Photons: Particles vs Waves

* review
- Planck explained blackbody spectrum by $E=nh\nu$, quantization of "oscillator modes"
- effective UV cutoff: higher frequencies do not have enough energy for 1st excited state
  \[
  \langle E \rangle \sim e^{-\frac{hv}{kT}}
  \]
- same for heat capacity: $C = \frac{3}{2}N_A k$
- $d =$ # degrees of freedom = frozen out at low temp.
- quantum, not continuous states
- only first step in development of "photon"

* history of particle vs wave theory of light
- Aristotle - disturbance of aether
- Democritus - indivisible atoms
- Alhazen 11th century - refraction/reflection particles
- Descartes 1630 - wave like
- Newton 1670 - corpuscular (different colors)
- Hooke, Huygen, Fresnel - mathematical waves
- Young 1803 - double slit interference
- Poisson 1818 - "spot" observed by Arago.
- Maxwell 1860's - wave equations for light
- Hertz 1887 - observed wave

* what is the difference between particle & wave?

<table>
<thead>
<tr>
<th>Particle</th>
<th>Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>position</td>
<td>local, point-like</td>
</tr>
<tr>
<td>D.O.F.</td>
<td>$\mathbf{r}$ vector</td>
</tr>
<tr>
<td>dynamics</td>
<td>$\mathbf{F} = m\mathbf{a}$</td>
</tr>
<tr>
<td>quantization</td>
<td>mass</td>
</tr>
<tr>
<td>sociality</td>
<td>individual</td>
</tr>
<tr>
<td>physical</td>
<td>$m$ (inertia)</td>
</tr>
<tr>
<td></td>
<td>collective motion of particles</td>
</tr>
<tr>
<td></td>
<td>$\Psi, \Phi$ properties of medium</td>
</tr>
<tr>
<td></td>
<td>spread out</td>
</tr>
<tr>
<td></td>
<td>$\Phi(\mathbf{r})$ field</td>
</tr>
<tr>
<td></td>
<td>$(\nabla^2 - \omega^2 \mathbf{r}) \Psi = 0$</td>
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<tr>
<td></td>
<td>modes</td>
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</tbody>
</table>
Physical dispersion interactions interference conservation

\[ E = \frac{\gamma}{m} \]

\[ \gamma = \frac{E}{mc} = \frac{\omega}{k} \]

\[ \mathbf{\nabla} \cdot (\mathbf{S} = \mathbf{E} \times \mathbf{A}, \mathbf{T}) = 0 \]

* Photoelectric Effect

Irray: discovered by Hertz while showing that light is an E&M wave

- a) negative charged electrons
- b) cutoff frequency \([\text{not intensifying!}]\)
- c) current proportional to intensity \([\text{no delay!}]\)
- d) energy of electron \(\propto\) frequency \([\text{not intensity!}]\)

"photon" of light "hv" releases electron against work function \(W\) of metal \(eV = hv - W\)

Millikan 1918 confirmed experimentally, didn't accept photons

* Compton effect

Irray: used Bragg diffraction to discover it!

Classical: \(I(\theta) \sim 1 + \cos^2 \theta\)

Rayleigh scattering \(\nu = \nu_0\)

Partical: elastic scattering, conservation of \(E, \mathbf{p}\)

\[ E^2 = (mc^2)^2 + (pc)^2 \quad \text{but} \quad \nu_0 = \frac{\partial E}{\partial p} = c \quad \text{so} \quad m_0c = 0 \quad E = pc = hv \]

\[ \mathbf{p} = \mathbf{p}' + \mathbf{p} \rightarrow \mathbf{P}^2 = (p - p')^2 \]

\[ \Delta E = 0 \rightarrow \quad hv + mc^2 = hv' + (m'c^2 + P'^2) \frac{1}{2} \]

\[ m^2 c^4 + P^2 c^2 = \left[ hv - hv' \right]^2 + m^2 c^4 + 2hv(v - v') m^2 c^2 \]
\[
\left( \frac{\hbar}{c} \right)^2 + \left( \frac{\hbar}{c} \right)'^2 - 2 \frac{\hbar}{c} \frac{\hbar}{c} \cos \theta \right) c^2 = h \left( v^2 + v'^2 - 2 v v' \right) + 2 h (v-v') m c^2
\]

\[h \: vv' (1 - \cos \theta) = mc^2 (v-v')\]

\[\chi' - \chi = \chi_c (1 - \cos \theta)\]

\[\frac{\hbar_c}{E} - \frac{\hbar_c}{E'} = \frac{\hbar}{mc} (1 - \cos \theta)\]

\[\frac{1}{E'} - \frac{1}{E} = \frac{1 - \cos \theta}{mc^2}\]

\[E' = \left( \frac{1}{E} + \frac{1 - \cos \theta}{mc^2} \right)^{-1}\]

*Example: \(^{60}\)Co \(\gamma\)-emission spectrum*
\[ E_T = E - E'_{\text{min}} = E - \frac{1}{\sqrt{E + \frac{m^2}{2}}} = \frac{E \left( \frac{1}{\sqrt{E + \frac{2m^2}{E}}} \right)^2 - 1}{\sqrt{E + \frac{2m^2}{E}}} \]

\[ m^2 = 0.511 \text{ MeV}. \]

\[ E_\gamma = 1.17 \text{ MeV}; \quad E_e = (1 + 0.218)^{-1} E_\gamma = 0.821; \quad E_\gamma = 960 \text{ keV} \]

\[ 1.33 \quad 0.192 \quad 0.839 \quad 1115 \text{ keV}. \]