Chapter 26
Special Relativity

Introduction
Time Dilation
Length Contraction
Mass Increase

$E=mc^2$

Just as solid ice can melt into liquid water, Mass is a frozen form of energy that can be converted into the more familiar energy of motion.

## $E=mc^2$: *A little bit of mass can yield enormous energy*

- The destruction of Hiroshima and Nagasaki was fueled by converting less than an ounce of matter into energy
- The energy consumed by New York City in a month is less than energy contained in a piece of paper
- A baseball running at 100 m.p.h contains enough energy to keep a car running continuously 65 m.p.h for about 5000 years
- ......
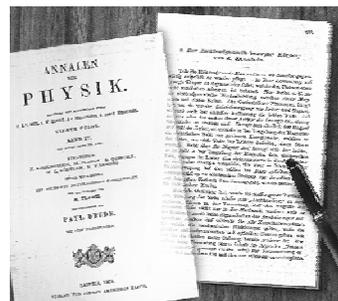
The 1905 paper on Special Relativity:
On the electrodynamics of moving bodies

This paper starts with a very simple example: **if a magnet is moved inside a coil a current is generated, if the magnet is kept fixed and the coil is moved again the same current is produced.**
This led Einstein to postulate that *the same laws of electrodynamics and optics will be valid for all frames of reference for which the laws of mechanics hold good*, which is known as the
Principle of Relativity
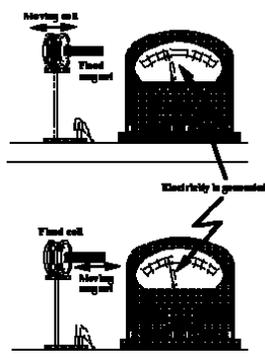
Relativity

Relativity links time and space, matter and energy, electricity and magnetism.

The theory of relativity was proposed in 1905 by Albert Einstein, and little of physical science since then has remained unaffected by his ideas.

"Common sense is the collection of prejudices acquired by age eighteen."
Albert Einstein (1952)

Please give up commonsense notions of space and time for 30 minutes!

Key Ideas

Two Principles of Relativity:
- The laws of physics are the **same** for all uniformly moving observers.
- The speed of light is the **same** for all observers.

Consequences:
- Different observers measure **different** times, lengths, and masses.
- Only **spacetime** is observer-independent.

All motion is relative to the observer; absolute motion does not exist.

Theory of relativity

The theory of relativity is concerned with physical consequences of the absence of a universal frame of reference.

The Special Theory of Relativity (1905):
Problems involving the motion of frames of reference at constant velocity with respect to one another.

The General Theory (1915):
Problems involving frames of references accelerated with respect to one another

Two Principles of the Special Theory of Relativity

1. The laws of physics are the **same** in all frames of references moving at constant velocity with respect to one another.
2. The speed of the light in free space has the **same** value for all observers, regardless of their state of motion or the state of motion of the source.

1st Principle of Relativity
The laws of physics are the same for all uniformly moving observers.

"Uniformly" = "with a constant velocity"

Implications:
- No such thing as "absolute rest".
- Any uniformly moving observer can consider themselves to be "at rest".

2nd Pricinple of Relativity
The speed of light in a vacuum is the same for all observers, regardless of their motion relative to the source.

Implications:
- The speed of light is a Universal Constant.
- We cannot send or receive information faster than the speed of light.

This has been experimentally verified in all cases.

Newton's Universe

- The universe keeps *absolute* time.
- Objects move through *absolute* space.
- Universe looks the *same* to all observers, regardless of how they move through it.

Einstein's Challenge

1905: Albert Einstein challenged Newton:
- We can only **compare** our view with that of other observers.
- All information we have is carried by **light**.
- But, light moves at a **finite speed**.

*The result is an **irreducible relativity** of our physical perspective.*

Spacetime

Newton's View:
- Space & Time are *separate* and *absolute*.
- Universe looks the same to all observers.

Einstein's View:
- Space & Time are **relative**.
- United by light into **Spacetime**.
- Only spacetime has an absolute reality independent of the observer.

Question: The theory of relativity is in conflict with
(a) experiment
(b) Newtonian mechanics
(c) electromagnetic theroy
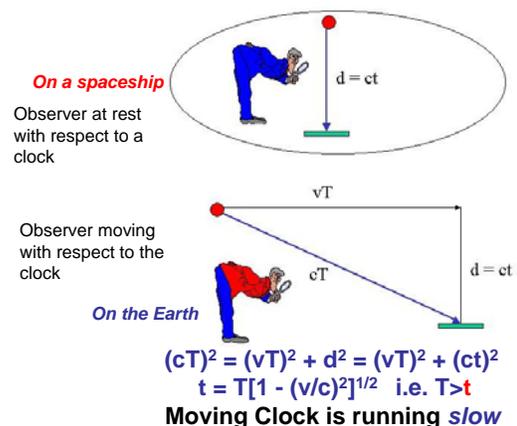(d) ordinary mathematcs

Answer: b

Question: According to the principle of relativity, the laws of physics are the same in all frames of reference
(a) at rest with respect to one another
(b) moving toward or away from one another at constant velocity
(c) moving parallel to one another at constant velocity
(d) all of the above

Answer: d

The Relativity of Time: A Thought Experiment

On a spaceship
Observer at rest with respect to a clock

$d = ct$

Observer moving with respect to the clock

On the Earth

vT

cT

$d = ct$

$(cT)^2 = (vT)^2 + d^2 = (vT)^2 + (ct)^2$
$t = T[1 - (v/c)^2]^{1/2}$ i.e. T>t
Moving Clock is running *slow*

Time Dilation

Moving clocks runs slowly

$$t = T[1 - (v/c)^2]^{1/2} \text{ or}$$
$$T = t/[1 - (v/c)^2]^{1/2}$$

(v has to be reasonably close to c)

Relative Time

The result is true for all clocks.

Conclusion: There is no *absolute time*

- Time passes at different rates for observers moving relative to each other.
- At speeds small compared to c, the difference is very small.

This has been verified experimentally using atomic clocks on airplanes and satellites.

Question: A young-looking woman astronaut has just arrived home from a long trip. She rushes up to an old gray-haired man and refers him as her son. How might this be possible?

Answer: Time dilation: Her clock and biological processes run slowly during her trip since she is moving relative to his rest frame, thus, she returned aged less than he did.

Question: If you were on a spaceship traveling at 0.5 c away from a star, at what speed would the starlight pass you?

Answer: The speed of light in vacuum is the same by all observers (2nd Principle). You would find that the starlight passes you at c.

Example: Find the speed relative to the earth of a spacecraft whose clock runs 1 s slow per day compared to a terrestrial clock.

Solution:

t=24hx60mim/hx60s/min=86,400 s
T=86,401 s
$T = t/[1 - (v/c)^2]^{1/2}$ or

$v=c [1 - (t/T)^2]^{1/2} =$
$=3x10^8$ m/s $(1-(86,400 \text{ s}/86,401 \text{ s})^2)$
$=1.44x10^6$ m/s

This is more than a thousand times faster than existing space craft.

Length Contraction

Observers on Earth: T=L/v

Observers on spacecraft: $t = T[1 - (v/c)^2]^{1/2}$

$$l=vt=vT[1 - (v/c)^2]^{1/2} =L[1 - (v/c)^2]^{1/2}$$
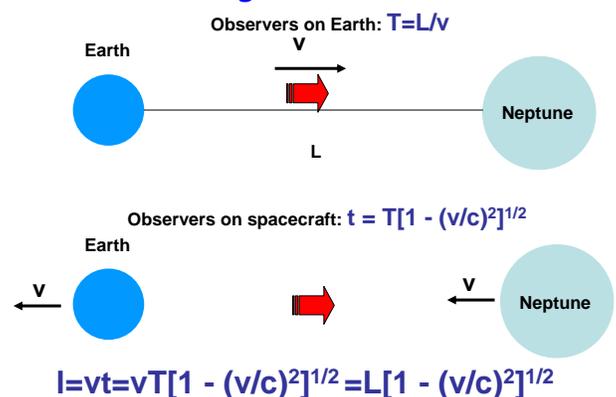
Length Contraction

Moving objects are shorter in the direction of motion than when at rest

$$l=L[1 - (v/c)^2]^{1/2}$$

Question: If you were traveling away from Earth at speed 0.5 c, would you notice a change in your heartbeat? Would your height and waistline change? What would observers on Earth using telescope say about you?

Answer: Since laws of physics are the same for all inertial observers, you would not notice any changes. However, observers on Earth watching you would say your heartbeat is slower, and you are thinner or shorter depending on which dimension of body is in the direction of motion.

Question: Suppose the speed of light were infinite. What would you happen to the relativistic predications of length contraction and time dilation?

Answer: We would not have to take into account the time light takes to reach us, so none of the relativistic effects would apply, i.e., the relativistic factor $(1-(v/c)^2)^{-1/2}$ would be equal to 1.

Mass Increase and Kinetic Energy

$$m=m_o/(1-v^2/c^2)^{1/2}$$

$$KE=mc^2-m_oc^2=m_oc^2/(1-v^2/c^2)^{1/2} – m_oc^2$$

Why nothing can travel faster than c?

Question: A spacecraft has left the earth and is moving toward Mars. An observer on the earth finds that, relative to measurements made when it was at rest, the spacecraft's

(a) length is greater
(b) mass is smaller
(c) clocks tick faster
(d) none of the above

Answer: d

Consequences of Relativity

Observers moving relative to each other:
- Do not measure the same times.
- Disagree on what events occur simultaneously.
- Do not measure the same lengths of objects.
- Do not measure the same masses for objects.

Other Consequences:
- Mass and Energy are **equivalent:** $E=mc^2$
- Massless particles **must** move at speed = c.

Essential Relativity

Two observers moving relative to each other experience the world **differently**:

- Both measure the **same** speed of light
- Both find the **same** physical laws relating distance, time, mass, etc.
- But, both measure **different** distances, times, masses, etc. applying those laws.

The key is the role of light

The World We Live in

All information about the Universe is carried by light
Speed of Light: c = 300,000 km/sec

Compared to everyday scales:
- 65 mph = 0.028 km/sec = 9.3×10^{-8} c
- light travel time in the lecture hall (front-to-back) = ~30 nanoseconds

Our everyday experience of the world is with phenomena at speeds much slower than that of light.

Light the Unifier

Because all information is carried by light at a finite speed, to satisfy the requirements of the two basic principles of Special Relativity:
- All uniformly moving observers see the same physical laws.
- All observers measure the same speed of light.

We unify otherwise disparate ideas:
- Space and Time are unified into Spacetime.
- Matter and Energy are Equivalent ($E=mc^2$)

Important Formulas

$$t = T[1 - (v/c)^2]^{1/2}$$

$$l = L[1 - (v/c)^2]^{1/2}$$

$$m = m_o/(1 - v^2/c^2)^{1/2}$$