University of Kentucky, Physics 361
EXAM 2, 2009-03-13 11:00–11:50

Instructions: The exam is closed book and timed (50 minutes).

\[ E = \hbar \omega \quad \Delta E \Delta t \geq \frac{\hbar}{4} \quad \hat{E} = i\hbar \frac{\partial}{\partial \phi} \]
\[ p = \hbar k \quad \Delta p \Delta x \geq \frac{\hbar}{2} \quad \hat{p} = -i\hbar \frac{\partial}{\partial p} \]
\[ L_z = \hbar m \quad \Delta L_z \Delta \phi \geq \frac{\hbar}{2} \quad \hat{L}_z = -i\hbar \frac{\partial}{\partial \phi} \]

\[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x,t) + V(x) \Psi(x,t) = i\hbar \frac{\partial}{\partial t} \Psi(x,t) \]
\[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + V(x) \psi(x) = E \psi(x) \]

\[
\begin{align*}
\hbar c &= 197 \text{ eV nm} \\
m_e c^2 &= 0.511 \text{ MeV} \\
k_e e^2 &= \frac{e^2}{4\pi \epsilon_0} = 1.44 \text{ eV nm} \\
E_n &= -Z^2 E_0/n^2 \\
E_0 &= m_e k_e^2 e^2 / 2\hbar^2 = 13.6 \text{ eV} \\
r_n &= n^2 a_0 / Z \\
a_0 &= \frac{\hbar^2}{m_e k_e^2} = 0.529 \text{ Å} = 0.1 \text{ nm} \\
\langle G \rangle &= \int_{-\infty}^{\infty} \psi^* \hat{G} \psi \, dx \\
\mu &= \frac{mM}{m+M} 
\end{align*}
\]

Part I—Short Answer

[6 pts] 1. In the following diagram, label the impact parameter and scattering angle. Draw the scattering trajectories starting at a) and c). Show the location of nucleus with an ‘×’. Draw a detector and label the corresponding differential cross section \(d\sigma\) being measured. Rank the three trajectories as having the smallest (1) to largest (3) differential cross section.

![Diagram](image)

[3 pts] 2. Using a picture and brief labels, show the analogy between tides on the Earth and hydrogen and deuterium atoms. Which atom has lower ground state energy?

[3 pts] 3. Why does the ideal harmonic oscillator have an infinite number of energy levels, while the finite square well has only a finite number of energy levels?

[2 pts] 4. What two energies are quantized in the Bohr model of atomic spectra?
5. Match each of the following potential functions (middle) to the corresponding energy level (top), and to the corresponding 4th state wave function (bottom).

6. Circle the wave function(s) which represent tunneling. What happens classically at the interface between positive and negative kinetic energy?

7. a) Draw the wave function for the 10th excited state of the harmonic oscillator potential. 
b) Show two features of the wave function which illustrate the correspondence principle.

8. How are momentum and position represented quantum mechanically?

9. a) Draw the node lines of the first few wave functions of the infinite square well on a disk. 
b) Indicate the degeneracy of each state.
Part II—Short Calculation

[5 pts] 10. Derive the dispersion relation between \( w \) and \( k \) starting from \( E = \frac{p^2}{2m} \). Compare this with terms in the time-dependent Schrödinger equation (circle the parts that are the same).

[10 pts] 11. Suppose Rutherford were able to peek at the alpha bullet-holes in his gold sample, and saw the result below. a) what would he measure for the nuclear cross section \( \sigma \)? b) List two errors in this supposition.

![Diagram showing alpha bullet-holes](image)

[10 pts] 12. a) Which of the following are eigenfunctions of the operator \( \frac{\partial^2}{\partial x^2} \)? b) Calculate the eigenvalue for each eigenfunction.
   i) \( e^{2ix} \)    ii) \( e^{-x} \)    iii) \( \cos(3x + 4) \)    iv) \( 3x + 4 \)    v) \( x^2 \)

Part III—Full Problems

[15 pts] 13. Show that \( \psi(x) = A \sin(k_n x) \) is a solution of the time-independent Schrödinger equation (TISE) for a particle in an infinite square well, with \( U(x) = 0 \) for \( 0 < x < L \) and \( U(x) = \infty \) for \( x < 0 \) or \( x > L \). Show that the boundary conditions are satisfied on both sides of the well, and determine the values of \( k_n \) and therefore \( E_n \). Normalize the wave functions.
14. Use the solutions above to construct wave functions $\psi_{nm}(x, y)$ of a particle in a 2-D infinite well with a square boundary, i.e. $U(x, y) = 0$ for $0 < x < L$, $0 < y < L$, and $U(x, y) = \infty$ elsewhere, and calculate the energy values $E_{nm}$. Draw the node lines for the six lowest energy states, and show which ones are degenerate. No need to do a full separation of variables.