1 Power Requirements of the Spin Flipper

Magnetic Field The spin flipper will be placed in a uniform static magnetic field pointing along the axis of the spin flipper. If this is the z-axis then the external field will be

$$\mathbf{B}_o = B_o\hat{\boldsymbol{z}} = 10\,\text{Gauss} \tag{1.1}$$

From the particle data booklet the neutron has a magnetic moment

$$\mu = g_n\mu_n = -1.9130427\mu_n \tag{1.2}$$

where g_n is the spin g-factor and where μ_n is the nuclear magneton given by

$$\mu_n = e\hbar/2m_p = 5.050783 \times 10^{-27}\text{J/T} = 3.152451258 \times 10^{-14}\text{MeV/T} \tag{1.3}$$

To determine the gyromagnetic ratio for the neutron begin with the general equation for the energy of a dipole in a magnetic field

$$E = -\boldsymbol{\mu} \cdot \mathbf{B} \tag{1.4}$$

with

$$\boldsymbol{\mu} = g_n \frac{e}{m_p}\mathbf{S} \tag{1.5}$$

then a scalar equation follows from

$$\omega_L = g_n \frac{e}{m_p} B \equiv \gamma_n B \tag{1.6}$$

where ω_L is the Larmor frequency and γ_n is the gyromagnetic ratio given by

$$\gamma_n = 1.83247165 \times 10^8 \frac{1}{sT} \tag{1.7}$$

In the 10 Gauss external field the Larmor precession frequency will be

$$\nu_L = 29.164692\,\text{kHz} \tag{1.8}$$

Neutrons entering the spin flipper will initially be characterized by a helicty of +1. This means they will be spin-polarizd along the direction of motion. Each spin will process around the transverse magnetic field inside the spin flipper at an appropriate rate for the spin to emerge on the other side of the flipper with a helicity −1.

The flux of neutrons through the spin flipper is composed of a distribution of de Broglie wavelengths in the approximate range $3\AA \leq \lambda \leq 6\AA$. For definiteness suppose a specific neutron has a de Broglie wavelength of $\lambda = 5\AA$. The mass of a neutron has a value

$$m_n = 1.67492729 \times 10^{-27}\text{kg} \tag{1.9}$$

This implies a velocity

$$v_n = 791.2068 \text{ m/s} \tag{1.10}$$

If the spin flipper has a length of 16 inches (40.64 cm), then the total time the neutron will be exposed to the field B_{rf} will be $\delta t \sim 5.1364 \times 10^{-4}$ seconds. Based on this time, the angular frequency (rate of neutron flip) must be

$$\omega_F = \pi/\delta t = 6.11626 \times 10^3 \,\text{rads/sec} \tag{1.11}$$

The value of the RF field is easily determined by the relation $\omega_F = \gamma_n B_{rf}$ but since the spin flippper is characterize by a field which will oscillate at frequency ω_L (i.e not constant and not rotating at this frequency) then the the appropriate equation for the magneitc field will be

$$\boxed{B_{rf}(t) = \frac{2\omega_F}{\gamma_n} \cdot e^{i\omega_L t} \sim 0.66754\, G \cdot e^{i\omega_L t}} \tag{1.12}$$

1.1 Components of the RCL Circuit

Power requirements for the spin flipper are related to the calculated impedance of the circuit. The essential idea is to drive the circuit at a frequency equal to the Larmor frequency of the neutron ω_L. Minimum power requiremnts are obtained by adding an appropriate capacitor in series with the spin flipper such that the Larmor frequency becomes the resonant frequency of the circuit.

Resistance: The inner cylinder of the spin flipper will be wrapped with solid aluminum wire while the outer cylinder will be wrapped with solid Copper wire. Resistance values per foot for each is given by

$$\rho_{cu} \sim 0.0066\,\Omega/ft \qquad \text{and} \qquad \rho_{al} \sim 0.0105\,\Omega/ft \tag{1.13}$$

The total resistance of the circuit can be written as

$$R_{tot} = \rho_{cu} \cdot \ell_{cu} + \rho_{al} \cdot \ell_{al} \tag{1.14}$$

where ℓ_{cu} is the total length of copper wire and ℓ_{al} is the total length of aluminum wire. These legnths can be calculated as follows:

$$\begin{aligned} \ell_{al}(surface) &= 64\,\text{wires} \cdot 16\,\text{in./wire} = 1,024\,\text{in.} \\ \ell_{al}(endcaps) &= 2\,\text{endcaps} \cdot 314.7\,\text{in.} = 629.4\,\text{in.} \\ \ell_{al} &= 137.78\ ft \end{aligned}$$

For the copper wire

$$\begin{aligned} \ell_{cu}(innersurface) &= 64 \times 16 \times 4 = 4,096\,\text{in.} \\ \ell_{cu}(outersurface) &= 62 \times 16 \times 4 = 3968\,\text{in.} \\ \ell_{cu}(endcaps) &= 8 \times 116\text{in./quadrant} = 928\,\text{in.} \\ \ell_{cu} &= 749.33\ ft \end{aligned}$$

With this data the total resistance of the circuit is calculated to be

$$R_{tot} = 6.3923\ \Omega \tag{1.15}$$

This is a theoretical estimate and does not take into account additional resistance from two solder joints which connect the individual Al and Cu wires.

Inductance: There are two ways to calculate the inductance of the spin fliper. A theoretical calculation follows by integrating the magnetic field over the cylinder volume. Another method is to add the flux through each each individual wire loop. Each calculation produces a result which is within a few percent of the other result. To obtain a value for the inductance it is first necessary to calculate the magnitude of the current $I(t)$ in the circuit.

The current supplied to the spin flipper is ultimately determined by the strength of the magnetic field required inside the inner cylinder to rotate the neutron spins. The amplitude of the field has already been calculated in equation (1.12) to be $B_{rf} = 0.66754$ G. The auxillary field is therefore

$$H_{rf} = B_{rf}/\mu_o = 53.12115\, A/m \tag{1.16}$$

The current in the individual wires can be determined from several of the currents given in Table 1 by dividing out the appropriate number of wires. However, a more fundamental calculation uses the spacing Δx_{in} between the individual wires which run along the endcaps of the inner cylinder:

$$I = H_{rf} \cdot \Delta x_{in} = (53.12115\, A/m) \cdot (0.009922\, m) = 0.527061\, mA \tag{1.17}$$

First Calculation: The total magnetic field energy in the coil in each of two regions is given by

$$E = \frac{\mu_o}{2} \int |\boldsymbol{H}|^2 dv \tag{1.18}$$

In the region $r \leq R_{in}$ the answer is almost trivial since the field is constant. One finds

$$E_{in} = \frac{\mu_o}{2} H_{rf}^2 \pi R_{in}^2 z_o = \frac{1}{2} L_{in} I^2 \tag{1.19}$$

implying

$$L_{in} = \frac{\mu_o \pi R_{in}^2 H_{rf}^2 z_o}{I^2} = 0.41073 \text{ mH} \tag{1.20}$$

In the region $R_{in} < r < R_{out}$ the integral is more complicated. The integral is

$$E_{out} = \frac{\mu_o}{2} \int_{R_{in}}^{R_{out}} \int_0^{2\pi} |\boldsymbol{H}_{out}|^2 dv \tag{1.21}$$

with the result:

$$E_{out} = \frac{\mu_o \pi H_{rf} z_o}{2} \left[\frac{R_{in}^2}{R_{out}^2 - R_{in}^2} \right]^2 \cdot \left[\frac{R_{out}^4}{R_{in}^2} - R_{in}^2 \right] = \frac{1}{2} L_{out} I^2 \tag{1.22}$$

Solving for L_{out} and writing the result in terms of L_{in} gives

$$L_{out} = 4 \cdot L_{in} = 1.64291 \text{ mH} \tag{1.23}$$

The total inductance is therefore

$$L = 2.05365 \text{ mH} \tag{1.24}$$

Second Calculation: The inductance of the inner cylinder can be determined by calculating the flux Φ_i through each individual aluminum wire loops and then summing over loops:

$$\Phi = \sum_{i=1}^{32} \Phi_i = L_{in} I \tag{1.25}$$

The flux Φ_i is given by the surface integral

$$\Phi_i = \int_s \mathbf{B}_{rf} \cdot d\mathbf{S} \tag{1.26}$$

but the magnetic field is constant in this region and points in the same direction as $d\mathbf{S}$, so

$$\Phi_i = B_{rf} \cdot S_i = \mu_o H_{rf} \cdot S_i \tag{1.27}$$

The inductance is therefore reduced to calculating the total area enclosed by the individual aluminum wire loops. Using $S = 5035.12\ in^2$ gives

$$L_{in} = \frac{\mu_o H_{rf} S}{I} = 0.41143 \text{ mH} \tag{1.28}$$

To calculate the inductance of the outer cylinder loops it is only necessary to note that the total flux through the top half of the inner cylinder runs entirely through the top half of the outer cylinder. The only difference is that this flux traverses four times as many wire loops (128 instead of 32). This means that $L_{out} = 4 \cdot L_{in}$ as before. the total inductance is therefore

$$L = 2.05714 \text{ mH} \tag{1.29}$$

and differs from the previous result by $\Delta = .17$ %.

Capacitance: The spin flipper by itself has a neglible capacitance. If the device is to operate at the Larmor frequency ω_L an external capacitor must be added in series. The value of the capacitance is easily determined from the resonance condition

$$\omega_L^2 = 1/LC \tag{1.30}$$

which implies a capacitance of

$$C = 14.501 \text{ nF} \tag{1.31}$$

1.2 RFSF as an RCL Circuit

The RFSF connected in series with an external capacitor behaves as a classsic RCL circuit. With an applied external voltage $V(t) = V_o e^{i\omega_L t}$ the steady differential equation can be written in terms of the charge $Q(t)$ on the capacitor:

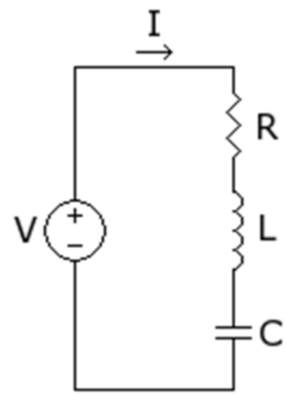

Figure 1: RCL Circuit

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V_o e^{i\omega_L t} \tag{1.32}$$

The general solution is

$$Q(t) = \frac{V_o/\omega}{\left[R^2 + (\omega L - \frac{1}{\omega C})^2\right]^{1/2}} e^{i(\omega_L t + \phi)} \tag{1.33}$$

where the phase is given by

$$\phi = \arctan\left[\frac{R}{\omega L - \frac{1}{\omega C}}\right] \tag{1.34}$$

The current follows by differentiating (1.33). Its magnitude is

$$I(t) = \frac{V_o}{\left[R^2 + (\omega L - \frac{1}{\omega C})^2\right]^{1/2}} \cdot \sin(\omega t + \phi) \tag{1.35}$$

and the average power supplied to the circuit over one cycle is $P_{avg} = \frac{1}{2}I(\omega)^2 R$ or

$$P_{avg} = \frac{V_o^2 R/2}{R^2 + (\omega L - \frac{1}{\omega C})^2} \tag{1.36}$$

If the spin flipper is operated at resonance then $V_o = IR = 3.369$ Volts. and

$$P_{avg} = \frac{V_o^2}{2R} = .8879\,\text{Watts} \tag{1.37}$$

Q-Value An important indicator for the RFSF is its Q-value which is defined by the relation:

$$Q = \omega_L \cdot \frac{\text{stored energy in the RFSF}}{\text{average power supplied at resonance}} \tag{1.38}$$

Using the relations

$$E = \frac{1}{2} L I^2 \qquad\qquad P_{avg} = \tfrac{1}{2} I^2 R$$

then the Q-value follows as

$$Q = \frac{\omega_L L}{R} = 58.872 \tag{1.39}$$

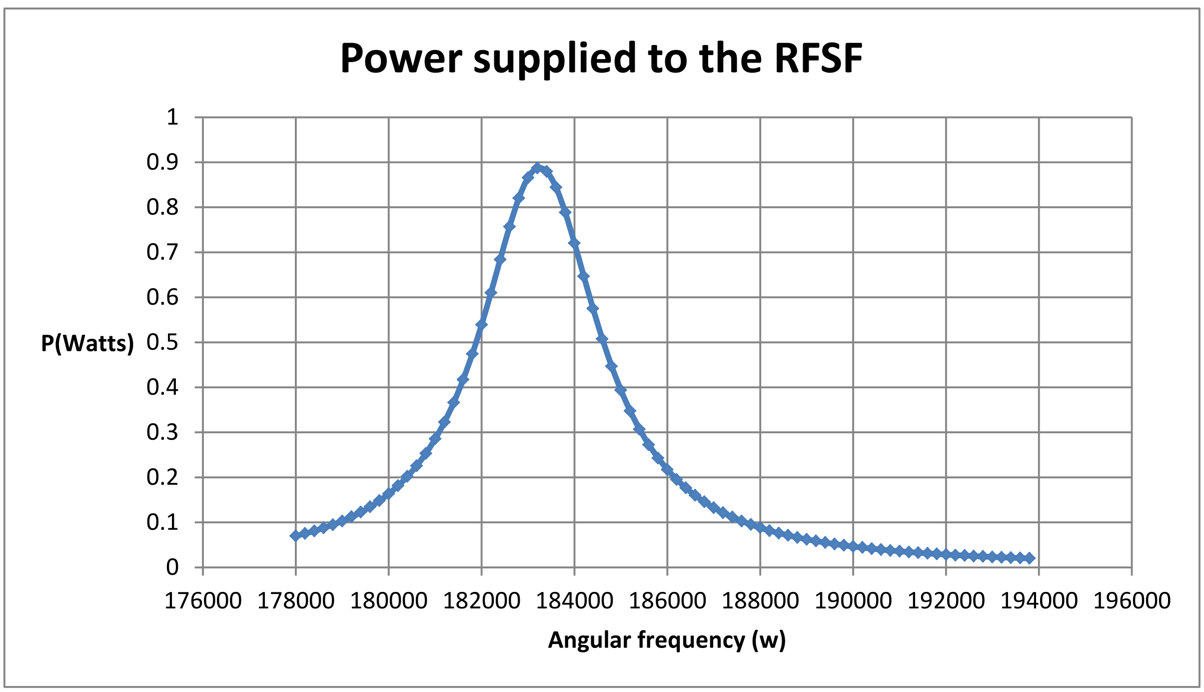

Figure 2: Power vs Frequency