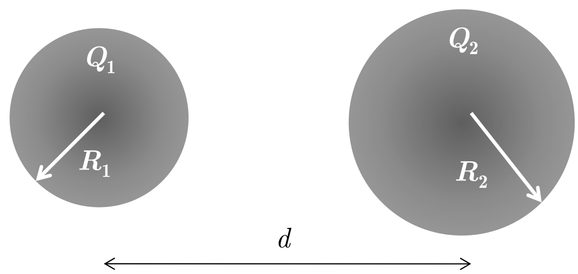
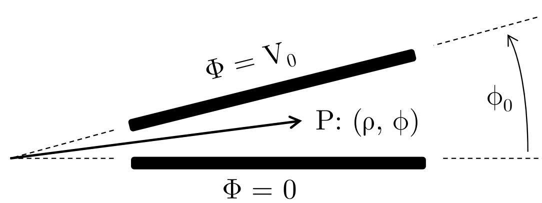
PHY 611 – Electromagnetic Theory I
Midterm Exam – Wednesday, October 17, 2012 – Due at 10:00 p.m.

Problem 1 [20 points]
Calculate the electrostatic interaction energy (i.e., you do not need to consider the self energies) of two non-overlapping, homogeneous spheres of total charge $Q_{1,2}$ and radius $R_{1,2}$ which are separated by a distance $d > R_1 + R_2$, as shown below.



Problem 2 [15 points]
Consider two large rectangular conducting plates which form an angle ϕ_0 with each other as shown below. One plate is held at a potential $\Phi = V_0 > 0$, while the other plate is grounded. Neglecting end effects, find the electric field $\vec{E}$ at some point (ρ, ϕ) in the region between the plates, and then find the charge densities σ on each of the plates (making sure to indicate their signs). Indicate with a sketch the orientation and direction of the $\vec{E}$ field lines.



Problem 3 [30 points]
Work parts (a), (b), and (c) of Problem 2.7 in Jackson (p. 87). Express your answer to (a) in terms of cylindrical coordinates. You do not need to work part (d).

Exam Continued — Over

Problem 4 [35 points]

Consider the two-dimensional Neumann boundary value problem shown below. There is no free charge anywhere. Instead of solving this problem via a Green function technique, we will solve this problem via separation of variables. Starting from a postulated solution $\Phi(x,y) = X(x)Y(y)$, derive an expression for the potential $\Phi(x,y)$ in the interior of the rectangle in the form of

$$\Phi(x,y) = \sum_n c_n \cdot (\text{basis functions in } x) \cdot (\text{basis functions in } y). \tag{1}$$

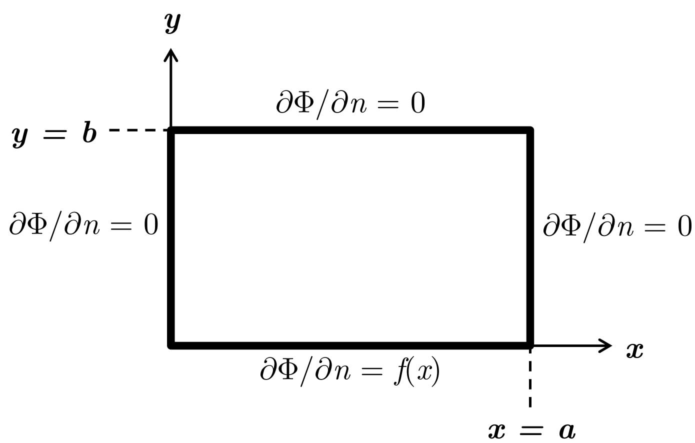
To receive full credit, you will need to: (a) determine the $X(x)$ and $Y(y)$ basis functions; (b) solve for the c_n expansion coefficients in terms of the boundary conditions; and (c) explicitly state what values of n are permitted in the sum (and justify why this is so). In particular, are there are any values of n that must be excluded based on a physics law?

Hint 1: Recall the two-dimensional version of Gauss's Law for a vector field $\vec{F}$ defined over a domain D with a closed boundary C,

$$\iint_D \vec{\nabla} \cdot \vec{F}\, da = \int_C \vec{F} \cdot \hat{n}\, d\ell.$$

Hint 2: Given the above hint, what then must the value of $\int_0^a f(x)dx$ be?

Hint 3: Recall that $\frac{d}{du}\sinh(u - u_0) = \cosh(u - u_0)$, where u_0 is some constant. This might be useful to recall in constructing your basis functions.



Potentially Useful Formulas

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

End of Exam