Optimization of the n-^{3}He PV Experiment

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A ballistic neutron transport simulation is being performed to optimize the measurement of hadronic parity violation in the reaction n + ^{3}He $\rightarrow$ p + t. The simulation will be used for the design of a combined ^{3}He scattering target and drift chamber to detect the recoil proton and triton.

1 Design Considerations

The goals of this simulation are to determine:

- the ionization response in each wire plane
- the sensitivity to the asymmetry in each wire plane
- the statistical error and correlations of the signals in each wire plane
- the requirements in digitizing the output signal
- the effective statistics $\delta^{-2}A = \beta N$ of the helicity asymmetry
- the feasibility of measuring a detector asymmetry to cancel beam fluctuations

The simulation has two main parts: simulating the neutron intensity and phase space at the end of the guide, and simulation the reaction and detection of ions in the ^{3}He chamber.

2 McStas Simulation

The present simulation is based on the output ntuple of the simulation reported in [1]. This ntuple was used as the event generator. The ntuple is normalized so that integral of the variable p_6 over the entire ntuple represents the number of neutrons exiting the supermirror bender polarizer during one pulse of the proton beam of power 2 MW. The neutron phase space at this point is specified by the variables $(\boldsymbol{x}, \boldsymbol{v}) = (x_6, y_6,\ v_{x6}, v_{y6}, v_{z6})$, with the weight p_6. Cuts are placed on the events to simulate the choppers ($C_{1,2}$) and collimator (C_{col}) in the guide, as described in [2], to generate the number N of neutrons out the end of the guide per 1.4 MW pulse:

$$N = \sum_{\boldsymbol{x},\boldsymbol{v}} n, \qquad \text{where} \qquad n = p_6 C_1 C_2 C_{col}(1.4/2.0) \tag{1}$$

is the weighted number from each event in the ntuple, and $\sum_{\boldsymbol{x},\boldsymbol{v}}$ is summed over events in the ntuple (Monte Carlo integral over neutron phase space).

For each ntuple event with phase $(\boldsymbol{x}, \boldsymbol{v})$, the detector response is integrated over the position z of the interaction vertex and the angle $\alpha = \cos(\theta)$ of the recoil proton. This introduces an additional weight factor

$$w_{z\alpha} = e^{-\rho\sigma z} \rho\sigma dz \cdot \tfrac{1}{2} d\alpha, \tag{2}$$

where ρ is the ^{3}He density, σ is the n + ^{3}He $\rightarrow$ t + p total cross section, and z is the distance from the entrance to the ion chamber.

For each vertex (z, α), we calculate the ionization distribution due to the proton and triton recoil energy using $\beta_{p,t}(x) = dn_{ion}/dx$, the ion density as a function of the distance the proton or triton has traveled. Since proton and triton ion tracks are indistinguishable except for their range, we add the two functions. The triton recoils opposite to the proton since the neutron has negligible energy. Changing coordinates and integrating over each ion chamber wire plane (detector) i bounded by $z_i < z' < z_{i+1}$, the ionization response for a single event is

$$\beta_i(z, \alpha) = \int_{z_i}^{z_{i+1}} dz' \left(\beta_p \left(\frac{z' - z}{\alpha} \right) + \beta_t \left(\frac{z' - z}{-\alpha} \right) \right), \tag{3}$$

corresponding to

$$n_i = \beta_i(z, \alpha)\, w_{z\alpha}\, n \tag{4}$$

ions detected in the wire plane i for that event. In the code, the integration is sampled over the entire range of $\beta_{t,p}$, adding ions to the corresponding wire plane i at each step along the track.

To extract the information described in the introduction, we need to tabulate the following variables over the ntuple (and vertex):

$$\begin{aligned}
N &\equiv \sum_{\boldsymbol{x},\boldsymbol{v}} n, && (5)\\
P^2 N \equiv F &\equiv \sum_{\boldsymbol{x},\boldsymbol{v}} p^2 n = \langle p^2 \rangle\, N, && (6)\\
\beta_i N \equiv N_i &\equiv \sum_{\boldsymbol{x},\boldsymbol{v},z,\alpha} n_i = \langle \beta_i(z, \alpha) \rangle\, N, && (7)\\
\alpha_i \beta_i N \equiv M_i &\equiv \sum_{\boldsymbol{x},\boldsymbol{v},z,\alpha} \alpha\, n_i = \langle \alpha \beta_i(z, \alpha) \rangle\, N, && (8)
\end{aligned}$$

where P is the neutron polarization, β_i and α_i are the ion yield normalized to the neutron flux and sensitivity to the physics asymmetry, respectively, in the wire plane i of the ion chamber. $\langle x \rangle$ is the weighted average of x over $\sum_{\boldsymbol{x},\boldsymbol{v},z,\alpha} w_{z\alpha} n$. We also tabulate the statistical uncertainty which requires a full covariance matrix, since each neutron reaction leaves an ion track spanning multiple wire planes. We assume that the most significant statistical error comes from the event shot noise, not ionization fluctuations. In this case the covariance matrix is

$$\delta^2 N_{ij} = \sum_{\boldsymbol{x},\boldsymbol{v},z,\alpha} \delta^2 n_{ij} = \sum_{\boldsymbol{x},\boldsymbol{v},z,\alpha} \frac{n_i n_j}{w_{z\alpha} n} = \langle \beta_i \beta_j \rangle N. \tag{9}$$

3 Helicity Asymmetry

For a single event $(\boldsymbol{x}, \boldsymbol{v}, z, \alpha)$, the helicity-dependent probability of scattering a proton into the angle $\alpha = \cos\theta$ is

$$n^{\pm} = n(1 \pm PA\alpha), \tag{10}$$

where $\pm P$ labels the spin polarization of the neutron and A is the physics asymmetry to be measured. Integrated over a neutron pulse, the helicity-dependent detector response is then

$$N_i^{\pm} = N_i(1 \pm PA\alpha_i), \tag{11}$$

with the covariance matrix $\delta^2 N_{ij}$ after summing over both helicity states. From this we extract individual detector helicity asymmetries,

$$A_i = \frac{N_i^+ - N_i^-}{N_i^+ + N_i^-} = PA\alpha_i. \tag{12}$$

The covariance matrix of detector asymmetries A_i is

$$\delta^2 A_{ij} = \frac{\delta^2 N_{ij}}{N_i N_j} = \frac{\langle \beta_i \beta_j \rangle}{\beta_i \beta_j N}. \tag{13}$$

Let the extracted physics asymmetry be $A_i' = A_i / P\alpha_i$. The average extracted asymmetry over all detectors must be covariantly weighted by the matrix w:

$$w_{ij} = \delta^{-2} A_{ij}' = P^2 \alpha_i \alpha_j \delta^{-2} A_{ij}, \tag{14}$$

$$A' \equiv \langle A_i' \rangle_w \equiv \frac{\sum_{ij} w_{ij} A_i'}{\sum_{ij} w_{ij}} = \frac{\sum_{ij} P\ \alpha_i A_j \delta^{-2} A_{ij}}{\sum_{ij} P^2 \alpha_i \alpha_j \delta^{-2} A_{ij}}. \tag{15}$$

The figure of merit equals the denominator, a measure of the effective statistics:

$$\delta^{-2} A' \ \equiv\ \sum_{ij} \delta^{-2} A_{ij}' \ =\ \sum_{ij} w_{ij} \ =\ \sum_{ij} P^2 M_i M_j \delta^{-2} N_{ij} \ \equiv\ \alpha P^2 N, \tag{16}$$

and α is an 'efficiency of statistics'. Note that $\alpha \leq \langle \alpha^2 \rangle = \frac{1}{3}$, because reactions with the proton emitted at 90° do not contribute to a measurement of the asymmetry. For complete correlation it degenerates into an overdetermined linear system, because each detector carries the same information. This corresponds to null eigenvalues in the matrix $\delta^2 N_{ij}$.

4 Detector Asymmetry

The last question we consider is whether it is feasible to measure a detector asymmetry separately for each neutron spin state, so that we do not need to normalize by the beam flux while making the asymmetry measurement. This is a question of how much of the backward scattering we are able to detect, or the leverage between detector sensitivities α_i. The perfect experiment would have $\alpha_1 = 1$, $\alpha_2 = -1$. For a given helicity, we wish to extract both N and $\Delta \equiv NA$ from the individual rates $N_i^{\pm}$, where the sign is fixed for each pulse. Given the response β_i and sensitivity α_i coefficients simulated above, we extract these from a covariant least squares fit to the form

$$N_i^{\pm} \approx \beta_i N \pm P\alpha_i\beta_i\Delta \qquad \text{or} \qquad \mathbf{N} \approx \mathbf{B}\mathbf{a}, \tag{17}$$

where

$$\mathbf{N} = \begin{pmatrix} N_0^{\pm} \\ N_1^{\pm} \\ \vdots \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} \beta_0 & \pm P\alpha_0\beta_0 \\ \beta_1 & \pm P\alpha_1\beta_1 \\ \vdots & \vdots \end{pmatrix}, \qquad \mathbf{a} = \begin{pmatrix} N \\ \Delta \end{pmatrix}, \tag{18}$$

using the formula

$$\chi^2 = \tfrac{1}{2}(\mathbf{N} - \mathbf{B}\mathbf{a})^T \delta^{-2}\mathbf{N}\,(\mathbf{N} - \mathbf{B}\mathbf{a}), \tag{19}$$

$$\nabla_a\chi^2 = \mathbf{B}^T \delta^{-2}\mathbf{N}\,(\mathbf{B}\mathbf{a} - \mathbf{N}) = 0, \tag{20}$$

where $\delta^{-2}\mathbf{N}$ acts as a least squares metric. The solution is

$$\mathbf{a} = \mathbf{B}^{-1}\mathbf{N} \equiv (\delta^2\mathbf{a}\,\mathbf{B}^T\delta^{-2}\mathbf{N})\,\mathbf{N}, \qquad \text{where} \qquad \delta^2\mathbf{a} = (\mathbf{B}^T\delta^{-2}\mathbf{N}\,\mathbf{B})^{-1} \tag{21}$$

is the covariance matrix. This is used to calculate the error in $A' = \Delta/N$ and thus the effective statistics $\delta^{-2}A' = \beta P^2 N$. A comparison of β from spin-asymmetries and detector-asymmetries will follow from the finished MC simulation.

References

[1] C. B. Crawford, unpublished technical note, "McStas Bender Optimization for the NPDG cold line", 2007-10-04. http://sns.phys.utk.edu/svn/npdg/trunk/simulations/sns/choppers/tex/pol_bender.pdf

[2] C. B. Crawford, R. Mahurin, unpublished technical note, "Opening angles for the SNS FnPB choppers", 2006-11-30. http://sns.phys.utk.edu/svn/npdg/trunk/simulations/sns/choppers/tex/choppers.pdf