Section 7.2.1 - Faraday's Law

* three experiments - one result!

a) moving loop in static B field (7.1)
b) static loop in moving B field
c) static loop in static changing B field

$\left.\begin{matrix}a\\b\end{matrix}\right\}$ change of reference frame (S.R.) (nonuniform field) ~ motional emf

$\left.\begin{matrix}b\\c\end{matrix}\right\}$ motion of flux lines irrelevant only net flux ~ Faraday's law

* different physics involved, both involving B fields

a) Lorentz force law
- moving charge in static field

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{E}_{eff} = \vec{v} \times \vec{B}$$

b,c) Faraday's law
- static charge in changing field

$$\mathcal{E}_E = \oint_{\partial S} \vec{E} \cdot d\vec{l} = \int_S \underbrace{\nabla \times \vec{E}}\cdot d\vec{a} = \int_S \underbrace{-\frac{\partial \vec{B}}{\partial t}}\cdot d\vec{a} = -\frac{d\Phi_B}{dt}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

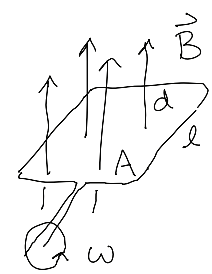
* Special Relativity
~ equivalence of E&M in different ref. frames
~ Lorentz transformations $\vec{E} \leftrightarrow \vec{B}$,
both components of $\vec{F} = \vec{E}\,dt + \vec{B}$

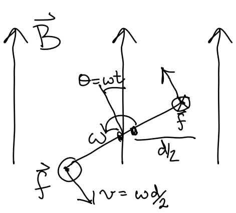
* Lenz's law
~ fields have "inertia"
~ it takes energy to build/destroy E,B
~ currents oppose change in fields

* Example of a) - AC generator

$$A(t) = A\cos(\omega t)$$

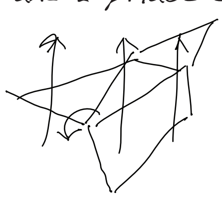
$$\mathcal{E} = -\frac{\partial \Phi}{\partial t} = AB\omega\sin(\omega t)$$

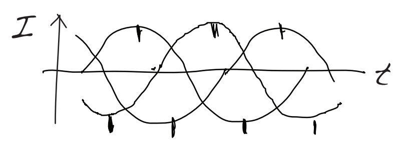




$$\mathcal{E} = \int \vec{v} \times \vec{B} = 2 \cdot \int \frac{\omega d}{2} B \sin\theta \cdot dl$$
$$= AB\omega\sin(\omega t)$$

~ 3-phase generator has 6 maxima of current per cycle
~ both 1-phase and 2-phase only have 2 ~ bicycle pedal problem





* Example 7.5

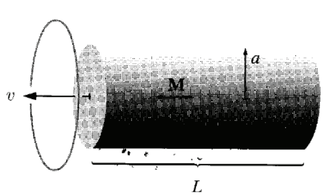


Figure 7.21

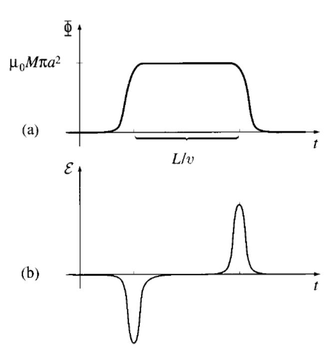


Figure 7.22

* Example 7.6

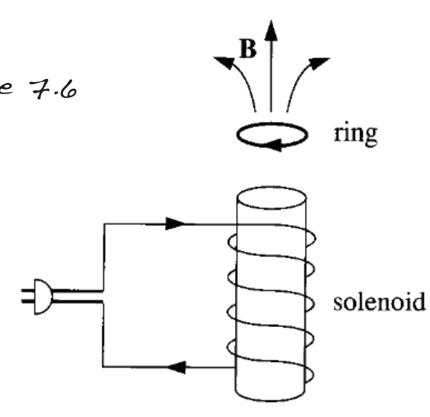


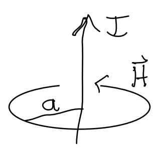
Figure 7.23

Section 7.2.2 - Induced Electric Field

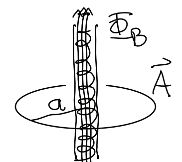
* three Ampere-like laws - one technique!

Ampere	Vector Potential		Faraday
$\nabla \times \vec{H} = \vec{J}$	$\nabla \times \vec{A} = \vec{B}$	$\longleftrightarrow$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
$\mathcal{E}_H = \Phi_J = I$	$\mathcal{E}_A = \Phi_B$	solution to $\nabla \times E \neq 0$ no potential!	$\mathcal{E}_E = -\frac{d\Phi_B}{dt}$

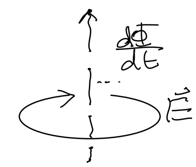
* with proper symmetry, each can be solved with Amperian loop



$$\vec{H} = \frac{I}{2\pi a}\hat{\phi}$$



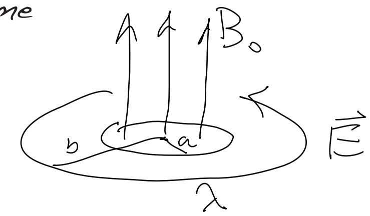
$$\vec{A} = \frac{\Phi_B}{2\pi a}\hat{\phi}$$



$$\vec{E} = -\frac{d\Phi_B}{2\pi a\, dt}$$

* Example 7.8: charge glued on a wheel

~ angular momentum from turning off field independent of time



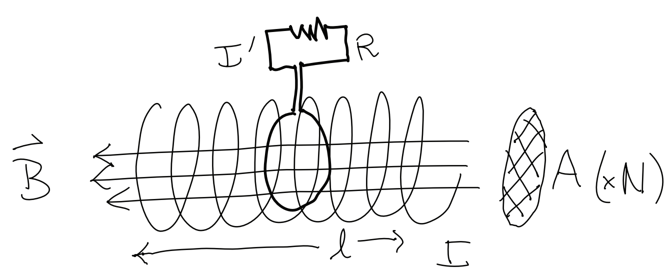
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt} = -\pi a^2 \frac{dB}{dt}$$

$$dL = N\, dt = b\lambda \oint \vec{E} \cdot d\vec{l}\, dt = -b\lambda\, \pi a^2 \frac{dB}{\cancel{dt}} \cancel{dt}$$

~ alternate approach: vector potential (momentum)

$$d\vec{p} = \vec{F} dt = q\vec{E} dt = -q \frac{d\Phi}{2\pi a\, \cancel{dt}} \cancel{dt} = -q\, d\vec{A}$$

* Problem 7.12: mutual inductance



$$n = N/l$$

$$\Delta B_t = \mu_0 K_s = \mu_0 n I$$

$$\Phi = BA = \underbrace{\frac{\mu_0 A}{l}} NI \equiv \frac{1}{\mathcal{R}} NI \equiv \mathcal{P} NI$$

'reluctance' 'permeance'

$$I'R = \mathcal{E} = -\frac{d\Phi}{dt} = \mathcal{P} N \frac{dI}{dt}$$