# Problem 9.5

The equilibrium separation of the $Rb^+$ and $Cl^-$ ions in RbCl is about 0.267 nm.

## Part a

Calculate the potential energy of attraction of the ions, assuming them to be point charges.

Assuming that $-ke^2/r = 0$ at infinite separation, the electrostatic potential energy at 0.267 nm for $Rb^+$ and $Cl^-$ is:

$$\begin{aligned} -\frac{ke^2}{r} &= -\frac{\left(8.98755179 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2\right)\left(1.60217646 \times 10^{-19}\mathrm{C}\right)^2}{2.67 \times 10^{-10}\mathrm{m}} \\ &= -5.393\,\mathrm{eV} \end{aligned}$$

## Part b

The ionization energy of rubidium is 4.18 eV, and the electron affinity of Cl is 3.62 eV. Find the dissociation energy, neglecting the energy of repulsion.

The ionization energy to form $Rb^+$ and $Cl^-$ is, then, $4.18\,\mathrm{eV} - 3.62\,\mathrm{eV} = 0.56\,\mathrm{eV}$, so the total energy required to dissociate RbCl (if we neglect the energy of repulsion) is $5.393\,\mathrm{eV} - 0.56\,\mathrm{eV} = 4.833\,\mathrm{eV}$.

## Part c

The measured dissociation energy is 4.37 eV. What is the energy due to repulsion of the ions?

If the measured dissociation energy at the equilibrium separation is 4.37 eV, the energy due to repulsion of the ions must be $4.833 - 4.37 = 0.463\,\mathrm{eV}$.

# Problem 9.6

Compute the Colomb energy of the KBr molecule at the equilibrium separation. Use that result to compute the exclusion-principle repulsion at $r_0$.

The equilibrium separation of KBr is $r_0 = 0.282\,\mathrm{nm}$, so the Colomb energy at equilibrium separation is $-ke^2/r = -5.393\,\mathrm{eV}$. The ionization energy for K is 4.34 eV, and the electron affinity for Br is 3.36 eV. So, the ionization energy is $4.34\,\mathrm{eV} - 3.36\,\mathrm{eV} = 0.98\,\mathrm{eV}$, giving us a total dissociation energy (neglecting the energy of repulsion) of $5.393\,\mathrm{eV} - 0.98\,\mathrm{eV} = 4.126\,\mathrm{eV}$. The measured dissociation energy of KBr is 3.94 eV, so, the exclusion-principle repulsion at $r_0$ is $4.126\,\mathrm{eV} - 3.94\,\mathrm{eV} = 0.186\,\mathrm{eV}$.

# Problem 9.7

If the exclusions-principle repulsion in Problem 9-6 is given by Equation 9-2, compute the coefficient $A$ and the exponent $n$.

Equation 9-2 [Tipler & Llewellyn, p. 365] is:

$$E_{ex} = \frac{A}{r^n}$$

At equilibrium separation, this gives us:

$$0.186\,\mathrm{eV} = \frac{A}{(0.282\,\mathrm{nm})^n}$$

[Tipler & Llewellyn, p. 367] At $r = r_0$, the net force on each ion must be zero because the potential energy has its minimum value at that point. This means that at $r = r_0$, the net Coloumb force $F_C$is equal in magnitude and opposite in sign to the exclusion-principle repulsive force, i.e.:

$$F_C = -\left(\frac{dU_c}{dr}\right)_{r=r_0} = \left(\frac{nA}{r^{n+1}}\right)_{r=r_0}$$

So, at $r = r_0$ we have that:

$$F_C = \frac{U_C\left(r_0\right)}{r_0} = \frac{ke^2}{r_0^2} = 18.107\,\text{eV/nm}$$

And:

$$\frac{nA}{r_0^{n+1}} = \frac{n}{r_0}\frac{A}{r^n} = \frac{n}{0.282\,\text{nm}}\left(0.186\,\text{eV}\right) = 18.107\,\text{eV/nm}$$

Or:

$$\begin{aligned} n &= 18.107\,\text{eV/nm}\left(\frac{0.282\,\text{nm}}{0.186\,\text{eV}}\right) \\ &= 27.453 \\ &\approx 27 \end{aligned}$$

So, now we can calculate $A$:

$$\begin{aligned} 0.186\,\text{eV} &= \frac{A}{\left(0.282\,\text{nm}\right)^{27}} \\ \left(0.186\,\text{eV}\right)\left(0.282\,\text{nm}\right)^{27} &= A \\ 2.668 \times 10^{-16}\text{eV} \cdot \text{nm}^{27} &= A \end{aligned}$$

# Problem 9.11

What kind of bonding mechanism would you expect for:

## Part a

The KCl molecule?

Ionic

## Part b

The $O_2$ molecule?

Covalent

## Part c

The $CH_4$ molecule?

Covalent

# Problem 9.37

A helium-neon laser emits light of wavelength 632.8 nm and has a power output of a 4 mW. How many photons are emitted per second by this laser?

We know that, for a photon, $E = hf$ and $\lambda f = c$, so $E = hc/\lambda$. Also, $P = \Delta W/\Delta t$. If $P = 4\,\text{mW}$, then we must have enough photons to equal $4\,\text{mJ/s}$. So, $E = hc/\lambda = 1.959\,\text{eV}$, and

$$\frac{4\,\text{mJ/s}}{1.959\,\text{eV/photon}} = 1.274 \times 10^{16}\text{photons/second}$$

# Problem 10.10

## Part a

Given a mean free path $\lambda = 0.4\,\text{nm}$ and a mean speed $\langle v\rangle = 1.17 \times 10^5\text{m/s}$ for the current flow in copper at a temperature of 300 K, calculate the classical value for the resistivity $\rho$ of copper.

The restivity is given by $\rho = \frac{m_e\langle v\rangle}{ne^2\lambda}$. In this case, we know $m_e$, $e$, and, for copper, $n = 8.47 \times 10^{22}\,\text{atoms/cm}^3$ [Tipler & Llewellyn, p. 423]. So, we have that:

$$\begin{aligned}\rho &= \frac{m_e\langle v\rangle}{ne^2\lambda} \\ &= \frac{\left(9.109 \times 10^{-31}\text{kg}\right)\left(1.17 \times 10^5\text{m/s}\right)}{(8.47 \times 10^{22}\,\text{atoms/cm}^3)\left(1.602 \times 10^{-19}\text{C}\right)^2(0.4\,\text{nm})} \\ &= 1.22549255 \times 10^{-7}\Omega\cdot\text{m}\end{aligned}$$

## Part b

The classical model suggests that the mean free path is temperature independent and that $\langle v\rangle$ depends on temperature. From this model, what would $\rho$ be at 100 K?

We have that $\langle v\rangle = \sqrt{\frac{8kT}{\pi m_e}}$. This depends only on known constants, except for $T$, the temperature. So, at 100 K we have:

$$\begin{aligned}\langle v\rangle &= \sqrt{\frac{8k\,(100\,\text{K})}{\pi m_e}} \\ &= 6.213 \times 10^4\text{m/s}\end{aligned}$$

The resistivity at this temperature is, then:

$$\begin{aligned}\rho &= \frac{m_e\langle v\rangle}{ne^2\lambda} \\ &= \frac{\left(9.109 \times 10^{-31}\text{kg}\right)\left(6.213 \times 10^4\text{m/s}\right)}{(8.47 \times 10^{22}\,\text{atoms/cm}^3)\left(1.602 \times 10^{-19}\text{C}\right)^2(0.4\,\text{nm})} \\ &= 6.50884052 \times 10^{-8}\Omega\cdot\text{m}\end{aligned}$$

# Problem 10.16

Find the average energy of the electrons at $T = 0\,\text{K}$ in:

### Part a

> copper ($E_F = 7.06\,\text{eV}$)

According to [Tipler & Llewellyn, p. 429], the average energy at $T = 0\,\text{K}$ is given by $\langle E \rangle = \frac{3}{5} E_F$. So, for copper, the average energy of the electrons is $\langle E \rangle = \frac{3}{5} * 7.06\,\text{eV} = 4.236\,\text{eV}$.

### Part b

> Li ($E_F = 4.77\,\text{eV}$)

Once again, the average energy at $T = 0\,\text{K}$ is given by $\langle E \rangle = \frac{3}{5} E_F$. So, for Li, the average energy of the electrons is $\langle E \rangle = \frac{3}{5} * 4.77\,\text{eV} = 2.862\,\text{eV}$.

## Problem 10.27

### Part a

> The energy gap between the valence band and the conduction band in silicon is 1.14 eV at room temperature. What is the wavelength of a photon that will excite an electron from the top of the valence band to the bottom of the conduction band?

For light, $E = hf = hc/\lambda$, so $\lambda = hc/E$. In this case, then, $\lambda = hc/1.14\,\text{eV} = 1088\,\text{nm}$.

### Part b

> Repeat the calculation in part (a) for germanium, for which the energy gap is 0.72 eV.

In this case, $\lambda = hc/0.72\,\text{eV} = 1722\,\text{nm}$.

### Part c

> Repeat the calculation in part (a) for diamond, for which the energy gap is 7.0 eV.

In this case, $\lambda = hc/7.0\,\text{eV} = 177.1\,\text{nm}$.

## Problem 11.27

> Show that the $\alpha$ particle emitted in the decay of the $^{232}$Th carries away 4.01 MeV, or 98 percent, of the total decay energy.

The $\alpha$ decay of the $^{232}$Th atom is $^{232}\text{Th} \rightarrow^{228} \text{Ra} + \alpha$. We know the total decay energy is $Q = 4.08\,\text{MeV}$ [Tipler & LLewellyn web site] and that, if the particle is at rest during the decay, that both the $\alpha$ and the $^{228}$Ra have the same momenta. The sum of the kinetic energy of these two particles is

$$Q = \frac{p^2}{2M_D} + \frac{p^2}{2M_{\text{He}}} = \frac{p^2}{2M_{\text{He}}} \left( 1 + \frac{M_{\text{He}}}{M_D} \right)$$

Where $M_D$ is the mass of the daughter atom, $^{228}$Rb, $M_{\text{He}}$ is the mass of the $\alpha$ particle, and $p$ is the common momenta of the two particles. Since $M_{\text{He}} = 4$ and $M_D = A - 4$, where $A$ is mass number of the parent nucleus, $^{232}$Th, and in this case $A = 232$. So, substituting $E$ for $\frac{p^2}{2M_{\text{He}}}$, the kinetic energy of the $\alpha$ particle, we get:

$$\begin{aligned} Q &= E\left(1 + \frac{M_{\text{He}}}{M_D}\right) \\ \frac{4.08\,\text{MeV}}{\left(1 + \frac{4}{232-4}\right)} &= E \\ 4.0097\,\text{MeV} &= E \end{aligned}$$

# Problem 11.34

$^{80}$Br can undergo all three types of $\beta$ decay.

## Part a

Write down the decay equation in each case.

For $\beta$ decay, we have:

$$\begin{aligned} \beta^-\text{decay:}\,^{80}_{35}\text{Br} &\rightarrow\,^{80}_{36}\text{Kr} + \beta^- + \bar{\nu}_e \\ \beta^+\text{decay:}\,^{80}_{35}\text{Br} &\rightarrow\,^{80}_{34}\text{Se} + \beta^+ + \nu_e \\ \text{electron capture:}\,^{80}_{35}\text{Br} &\rightarrow\,^{80}_{34}\text{Se} + \nu_e \end{aligned}$$

## Part b

Compute the decay energy for each case.

For $\beta^-$decay, the decay energy is given by $\frac{Q}{c^2} = M_P - M_D$, where $M_P$ is the mass of the parent particle and $M_D$ is the mass of the daughter particle. So, in this case we have $Q/c^2 = 79.918528\,\text{u} - 79.916377\,\text{u} = 0.002151\,\text{u} = 2.004\,\text{MeV}/c^2$.

For $\beta^+$ decay, the decay energy is given by $\frac{Q}{c^2} = M_P - (M_D + 2m_e)$. So, we have $Q/c^2 = 79.918528\,\text{u} - (79.916519\,\text{u} - 2m_e) = 0.003106\,\text{u} = 2.893\,\text{MeV}/c^2$

Finally, for electron capture, the decay energy is calculated in the same way as for $\beta^-$, which gives $Q/c^2 = 79.918528\,\text{u} - 79.916519\,\text{u} = 0.002009\,\text{u} = 1.871\,\text{MeV}/c^2$.

# Problem 11.51

Write three different reactions that could produce the products:

## Part a

$n +^{23}$ Na

$$\begin{aligned} \alpha + {}^{20}\mathsf{F} \rightarrow {}^{23}\mathsf{Na} + n + Q \\ d + {}^{22}\mathsf{Ne} \rightarrow {}^{23}\mathsf{Na} + n + Q \\ {}^{11}\mathsf{B} + {}^{13}\mathsf{C} \rightarrow {}^{23}\mathsf{Na} + n + Q \end{aligned}$$

In each case, $Q = (m_x + m_X - m_y - m_Y)\, c^2$, so we get for the first reaction:

$$\begin{aligned} Q &= \left(m_\alpha + m_{\mathsf{Fl}} - m_n - m_{\mathsf{Na}}\right) c^2 \\ &= (4.00150617\,\mathsf{u} + 19.999982\,\mathsf{u} - 1.008665\,\mathsf{u} - 22.989767\,\mathsf{u})\, c^2 \\ &= 2.847\,\mathsf{MeV} \end{aligned}$$

For the second reaction:

$$\begin{aligned} Q &= \left(m_d + m_{\mathsf{Ne}} - m_n - m_{\mathsf{Na}}\right) c^2 \\ &= (2.01355321270\,\mathsf{u} + 21.991383\,\mathsf{u} - 1.008665\,\mathsf{u} - 22.989767\,\mathsf{u})\, c^2 \\ &= 6.059\,\mathsf{MeV} \end{aligned}$$

For the third reaction, we get:

$$\begin{aligned} Q &= \left(m_{\mathsf{B}} + m_{\mathsf{C}} - m_n - m_{\mathsf{Na}}\right) c^2 \\ &= (11.009305\,\mathsf{u} + 13.003355\,\mathsf{u} - 1.008665\,\mathsf{u} - 22.989767\,\mathsf{u})\, c^2 \\ &= 13.25\,\mathsf{MeV} \end{aligned}$$

So, the final reactions are:

$$\begin{aligned} \alpha + {}^{20}\mathsf{Fl} \rightarrow {}^{23}\mathsf{Na} + n + 2.847\,\mathsf{MeV} \\ d + {}^{22}\mathsf{Ne} \rightarrow {}^{23}\mathsf{Na} + n + 6.059\,\mathsf{MeV} \\ {}^{11}\mathsf{B} + {}^{13}\mathsf{C} \rightarrow {}^{23}\mathsf{Na} + n + 13.25\,\mathsf{MeV} \end{aligned}$$

## Part b

$p + {}^{14}\mathsf{C}$

$$\begin{aligned} \alpha + {}^{11}\mathsf{B} \rightarrow {}^{14}\mathsf{C} + p + Q \\ d + {}^{13}\mathsf{C} \rightarrow {}^{14}\mathsf{C} + p + Q \\ {}^{6}\mathsf{Li} + {}^{9}\mathsf{Be} \rightarrow {}^{14}\mathsf{C} + p + Q \end{aligned}$$

In each case, $Q = (m_x + m_X - m_y - m_Y)\, c^2$, so we get for the first reaction:

$$\begin{aligned} Q &= \left(m_\alpha + m_{\mathsf{B}} - m_p - m_{\mathsf{C}}\right) c^2 \\ &= (4.00150617\,\mathsf{u} + 11.009305\,\mathsf{u} - 1.00727646688\,\mathsf{u} - 14.003242\,\mathsf{u})\, c^2 \\ &= 0.2727\,\mathsf{MeV} \end{aligned}$$

For the second reaction:

$$\begin{aligned} Q &= \left(m_d + m_{\mathsf{C}} - m_p - m_{\mathsf{C}}\right) c^2 \\ &= (2.01355321270\,\mathsf{u} + 13.003355\,\mathsf{u} - 1.00727646688\,\mathsf{u} - 14.003242\,\mathsf{u})\, c^2 \\ &= 5.952\,\mathsf{MeV} \end{aligned}$$

For the third reaction, we get:

$$\begin{aligned} Q &= \left(m_{\mathsf{Li}} + m_{\mathsf{Be}} - m_n - m_{\mathsf{Na}}\right) c^2 \\ &= (6.015121\,\mathsf{u} + 9.012174\,\mathsf{u} - 1.00727646688\,\mathsf{u} - 14.003242\,\mathsf{u})\, c^2 \\ &= 15.63\,\mathsf{MeV} \end{aligned}$$

So, the final reactions are:

$$\begin{aligned} \alpha + {}^{11}\mathsf{B} &\rightarrow {}^{14}\mathsf{C} + p + 0.2727\,\mathsf{MeV} \\ d + {}^{13}\mathsf{C} &\rightarrow {}^{14}\mathsf{C} + p + \;\; 5.952\,\mathsf{MeV} \\ {}^{6}\mathsf{Li} + {}^{9}\mathsf{Be} &\rightarrow {}^{14}\mathsf{C} + p + \;\; 15.63\,\mathsf{MeV} \end{aligned}$$

## Part c

$d + {}^{31}\mathsf{P}$

$$\begin{aligned} \alpha + {}^{29}\mathsf{Si} &\rightarrow {}^{31}\mathsf{P} + d + Q \\ {}^{21}\mathsf{Ne} + {}^{12}\mathsf{C} &\rightarrow {}^{31}\mathsf{P} + d + Q \\ {}^{17}\mathsf{O} + {}^{16}\mathsf{O} &\rightarrow {}^{31}\mathsf{P} + d + Q \end{aligned}$$

In each case, $Q = (m_x + m_X - m_y - m_Y)\, c^2$, so we get for the first reaction:

$$\begin{aligned} Q &= \left(m_\alpha + m_{\mathsf{Si}} - m_d - m_{\mathsf{P}}\right) c^2 \\ &= (4.00150617\,\mathsf{u} + 28.976495\,\mathsf{u} - 2.01355321270\,\mathsf{u} - 30.973762\,\mathsf{u})\, c^2 \\ &= -8.676\,\mathsf{MeV} \end{aligned}$$

For the second reaction:

$$\begin{aligned} Q &= \left(m_{\mathsf{Ne}} + m_{\mathsf{C}} - m_d - m_{\mathsf{P}}\right) c^2 \\ &= (20.993841\,\mathsf{u} + 12\,\mathsf{u} - 2.01355321270\,\mathsf{u} - 30.973762\,\mathsf{u})\, c^2 \\ &= 6.079\,\mathsf{MeV} \end{aligned}$$

For the third reaction, we get:

$$\begin{aligned} Q &= \left(m_{{}^{17}\mathsf{O}} + m_{{}^{15}\mathsf{O}} - m_d - m_{\mathsf{P}}\right) c^2 \\ &= (16.999132\,\mathsf{u} + 15.994915\,\mathsf{u} - 2.01355321270\,\mathsf{u} - 30.973762\,\mathsf{u})\, c^2 \\ &= 6.271\,\mathsf{MeV} \end{aligned}$$

So, the final reactions are:

$$\begin{aligned} \alpha + {}^{29}\mathsf{Si} &\rightarrow {}^{31}\mathsf{P} + d - 8.676\,\mathsf{MeV} \\ {}^{21}\mathsf{Ne} + {}^{12}\mathsf{C} &\rightarrow {}^{31}\mathsf{P} + d + 6.079\,\mathsf{MeV} \\ {}^{17}\mathsf{O} + {}^{16}\mathsf{O} &\rightarrow {}^{31}\mathsf{P} + d + 6.271\,\mathsf{MeV} \end{aligned}$$