If medium is linear (i.e., response of system to an external applied $\vec{E}$ scales with $\vec{E}$), so no "ferroelectric" effects, (electric hysteresis) and assuming medium is isotropic (i.e., same everywhere): and independent of direction of external applied $\vec{E}$

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} \qquad \text{"induced polarization"}$$

$\chi_e$: "electric susceptibility"

$$\Rightarrow \vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E} = \varepsilon_0 (1+\chi_e) \vec{E} \equiv \varepsilon \vec{E}$$

$\varepsilon \equiv (1+\chi_e)\varepsilon_0$ : "electric permittivity"

$\frac{\varepsilon}{\varepsilon_0} \equiv$ "dielectric constant", or "relative electric permittivity"

$$\Rightarrow \vec{\nabla} \cdot \vec{D} = \vec{\nabla} \cdot \varepsilon \vec{E} = \varepsilon \vec{\nabla} \cdot \vec{E} = \rho \Rightarrow \vec{\nabla} \cdot \vec{E} = \rho/\varepsilon \;//$$

All problems in a medium with $\varepsilon$ are reduced to same problems as before just now with $\varepsilon_0 \to \varepsilon$. $\varepsilon$ accounts for macroscopic nature of the problem (i.e., response of medium to $\vec{E}$ field)

<u>At boundary between two different media:</u>

② $\varepsilon_2$ $\quad \uparrow \hat{n}$ : from ① to ②

① $\varepsilon_1$

$$\left\{ \begin{array}{l} \bullet\ (\vec{D}_2 - \vec{D}_1) \cdot \hat{n} = \sigma \\ \bullet\ (\vec{E}_2 - \vec{E}_1) \times \hat{n} = 0 \end{array} \right.$$

can be derived in same way as for $\vec{E}$ (see p. 13)

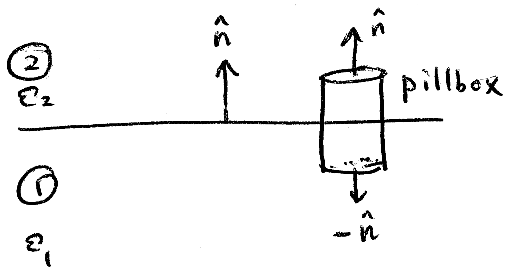
discontinuity in normal component of $\vec{D}$

tangential component of $\vec{E}$ continuous across an interface

• $\sigma$ = macroscopic surface charge density on surface at interface (not including any polarization charge)

$\Rightarrow$ solve boundary-value problems just as before, just using these boundary conditions on $\vec{D}_n$ and $\vec{E}_t$

What is
Polarization charge density?




$$\text{Recall we have:}\quad \vec{\nabla}\cdot\vec{E} = \overset{\text{macroscopic charge density}}{\frac{\rho}{\varepsilon_0}} - \overbrace{\frac{1}{\varepsilon_0}\,\vec{\nabla}\cdot\vec{P}}^{\text{polarization charge, } \rho_{pol}}$$

$$\equiv \frac{\rho}{\varepsilon_0} + \underbrace{\frac{\rho_{pol}}{\varepsilon_0}}_{\text{look at this over our infinitesimal pill box}} \quad ; \quad \rho_{pol} \equiv -\vec{\nabla}\cdot\vec{P}$$

$$\int_V d^3x\, \frac{\rho_{pol}}{\varepsilon_0} = \frac{-1}{\varepsilon_0}\int_V d^3x\, \vec{\nabla}\cdot\vec{P}$$

$$= -\frac{1}{\varepsilon_0}\int_S d\vec{a}\cdot\vec{P}$$

$$= -\frac{1}{\varepsilon_0}\left[\left(\vec{P}_2 - \vec{P}_1\right)\cdot\hat{n}\right]\cdot \quad da$$

$\Rightarrow$ if $\vec{P}_2 \neq \vec{P}_1$, i.e., if $\varepsilon_1 \neq \varepsilon_2$, then there is a polarization surface charge density of:

$$\underline{\underline{\sigma_{pol} = -\left(\vec{P}_2 - \vec{P}_1\right)\cdot\hat{n}}}$$

# MAGNETOSTATICS

Difference (major) between electrostatics and magnetostatics:

no free magnetic charges (monopoles)

**But!** see Blas Cabrera, Phys. Rev. Lett, 48, 1378 (1982),

single candidate magnetic monopole event! [no subsequent confirmation]

In Jackson: e.g., $\oint_c \vec{E}\cdot d\vec{l} = -\partial_t \oint_S \vec{B}\cdot d\vec{a}$

$\vec{B}$ : "magnetic flux density" or "magnetic induction" in any medium

$\vec{H}$: "magnetic field"

But! See Griffiths for discussion of whether $\vec{B}$ or $\vec{H}$ is the "magnetic field"

**Later on:** $\vec{B} = \mu\vec{H}$, $\mu$: "magnetic permeability"; $\mu = \mu_0(1+\chi_m)$

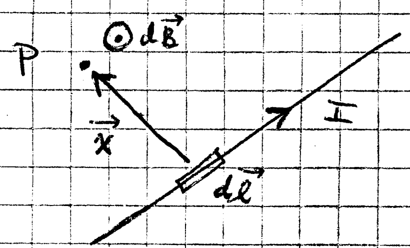
$\chi_m$: "magnetic susceptibility"

$\vec{J}$: "current density" $\left[\dfrac{C}{m^2\text{-}s} \equiv \dfrac{A}{m^2}\right]$ wire $\otimes \vec{J}$

current $\vec{I}$: Amperes

($\vec{J}$ integrated over cross-sectional area)

Biot-Savart Law:




$d\vec{l}$: infinitesimal current element, in direction of current flow

$\vec{x}$: from $d\vec{l}$ element to observation point P

$$d\vec{B} = kI\,\frac{(d\vec{l}\times\vec{x})}{|\vec{x}|^3} \quad \text{(inverse square law)}$$

$$\textbf{SI:}\quad k = \frac{\mu_0}{4\pi} = 10^{-7}\,\frac{N}{A^2}$$

$$\Rightarrow [\vec{B}] = \frac{N}{A\cdot m} = \frac{N}{C}\cdot\left(\frac{s}{m}\right) \equiv \text{Tesla!}$$

$\left(\frac{s}{m}\right)$: 1/velocity, by dimensional analysis

Recall: $[E] = \dfrac{N}{C}$ $(\vec{F} = q\vec{E})$ ($\rightarrow$ units of $c\vec{B}$ and $\vec{E}$ same!)



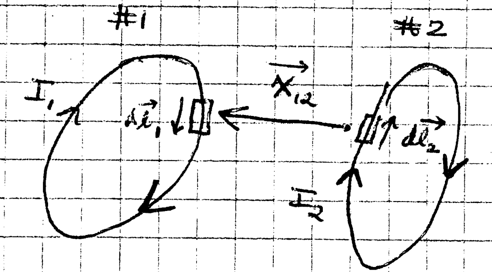
$\Rightarrow$ As find later, $c\vec{B}$ and $\vec{E}$ form components of $F^{\mu\nu}$ (field-strength tensor, in relativistic electrodynamics)

Ampere's Law: force experienced by a current element $I_1 d\vec{l}_1$ in presence of external magnetic induction $\vec{B}$

$$d\vec{F} = I_1 \, d\vec{l}_1 \times \vec{B}$$

If $\vec{B}$ due to closed current loop #2 with current $I_2$, then total force $\vec{F}$ experienced by a closed current loop #1 with current $I_1$ is:

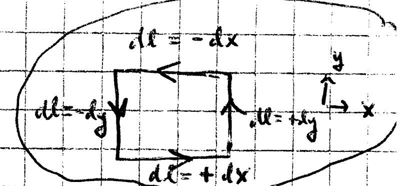
$$d\vec{F}_{12} = I_1 \, d\vec{l}_1 \times \oint_{\#2} \frac{\mu_0}{4\pi} I_2 \frac{d\vec{l}_2 \times \vec{x}_{12}}{|\vec{x}_{12}|^3}$$




$$\Rightarrow \vec{F}_{12} = \oint_{\#1} I_1 \, d\vec{l}_1 \times \oint_{\#2} \frac{\mu_0 I_2}{4\pi} \frac{d\vec{l}_2 \times \vec{x}_{12}}{|\vec{x}_{12}|^3}$$

$$= \frac{\mu_0 I_1 I_2}{4\pi} \oint_{\#1}\oint_{\#2} \frac{d\vec{l}_1 \times (d\vec{l}_2 \times \vec{x}_{12})}{|\vec{x}_{12}|^3}$$

using: $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$



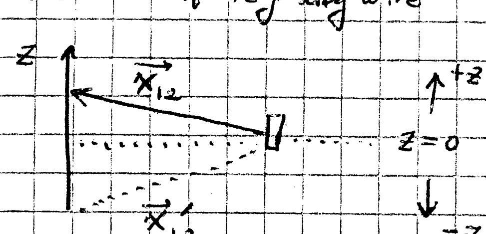

$$\Rightarrow d\vec{l}_1 \times (d\vec{l}_2 \times \vec{x}_{12}) = \underbrace{(d\vec{l}_1 \cdot \vec{x}_{12})}\, d\vec{l}_2 - (d\vec{l}_1 \cdot d\vec{l}_2)\, \vec{x}_{12}$$

in $\oint_{\#1} (d\vec{l}_1 \cdot \vec{x}_{12})\, d\vec{l}_2 = d\vec{l}_2 \oint_{\#1} \left[ dl_{1x} x_{12} + dl_{1y} y_{12} + dl_{1z} z_{12} \right]$

$dl_i$ in direction of current

$= 0$ (for closed loop)

or also for "infinitely long wire"



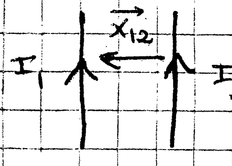

for each $\vec{x}_{12}$ with $z > 0$, there is a $\vec{x}_{12}'$ with $z < 0$, with $z_{12}$ of opposite sign $\Rightarrow \int_{\#1} (\ldots) = 0$

$\Rightarrow$

$$\underbrace{\vec{F}_{12} = - \frac{\mu_0 I_1 I_2}{4\pi} \oint_{\#1}\oint_{\#2} \frac{(d\vec{l}_1 \cdot d\vec{l}_2)\, \vec{x}_{12}}{|\vec{x}_{12}|^3}}$$

Ampere's Law of Force for Two Current Loops

If $d\vec{l}_1 \cdot d\vec{l}_2 > 0$ (i.e., currents in the same direction), $\vec{F}_{12} \propto -\vec{x}_{12}$




$\Rightarrow$ force on #1 towards #2

$\Rightarrow$ attractive

[if currents in opposite direction, $d\vec{l}_1 \cdot d\vec{l}_2 < 0 \Rightarrow$ force is repulsive!]