PHY 611 – Electromagnetic Theory I
Problem Set 5
Problems 1–5 Due: Wednesday, October 31 at 10:00 a.m. at the start of class
Problems 6–8 Due: Monday, November 5 at 10:00 a.m. at the start of class

References Policy: On the first page of your submitted work, please list all of your collaborators and any references/sources that you consulted (e.g., textbooks, journal articles, web pages, etc.).

Problem 1 [15 points]: Boundary-Value Problem in Spherical Coordinates
A conducting spherical shell of radius R is divided into eight equal sectors by a set of planes, with the z-axis their common line of intersection. You can think of such a configuration as being the "peel" on a perfectly-spherical "orange" (i.e., the fruit) consisting of eight equal "orange sections". Now suppose that these eight sectors are electrically insulated from each other and are alternately maintained at potentials $+V$ and $-V$ as one moves around the shell in the azimuthal coordinate ϕ. Find the potential $\Phi(r, \theta, \phi)$ in the regions: (a) external to the sphere; and (b) inside the sphere. You only need to determine the lowest-order term in ℓ (there will be two terms in this ℓ). Verify that all of your results are real-valued as they should be (i.e., not complex-valued).

Problem 2 [5 points]: Boundary-Value Problem in Cylindrical Coordinates
The axis of a semi-infinite cylinder of radius R is located on the z-axis. The cylinder extends from $z = 0$ to $z \to \infty$, with the "bottom lid" at $z = 0$ held at a potential V. The side walls are held at ground. As the cylinder is semi-infinite, it has no "top lid". Show that the potential inside of the cylinder is

$$\Phi(r, z) = \frac{2V}{R} \sum_n \frac{e^{-k_{0n} z}}{k_{0n}} \frac{J_0(k_{0n} r)}{J_1(k_{0n} R)},$$

where $k_{0n} = x_{0n}/R$, and x_{0n} denotes the n^{th} root of J_0.

Problem 3 [15 points]: Boundary-Value Problem in Cylindrical Coordinates
Now we will consider a finite-length cylindrical shell. Again, let the cylinder be of radius R, and now let the length of the cylinder be L, with the bottom and top surfaces located at $z = 0$ and $z = L$, respectively. The bottom and the top surfaces are held at ground. The side walls of the cylindrical shell are held at a potential $V(\phi, z)$, which is some function of ϕ and z. Find an expression for the potential inside of the cylinder.

Problem 4 [15 points]: Expansion of Green Function in Cylindrical Coordinates
In class we worked through the expansion of the Green function (for a point charge) in cylindrical coordinates. Complete this exercise by deriving Eqs. (3.149), (3.150), (3.151), and (3.152) in Jackson.

Problem 5 [10 points]: Eigenfunction Expansion of Dirichlet Green Function
Consider a rectangular box with walls defined by $x = \pm a/2$, $y = \pm a/2$, and $z = \pm a/2$.

(a) Use the eigenfunction method to derive the Dirichlet Green function for the interior problem.

(b) Suppose there is a point charge Q at the center of the box, and that all six sides of the box are grounded. Derive an expression for the induced charge density on the $z = a/2$ face of the box.

Problem 6 [15 points]: Method of Images at Boundary Between Dielectrics
Read Section 4.4 of Jackson on methods for the solution of boundary-value problems with dielectrics. Now consider the two half spaces defined by $z > 0$ and $z < 0$. The electric permittivity of the half space defined by $z > 0$ is ϵ_0, while that in the $z < 0$ half space is ϵ. A point charge q is located in the $z > 0$ half space at a distance a above the boundary (defined by the $z = 0$ plane) between the two half spaces.

(a) Calculate the force on the charge q.

(b) Calculate the polarization surface charge density at the boundary on the $z = 0$ plane.

(c) Calculate the force on the polarization surface charge density due to q.

Problem 7 [10 points]: Multipole Expansion
Work Problem 4.7 in Jackson. For part (c), instead of assuming a nucleus with a quadrupole moment of $Q = 10^{-28}$ m^2, look up the quadrupole moment of the deuteron and carry out the calculation for its particular value of Q.

Problem 8 [15 points]: Electric Field "Shielding"
Consider two concentric spheres of radii a and $b > a$ which are placed in an originally uniform electric field. Suppose the region between the two spheres, $a < r < b$, is filled with a dielectric medium of electric permittivity ϵ; the permittivity of the regions $r < a$ and $r > b$ is ϵ_0. Derive an expression which will tell you how well the region $r < a$ is "shielded" from the external electric field.