Physics 213: General Physics

Fall 2004

9:30 AM Lecture

Midterm I Solutions

Tuesday, September 21, 2004

Chem-Phys 153

Name (print): ____________________

Signature: ____________________

Student Number: ____________________

Your Seat Number (on back of chair): ____________________

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1. Immediately enter the requested information on this cover page. Do not turn this cover page over until you are told to do so.

2. This is a closed book exam. I have provided a list of necessary formulas at the back. You may use a calculator if you wish. Do not use any scratch paper. Use the blank backs of pages if you need to. Do not consult with your classmates nor look at their papers, THIS IS CHEATING.

   - Show ALL of your work, including diagrams and equations, and place a box around each $\boxed{\text{final answer.}}$ Problems will be graded for orderliness, completeness, and coherence as well as for correctness. Justify the use of any formulas you use. Partial credit will be awarded. A correct numerical final answer with no intermediate steps shown will *not* be given full credit.

Do not write below this line

Problems	Your grade	Maximum possible
A		30
B		40
C		30
Total		100

A. [30 points; **show the details of your calculations**; put a box around each final $\boxed{\text{answer}}$.]

A conducting sphere of radius $R_1 = 1cm$ is surrounded by a concentric conducting shell. The inner and outer radii of the shell are $R_2 = 8cm$ and $R_3 = 10cm$. The sphere contains a total charge of $Q_1 = -9nC$, while the outer shell contains a total charge of $Q_2 = +25nC$. See figure shown.

1. [6 points] Part of Q_2 will be on the inner surface of the shell, call it Q_2'. Part of it will be on the outer surface, call it Q_2''. Find both Q_2' and Q_2''. **Explain your logic clearly**.

   Answer:

   By charge conservation we know that

   $Q_2' + Q_2'' = Q_2$ (2 points)

   Since there cannot be an electric field inside the metal in equilibrium, all the field lines ending on the negative charge Q_1 must start from the charge on the inner surface of the shell. This means that

   $Q_2' = -Q_1 = +9nC$ (2 points)

   Therefore

   $Q_2'' = Q_2 - Q_2' = Q_1 + Q_2 = 16nC$ (2 points)

2. [6 points] Find the magnitude of the electric field at a distance $r = 10.5cm$ from the center of the sphere. Which direction is it pointing?

Answer:

Since r is outside all the shells we have to use the outside formula for the field for all three charges Q_1, Q_2', and Q_2''.

$|\vec{E}(r)| = |\frac{kQ_1}{r^2} + \frac{kQ_2'}{r^2} + \frac{kQ_2''}{r^2}|$

Since $Q_1 + Q_2' = 0$ the first two terms cancel, and we have

$|\vec{E}(r)| = \frac{k|Q_2''|}{r^2} = 1.306 \times 10^4 N/C$ (4 points)

Since Q_2'' is positive the field lines point radially outwards.
(2 points)

3. [6 points] Find the electric potential at $r = 10.5cm$ from the center of the sphere.

Answer:

Use the outside formula for the potential due to all three shells

$V(r) = \frac{kQ_1}{r} + \frac{kQ_2'}{r} + \frac{kQ_2''}{r} = \frac{kQ_2''}{r} = 1.371 \times 10^3 Volts$ (6 points)

4. [6 points] Find the electric field at a distance $r = 4.5cm$ from the center of the sphere. Which direction is it pointing?

Answer:

Since r is now inside the the outer shells, only the inmost shell with charge Q_1 contributes to the field. Thus

$|\vec{E}(r)| = \frac{k|Q_1|}{r^2} = 4 \times 10^4 N/C$ (4 points)

Since Q_1 is negative, the field points radially inwards.

(2 points)

5. [6 points] Find the electric potential at a distance $r = 4.5cm$ from the center of the sphere.

Answer:

We must now use the inside formula for the shells with Q_2' and Q_2'', but use the outside formula for the shell with Q_1. Thus

$V(r) = \frac{kQ_1}{r} + \frac{kQ_2'}{R_2} + \frac{kQ_2''}{R_3} = 652.5Volts$ (6 points)

B. [40 points; **show the details of your calculations**; put a box around each final $\boxed{\text{answer}}$.]

This is problem 22 of Chapter 17, one of your practice problems. In the Bohr model of the hydrogen atom, an electron (charge $-e = -1.6 \times 10^{-19}C$) orbits a proton (the nucleus with charge $+e = +1.6 \times 10^{-19}C$) in a circular orbit of radius $r = 0.53 \times 10^{-10}m$. Assume that the proton is stationary, while the electron moves in a circular orbit at constant speed.

1. [10 points] What is the electric potential at the electron's orbit due to the proton?

   Answer: This is just the potential of a point charge with $Q = +e$ at a distance r from it

   $V(r) = \frac{ke}{r} = 27.17Volts$

   (10 points)

2. [10 points] What is the kinetic energy of the electron? (Hint: The force needed to keep the electron moving in uniform circular motion is the centripetal force mv^2/r).

Answer: The force needed to keep the electron moving in a circle has to come from the Coulombic attraction between the electron and the proton. Thus

$\frac{ke^2}{r^2} = \frac{mv^2}{r}$ (4 points)

$\Rightarrow \frac{1}{2}mv^2 = \frac{ke^2}{2r} = 2.1735 \times 10^{-18} Joules$ (6 points)

3. [10 points] What is the total energy of the electron in this orbit? (Hint: Add the kinetic energy you calculated above to the potential energy).

   Answer: Total energy is the sum of the potential energy and the kinetic energy. We just found the kinetic energy. The potential energy of the electron at point r is

   $PE(r) = -eV(r) = -\frac{ke^2}{r} = -4.347 \times 10^{-18} Joules$ (5 points)

   Adding the KE we find the total energy to be

   $TE = PE + KE = -2.1735 \times 10^{-18} Joules$ (5 points)

4. [10 points] What is the ionization energy – that is, the energy required to remove the electron from the atom and take it to infinity?

Answer: When the electron and proton are infinitely separated, their potential energy is zero, and if they are at rest the kinetic energy is zero as well. So the total energy when they are separated is 0. (5 points)

Thus, the energy needed to be given to the electron in orbit must be enough to raise its energy from the negative value it has to 0. So the ionization energy is

$2.1735 \times 10^{-18} Joules$ (5 points)

C. [30 points] This is problem 48, Chapter 17, one of your practice problems. A parallel plate capacitor is isolated with a charge $\pm Q$ on each plate. The initial separation of the plates is d.

1. [10 points] If the separation of the plates if halved and a dielectric (with constant K) is inserted in place of air, by what factor does the energy stored in the capacitor change? (Hint: Take it step by step, first halving the distance and then inserting the dielectric).

   Answer: The key to this question is that since the capacitor is electrically isolated, the charge on the plates cannot change. Let the area of the plates be A. Then the initial capacitance is

   $C = \frac{\varepsilon_0 A}{d}$

   and the initial energy stored is

   $U = \frac{Q^2}{2C}$

   In Step 1, we halve the distance, so that $d' = d/2$. The charge doesnt change, and the new capacitance is

   $C' = \frac{\varepsilon_0 A}{d/2} = 2C$ (2 points)

   which implies that

   $U' = U/2$ (3 points)

   In Step 2, we insert a dielectric of constant K between the plates. We know that the electric field falls to

   $E' = E/K$

   which means that the capacitance increases by a factor of K.

   $C'' = KC' = 2KC$ (2 points)

   Thus, since the charge is constant, the final energy stored must be

   $U'' = \frac{U}{2K}$ (3 points)

2. [10 points] To what do you attribute the change in stored potential energy? Where did the energy come from or go to?

   Answer: In Step 1, as the plates come closer together the electric field is doing positive work on the plates. To keep them from slamming together an external force must be applied to to negative work on the system, thus reducing its energy. (5 points)

   In Step 2, as the dielectric is inserted, since the energy is decreasing, the capacitor "wants" to suck in the dielectric. Once again, an external force must be applied to keep the dielectric from accelerating into the capacitor. This external force does negative work on the system, reducing its energy. (5 points)

3. [10 points] How does the new value of the electric field between the plates compare to the old value?

   Answer: Already part of (1)

   $E'' = E/K$ (10 points)

SOME RELEVANT FORMULAS

Electric Forces and Electric Fields

Coulomb's Law, says that the force on charge q_1 due to charge q_2 is

$$\vec{F}_{12} = \frac{kq_1q_2}{{r_{12}}^2}\hat{r}_{12} \tag{1}$$

where $\vec{r}_1$ and $\vec{r}_2$ are the vector positions of the charges, $r_{12} = |\vec{r}_1 - \vec{r}_2|$ is the distance between the two charges, and

$$\hat{r}_{12} = \frac{\vec{r}_1 - \vec{r}_2}{r_{12}} \tag{2}$$

The Coulomb force constant is $k = 8.98755 \times 10^9 Nm^2/C^2$. You may use either this value or the more approximate $k = 9 \times 10^9 Nm^2/C^2$.

To find the electric force of a configuration of charges on a given charge, use superposition to add the individual forces from each of the charges **vectorially**.

The electric field $\vec{E}(\vec{r})$ at position $\vec{r}$ is defined as the force on a $+1C$ test charge placed at $\vec{r}$. The force on an arbitrary charge q is $q\vec{E}$.

For a thin spherical layer of charge Q_{layer} and radius R_{layer}, the field outside the layer is the same as that due to a point charge of size Q_{layer} at the center of the sphere. The magnitude of the field outside the layer at a distance r from the center of the layer is

$$E(\vec{r}) = \frac{kQ_{layer}}{r^2} \tag{3}$$

The field inside the spherical layer is zero. When a number of spherical layers are present, simply superpose the fields from all the layers.

The electric field inside a conductor at equilibrium is zero. The field at the surface of a conductor with a surface charge density Q/A is

$$E_{surface} = \frac{Q}{A\epsilon_0} \tag{4}$$

where ϵ_0 is the permittivity of free space and has the value

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85419 \times 10^{-12} C^2/Nm^2 \tag{5}$$

Electric Potential

The electric force is conservative, and therefore has a potential energy PE as a function of position. The electric potential difference V_{ab} is the potential energy difference of a $+1C$ test charge between a and b. This is also the work done **by** the electric field in moving a $+1C$ charge from a to b.

$$V_{ab} = V_a - V_b = (PE_a - PE_b)/q \tag{6}$$

The potential difference between the plates of a parallel plate capacitor is

$$V_{ab} = Ed \tag{7}$$

where d is the distance between the plates.

The potential due to a point charge Q measured at a distance r from it is

$$V(r) = \frac{kQ}{r} \tag{8}$$

The potential due to a spherical layer of charge Q_{layer} and radius R_{layer} depends on whether the point where it is measured outside or inside the layer. Outside (for $r > R_{layer}$) it is the same as due to a point charge of Q_{layer} located at the origin of the layer

$$V(r) = \frac{kQ_{layer}}{r} \qquad ; r > R \tag{9}$$

Inside the layer, the potential is constant

$$V(r) = \frac{kQ_{layer}}{R_{layer}} \qquad ; r < R \tag{10}$$

If there are many spherical layers simply use superposition to find the total potential, being careful to check for each layer whether the given point $\vec{r}$ is inside or outside.

Capacitance

A capacitor is a device for storing charge, $+Q$ on one plate, and $-Q$ on the other. The charge Q is proportional to the potential difference V_{ab} between the plates, the constant of proportionality being the capacitance C:

$$Q = CV_{ab} \tag{11}$$

For a parallel plate capacitor of plate area A and distance between the plates d the capacitance is

$$C = \frac{\epsilon_0 A}{d} \tag{12}$$

When two capacitors which are charged to different potentials are connected together, the charge redistributes itself insuch a way that they end up having the same potential difference between their plates in the final state.

A dielectric of dielectric constant K, when inserted between the plates of a parallel plate capacitor, gets polarized and reduces the total field by a factor K. Consequently, the capacitance is

$$C = \frac{K\epsilon_0 A}{d} \tag{13}$$

Electric Current

The electric current I is the amount of charge crossing an imaginary surface in a conductor every second.

Ohm's law says that the voltage (difference of electric potential) across a resistor is proportional to the current flowing in it, with the constant of proportionality being the resistance

$$V = IR \tag{14}$$

The resistance of a wire of length L and cross-sectional area A is

$$R = \frac{\rho L}{A} \tag{15}$$

where ρ is the resistivity of the wire.

The power delivered, or energy delivered per unit time, to the resistor is

$$P = VI = I^2R \tag{16}$$