Matter Waves

- de Broglie wavelength.

$\hbar = \frac{h}{2\pi}$ $\hbar c = 197\,\text{eV}\cdot\text{nm}$

E	f
p	λ

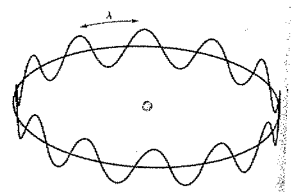
$E = hf = \hbar\omega$ $\omega = 2\pi f =$ "angular frequency"

$p = h/\lambda = \hbar k$ $k = \frac{2\pi}{\lambda} =$ wave number (spatial ang. frequency)

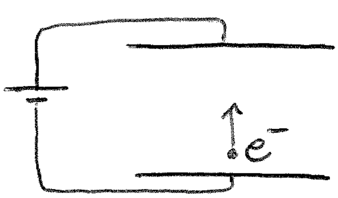
- interpretation of Bohr quantization condition

$$L = n\hbar$$
$$rp = n\frac{h}{2\pi}$$
$$2\pi r = n\lambda$$

FIGURE 6.1
If an electron wave — whatever it may be — is pictured as circling around the atomic nucleus, its wavelength λ must fit an integer number of times into the circumference.



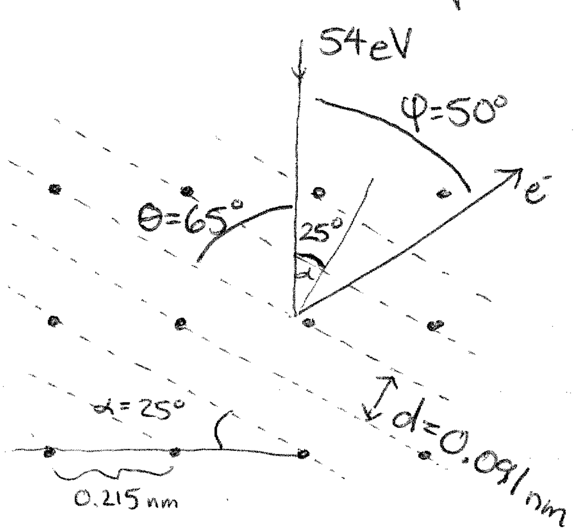
- example: wavelength of 54eV electron



$$E = \tfrac{1}{2}mv^2 = 54\,\text{eV}$$
$$v = \sqrt{\frac{2E}{m}}$$
$$= \sqrt{\frac{2\cdot 54\,\text{eV}}{0.511\times10^{6}\,\text{eV}/c^2}}$$
$$= 0.0145c$$
$$= 4.4\times10^{6}\,\text{m/s}$$

$$p = mv$$
$$= 0.511\,\text{MeV}/c^2 \cdot 0.0145c$$
$$= 7.42\,\text{keV}/c$$
$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1240\,\text{eV}\cdot\text{nm}}{7420\,\text{eV}}$$
$$= \boxed{0.167\,\text{nm}}$$

- Davisson-Germer experiment



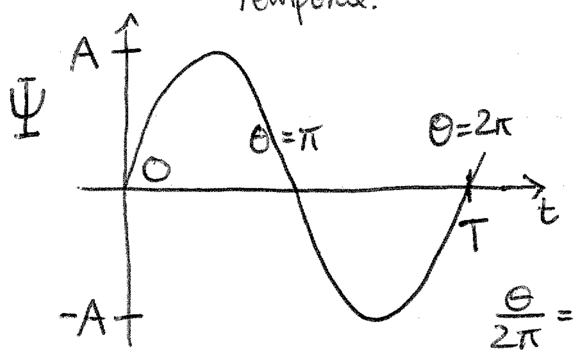
$$n\lambda = 2d\sin\theta$$
$$= 2\cdot(0.215\,\text{nm}\cdot\sin 25°)\cdot\sin(65°)$$
$$= 2\cdot 0.091\,\text{nm}\cdot\sin 65°$$
$$= \boxed{0.165\,\text{nm}}$$

Bragg scattering of an electron!

- neutron scattering
- electron microscopes

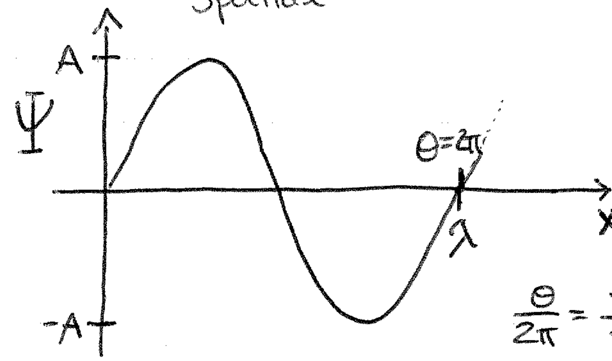
Wave function $\Psi(x,t)$

temporal



$$\frac{\theta}{2\pi} = \frac{t}{T}$$

spatial



$$\frac{\theta}{2\pi} = \frac{x}{\lambda}$$

$$\begin{aligned}\Psi &= A\sin(\theta)\\ &= A\sin\left(\frac{2\pi}{T}\cdot t\right)\\ &= A\sin(\omega t)\end{aligned}$$

$$\begin{aligned}\Psi &= A\sin(\theta)\\ &= A\sin\left(\frac{2\pi}{\lambda}\cdot x\right)\\ &= A\sin(kx)\end{aligned}$$

- putting the two together,

$$\Psi(x,t) = A\sin(kx-\omega t) \quad \text{or} \quad \cdots\cos\cdots$$

or

$$\begin{aligned}\Psi(x,t) &= A e^{i(kx-\omega t)}\\ &= A e^{i/\hbar(p\cdot x - E\cdot t)}\end{aligned}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$