E5= -1.404
E4= -1.744
E3=-10.74
E2=-10.86
E1=-10.975

#2. * Schrödinger's Time-Independent Equation:

a) $\hat{T}\psi + \hat{V}\psi = E\psi$

$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + 0\cdot\psi = E\psi$

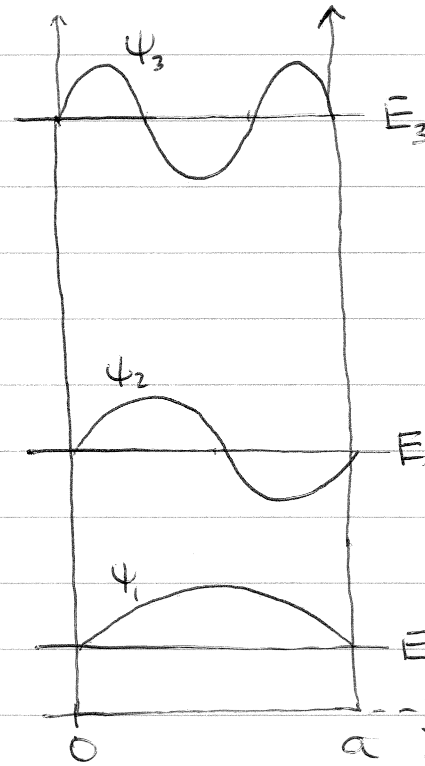
let $\psi = \cos kx$ or $\sin kx$

$\psi' = -k\sin kx$ or $k\cos kx$

$\psi'' = -k^2\cos kx$ or $-k^2\sin kx = -k^2\psi$

so $\frac{-\hbar^2}{2m}(-k^2)\psi = E\psi$

$E = \frac{(\hbar k)^2}{2m}$ ie $\frac{p^2}{2m} = T$



* Boundary conditions:

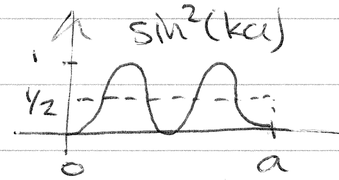
$\psi(x) = A\cos(kx) + B\sin(kx)$

i) $\psi(0) = A\cos(0) + B\sin(0) = A = 0$

ii $\psi(a) = B\sin(ka) = 0$

$k_n a = n\pi$

* normalization:



$\int_0^a \psi^*\psi\,dx = \int_0^a B^2\sin^2(ka)\,dx = 1$

$= B^2\int_0^a \frac{1}{2}dx = B^2\frac{a}{2} = 1$

$B = \sqrt{\frac{2}{a}}$

* solution: $\psi_n = \sqrt{\frac{2}{a}}\sin(k_n x)$

$E_n = \frac{(\hbar k_n)^2}{2m} = \frac{\hbar^2\pi^2}{2ma^2}n^2$ where $k_n = \frac{n\pi}{a}$

b) $\Delta E_{21} = \frac{\hbar^2\pi^2}{2ma^2}(2^2 - 1^2) = \frac{3\hbar^2\pi^2}{2ma^2}$ inf well

$\Delta E_{21} = +E_0\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}E_0$ hydrogen atom 121.5 nm

so $a = \frac{2\pi\hbar}{\sqrt{2mE_0}} = \frac{hc}{\sqrt{2mc^2\cdot E_0}} = \frac{1240\ \cancel{\text{MeV fm}}\ \text{eV}\cdot\text{nm}}{\sqrt{2\cdot 0.511\ \text{MeV}\cdot 13.6\text{eV}}} =$ 0.332 nm

c) $\Delta E_{31} = \frac{\hbar^2\pi^2}{2ma^2}(3^2 - 1^2) = \frac{1}{4}E_0\cdot 8 = 2E_0 = 27.2\text{eV}$ inf. well 45.56 nm

$\Delta E_{31} = E_0\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \frac{8}{9}E_0 = 12.09\text{eV}$ hydrogen 102.55 nm

d) the ionization energy is infinite (infinite well!)

e) the energy levels get farther apart, not closer together.
the orbitals do not get larger radius with higher "n"

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Solution to HW #5

#3.

$B = 2.225$ MeV

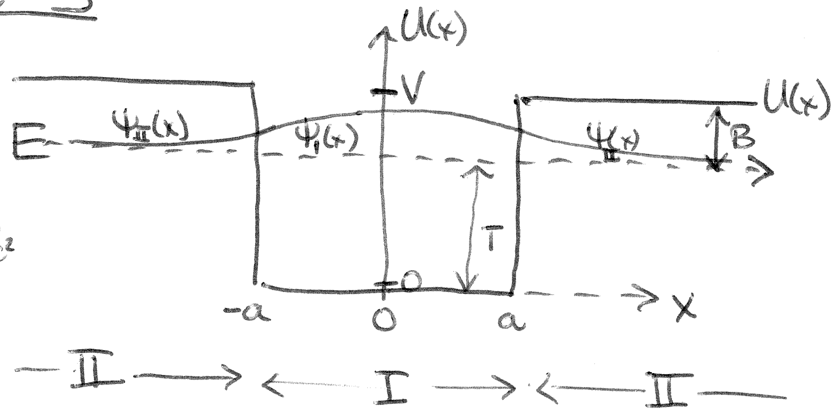
$a =$ ~~0.96 fm~~ 2.14 fm

$m = \frac{m_n \cdot m_p}{m_n + m_p} \sim \frac{1}{2} m_p = 469\,\mathrm{MeV}/c^2$

30

$\psi_I = A\cos(kx)$

$\psi_{II} = B e^{-\kappa x}$



a) $\psi'' = \frac{-2m}{\hbar^2}(E-U)\psi$

I: $-Ak^2\cos(kx) = \frac{-2m}{\hbar^2}(E-0)\cdot A\cos(kx)$

$$\boxed{\frac{\hbar^2 k^2}{2m} = E = V - B}$$

II: $B\kappa^2 e^{-\kappa x} = \frac{-2m}{\hbar^2}(E-V)\cdot Be^{-\kappa x}$

$$\boxed{\frac{\hbar^2 \kappa^2}{2m} = E - V = B}$$

boundary conditions:

a) $\psi_I(a) = \psi_{II}(a)$

$A\cos(ka) = Be^{-\kappa a}$

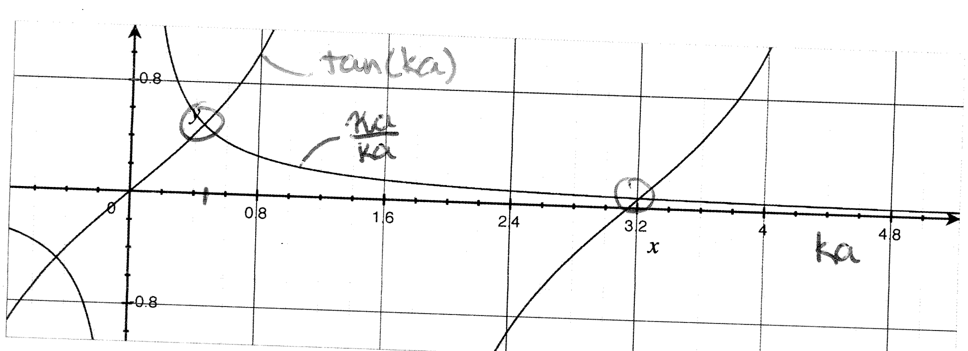
b) $\psi_I'(a) = \psi_{II}'(a)$

$-A\cdot k\sin(ka) = -B\kappa e^{-\kappa a}$

$$\frac{A k\sin(ka) = B\kappa e^{-\kappa a}}{A\cos(ka) = B\cdot e^{-\kappa a}} \Rightarrow \boxed{\tan(ka) = \frac{a\kappa}{ka}}$$

$$a\kappa = \frac{a}{\hbar c}\sqrt{2mc^2 \cdot B} = \frac{\cancel{0.96\,\mathrm{fm}}\ 2.14\,\mathrm{fm}}{197\,\mathrm{MeV}\cdot\mathrm{fm}}\sqrt{2\cdot 469\,\mathrm{MeV}\cdot 2.225\,\mathrm{MeV}} = \boxed{\cancel{0.2223}}\ 0.4957$$

$\frac{1}{a\kappa}$ is how far the wavefunction extends outside of the well $\sim 5\times$!



from the plot, the first crossing is at

$\boxed{ka = \cancel{0.455}}$ 0.6508

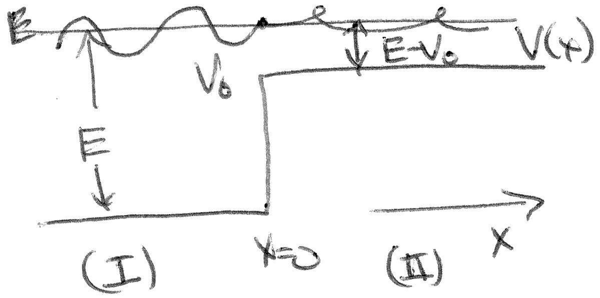
thus $V = \frac{\hbar^2 k^2}{2m} + B = \frac{\left(197\,\frac{MeV\cdot fm}{c}\cdot\frac{0.455}{\cancel{0.964} fm}\right)^2}{2\cdot 469\,MeV/c^2} + 2.225\,MeV$ 2.14

$= \boxed{\cancel{11.1\,MeV}}$ 6.06 MeV

in 3 dimensions the potential is ~ 50 MeV.
this is 818926 x greater than E_0
for the hydrogen atom!
thus, nuclear radiation occurs as γ-rays, not visible light.

c) from the plot, the second excited state is much higher energy, such that the particle would no longer be bound.
the same is true if one were to look at the solutions using $\Psi_1(x) = A\sin(kx)$.
∴ The deuteron has no excited states!
It is very loosely bound.

#4



I: $\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi_I + 0\cdot\psi_I = E\psi_I$

$\psi_1 = A\underset{\text{incident}}{e^{ik_1x}} + B\underset{\text{reflected}}{e^{-ik_1x}}$

$\frac{-\hbar^2\cdot -k_1^2}{2m} = E \qquad E = \frac{\hbar^2k_1^2}{2m}$

II: $\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi_{II} + V_0\psi_{II} = E\psi_{II}$

$\psi_2 = C\underset{\text{transmitted}}{e^{ik_2x}} + \underbrace{De^{-ik_2x}}_{\text{nowhere to come from!}}$

$E - V_0 = \frac{\hbar^2k_2^2}{2m}$

boundary conditions:

$\psi_1(0) = \psi_{II}(0)$: $Ae^{ik_1\cdot 0} + Be^{-ik_1\cdot 0} = Ce^{ik_2\cdot 0}$

$$A + B = C \qquad (1)$$

$\psi_1'(0) = \psi_{II}'(0)$: $ik_1Ae^{ik_1\cdot 0} + -ik_1Be^{-ik_1\cdot 0} = ik_2Ce^{ik_2\cdot 0}$

$$k_1A - k_1B = k_2\cdot C. \qquad (2)$$

multiply (1) by k_2:

$$\begin{array}{rl} & k_2A + k_2B = k_2C \\ (-) & k_1A - k_1B = k_2C \\ \hline & (k_2-k_1)A + (k_2+k_1)B = 0 \end{array}$$

$R = \left(\frac{B}{A}\right)^2 = \left(\frac{k_1-k_2}{k_1+k_2}\right)^2$

$= \left(\frac{\sqrt{E}-\sqrt{E-V_0}}{\sqrt{E}+\sqrt{E-V_0}}\cdot\underbrace{\frac{\sqrt{E}-\sqrt{E-V_0}}{\sqrt{E}-\sqrt{E-V_0}}}_{\text{rationalize denominator}}\right)^2$

$= \left(\frac{(\sqrt{E}-\sqrt{E-V_0})^2}{E-(E-V_0)}\right)^2 = $

$$\boxed{R = \left(\sqrt{E/V_0} - \sqrt{E/V_0 - 1}\right)^4}$$

multiply (1) by k_1:

$$\begin{array}{rl} & k_1A + k_1B = k_1C \\ (+) & k_1A - k_1B = k_2C \\ \hline & 2k_1A = (k_1+k_2)C \end{array}$$

$T = \frac{k_2}{k_1}\left(\frac{C}{A}\right)^2 = \frac{4k_1k_2}{(k_1+k_2)^2}$

$= \frac{4\sqrt{E}\sqrt{E-V_0}}{(\sqrt{E}+\sqrt{E-V_0})^2}\frac{(\sqrt{E}-\sqrt{E-V_0})^2}{(\sqrt{E}-\sqrt{E-V_0})^2}$

$$\boxed{T = 4\sqrt{E/V_0}\sqrt{E/V_0 - 1}\left(\sqrt{E/V_0} - \sqrt{E/V_0 - 1}\right)^2}$$

$T + R = \left(\frac{k_1-k_2}{k_1+k_2}\right)^2 + \frac{4k_1k_2}{(k_1+k_2)^2} = \frac{k_1^2+2k_1k_2+k_2^2}{(k_1+k_2)^2}$

$\boxed{T+R = 1}$

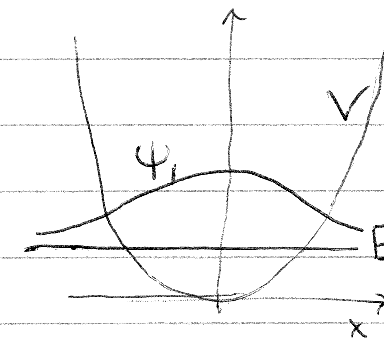
#6-3. $\psi(x) = Ae^{-x^2/2L^2}$ $\qquad E = \frac{\hbar^2}{2mL^2}$

$\psi'(x) = -\frac{x}{L^2} Ae^{-x^2/2L^2} = \frac{-x}{L^2}\psi(x)$

$\psi''(x) = \left(\frac{x^2}{L^4} - \frac{1}{L^2}\right)\psi(x)$

a) $\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V(x)\cdot\psi = E\cdot\psi$

$\frac{-\hbar^2}{2m}\left(\frac{x^2}{L^4} - \frac{1}{L^2}\right)\psi + V\cdot\psi = \frac{\hbar^2}{2mL^2}\cdot\psi$



$V(x) = \frac{\hbar^2}{2mL^4}x^2$

b) $F = -\frac{\partial U}{\partial x} = -\frac{\hbar^2}{mL^4}x = -kx$

Hooke's law, the spring force → harmonic oscillato

#6-4 a) $T = E - V = \frac{\hbar^2}{2mL^2} - \frac{\hbar^2}{2mL^2}\frac{x^2}{L^2} = \frac{\hbar^2}{2mL^2}\left(1 - \left(\frac{x}{L}\right)^2\right)$

b) $T=0$ when $1-\left(\frac{x}{L}\right)^2 = 0$ or $x = \pm L$

c) classically, let ω = oscillation angular frequency.

$x = A\sin(\omega t)$ $\qquad F = ma$

$\frac{dx}{dt} \equiv \dot{x} = +\omega\cdot A\cos(\omega t)$ $\Rightarrow$ $-kx = m(-\omega^2)x$

$\frac{d^2x}{dt^2} \equiv \ddot{x} = -\omega^2 A\sin(\omega t) = -\omega^2 x$ $\qquad k = m\omega^2$

thus $V = \frac{1}{2}kx^2 = \frac{1}{2}(m\omega^2)x^2 = \left(\frac{\hbar^2}{2mL^4}\right)x^2$

and $L^2 = \frac{\hbar}{m\omega}$

thus $\psi_1 = Ae^{-\frac{m\omega x^2}{2\hbar}}$ $\qquad E_1 = \frac{\hbar^2}{2m}\cdot\frac{m\omega}{\hbar} = \frac{1}{2}\hbar\omega$.

in general, $E_n = \left(\frac{1}{2}+n\right)\hbar\omega$ for $n = 0, 1, 2 \ldots$

#6-10 As in #2, $\psi_1 = A\sin(k_1 x)$ where $\sin(k_1 L) = 0$ $\qquad k_n L = n\pi$

$\int_0^L |\psi|^2 dx = A^2\frac{L}{2} = 1$ so $A = \sqrt{\frac{2}{L}}$ $\qquad k_1 = \frac{\pi}{L}$

a) $P(L/2 < x < L/2 + 0.002L) = \left(\sqrt{\frac{2}{L}}\right)^2\sin^2\left(\frac{\pi}{L}\cdot\frac{L}{2}\right)\cdot(0.002L) = 0.004$

b) $P(2L/3 < x < 2L/3 + 0.002L) = \frac{2}{L}\sin^2\left(\frac{\pi}{L}\frac{2L}{3}\right)\cdot(0.002L) = 0.003$

c) $P(L - 0.002L < x < L) \approx \sin^2\left(\frac{\pi}{L}\cdot L\right) \approx 0$.