Alternate derivation of ∇^2 (cyl)

* cartesian

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} \qquad \frac{\partial \hat{x}}{\partial x} = 0 \quad \frac{\partial \hat{y}}{\partial x} = 0 \quad \frac{\partial \hat{y}}{\partial x} = 0 \quad \frac{\partial \hat{y}}{\partial y} = 0.$$

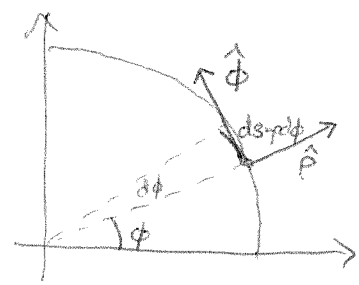
$$\nabla^2 = \nabla \cdot \nabla = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y}\right)\cdot\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y}\right)$$

$$= \hat{x}\cdot\hat{x}\frac{\partial^2}{\partial x^2} + \cancel{2\hat{x}\cdot\hat{y}}\frac{\partial^2}{\partial x \partial y} + \hat{y}\cdot\hat{y}\frac{\partial^2}{\partial y^2}$$

$$\boxed{\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}}$$

* cylindrical

$$\boxed{\begin{aligned} x &= \rho\cos\phi \\ y &= \rho\sin\phi \end{aligned}} \qquad \begin{aligned} \hat{\rho} &= \hat{x}\cos\phi + \hat{y}\sin\phi \\ \hat{\phi} &= -\hat{x}\sin\phi + \hat{y}\cos\phi \end{aligned}$$



$$\frac{\partial \hat{\rho}}{\partial \rho} = 0 \qquad \frac{\partial \hat{\rho}}{\partial \phi} = \frac{\partial}{\partial \phi}(\hat{x}\cos\phi + \hat{y}\sin\phi) = -\hat{x}\sin\phi + \hat{y}\cos\phi = \hat{\phi}$$

$$\frac{\partial \hat{\phi}}{\partial \rho} = 0 \qquad \frac{\partial \hat{\phi}}{\partial \phi} = \frac{\partial}{\partial \phi}(-\hat{x}\sin\phi + \hat{y}\cos\phi) = -\hat{x}\cos\phi - \hat{y}\sin\phi = -\hat{\rho}$$

$$\nabla = \hat{\rho}\frac{\partial}{\partial \rho} + \hat{\phi}\frac{\partial}{\partial s} = \hat{\rho}\frac{\partial}{\partial \rho} + \hat{\phi}\frac{1}{\rho}\frac{\partial}{\partial \phi}$$

$$\nabla^2 = \left(\hat{\rho}\frac{\partial}{\partial \rho} + \hat{\phi}\frac{1}{\rho}\frac{\partial}{\partial \phi}\right)\cdot\left(\hat{\rho}\frac{\partial}{\partial \rho} + \hat{\phi}\frac{1}{\rho}\frac{\partial}{\partial \phi}\right)$$

$$= \hat{\rho}\frac{\partial}{\partial \rho}\cdot\hat{\rho}\frac{\partial}{\partial \rho} + \hat{\rho}\underbrace{\frac{\partial}{\partial \rho}\cdot\hat{\phi}}_{0}\frac{1}{\rho}\frac{\partial}{\partial \phi} + \hat{\phi}\frac{1}{\rho}\underbrace{\frac{\partial}{\partial \phi}\cdot\hat{\rho}}_{\hat{\phi}}\frac{\partial}{\partial \rho} + \hat{\phi}\frac{1}{\rho}\underbrace{\frac{\partial}{\partial \phi}\cdot\hat{\phi}}_{-\hat{\rho}}\frac{1}{\rho}\frac{\partial}{\partial \phi}$$

$$= \hat{\rho}\cdot\hat{\rho}\frac{\partial^2}{\partial \rho^2} + \hat{\phi}\cdot\hat{\phi}\frac{1}{\rho}\frac{\partial}{\partial \rho} + \hat{\phi}\cdot\hat{\phi}\frac{1}{\rho^2}\frac{\partial^2}{\partial \phi^2}$$

– all other parts from the product rule are zero.

$$= \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \phi^2}$$