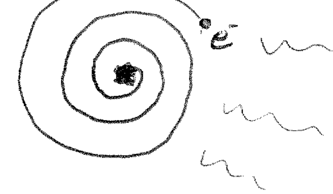
HW#3: ch4: #4, 8, 11, 16, 41, 49, 56

Bohr model of the H atom

- why is the sky blue?
  - classical E&M scattering
  - classical electron orbits would radiate, spiral inward
  - no spectra! (J.J Thompson model unsuccessful)

gravity: $F = G\frac{m_1 m_2}{r^2}$

electric: $F = k\frac{q_1 q_2}{r^2}$ $\quad k = \frac{1}{4\pi\epsilon_0}$



- E vs r : "Virial Theorem" relates T = kinetic energy, V = potential energy

if $V = kr^n$

then $F = \frac{dV}{dr} = nkr^{n-1} = \frac{mv^2}{r}$ "centripetal acceleration"

so $nkr^n = mv^2$

or $\boxed{n\langle V \rangle = 2\langle T \rangle}$ for average kinetic & potential energy.

for the atom $n=-1$ so $V = -\frac{kZe^2}{r}$ $\quad T = \frac{1}{2}\frac{kZe^2}{r}$

total energy $E = T+V = \boxed{-\frac{1}{2}\frac{kZe^2}{r}}$ $\quad$ also $\boxed{v^2 = \frac{kZe^2}{m_e}\cdot\frac{1}{r}}$

- Bohr model postulates
  1) stationary states (stable orbits of energy: E_n)
  2) quantum transitions & photons
     $E_{nm} = E_n - E_m = hf$ (frequency condition).
  3) correspondence principle.
     $\Rightarrow L \equiv m_e v r = n\hbar$ $\quad (\hbar \equiv \frac{h}{2\pi})$

"old quantum theory"
- explains hydrogen-like ions, but not even He atom

Bohr radius, energy

- from $mvr = n\hbar$ and $v^2 \propto \frac{1}{r}$

$$r_n = \frac{n^2}{Z}\cdot\underbrace{\frac{\hbar c}{m_e c^2}}_{2\pi\lambda_c}\cdot\underbrace{\frac{\hbar c}{ke^2}}_{1/\alpha} = \frac{n^2}{Z}\underbrace{a_0}_{\text{Bohr radius} = 0.529\,\text{Å}}$$

$$v_n = \frac{Z}{n}\cdot\underbrace{\frac{ke^2}{\hbar c}}_{\alpha = \frac{1}{137.036}}\cdot c$$

α = "fine structure constant" Sommerfeldt

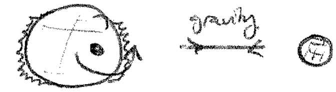
$$E_n = -\frac{Z^2}{n^2}\cdot\underbrace{\left(\frac{m_e c^2}{2}\right)\cdot\left(\frac{ke^2}{\hbar c}\right)^2}_{E_0 = 13.6\,\text{eV}}$$

$E_0 = 13.6\,\text{eV}$ "ionization energy"

note: ~~$k_B(300K) \ll$~~ $\underbrace{ke^2 \ll \hbar c}_{\text{quantum mechanical}} \underbrace{\ll m_e c^2 \cdot a_0}_{\text{non-relativistic}}$

Details of Bohr model

- reduced mass – what causes tides?



$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ but $p_1 = -p_2$

$E_{tot} = \frac{p^2}{2M} + \frac{p^2}{2m} = \frac{p^2}{2\mu}$ $\quad \mu = \frac{mM}{m+M} < m$ "reduced mass"

- correspondence principle

$hf = E_{n_i} - E_{n_f} = -E_0 Z^2\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$

$\frac{hc}{\lambda} = \underbrace{\frac{E_0 Z^2}{hc}}_{R_\infty}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \Rightarrow R_\infty = \frac{E_0}{hc} = \frac{m k e^2}{4\pi c \hbar^3} = \frac{(m_e c^2)\alpha^2}{4\pi(\hbar c)}$

$= 1.097 \times 10^{-2}/\text{nm}$

note: as $n \to \infty$ $\quad f = \frac{c}{\lambda} = \frac{-E_0 Z^2}{h}\underbrace{\left(\frac{1}{(n+1)^2} - \frac{1}{n^2}\right)}_{-2/n^3} \approx f_{rev} = \frac{v_n}{2\pi r_n} = \frac{Z^2}{n^3}\cdot\frac{\alpha^2 mc^2}{h}$

radiation frequency ~ orbital frequency

- ## Experimental Evidence of Bohr model

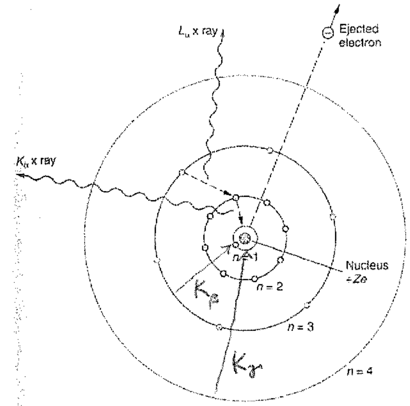
- X-ray spectra. – same atomic transition formula. except for screening.

$Z \to Z - b$

so $f^{1/2} = A_n(Z-b)$

$A_n = cR_\infty\left(1 - \frac{1}{n^2}\right)$

b = screening factor



- Auger electrons – Auger spectrum. emitted instead of photons.

- Franck-Hertz experiment

electrons used to excite atomic transitions.

- ~~Energy~~ Electron Energy Loss Spectoscopy (EELS)

inelastic scattering.