* What does “operate to observe” mean?
  - derives from Born probability amplitude interpretation
    and the machinery of probability: PDF, expectation value

\[ |\psi(x)|^2 = \psi^* \psi \quad \text{probability amplitude} \]
\[ \langle \psi | \psi \rangle = \int dx \psi^* \psi = 1 \quad \text{probability density (PDF)} \]
\[ \text{normalization} \]
“unitary operators evolve (rotate) from one state to another”

* expectation value - weighted average
  note the clean & consistent notation!

\[ \langle Q(x) \rangle = \int dx P(x) \cdot Q(x) = \int dx \psi^*(x) Q(x) \psi(x) \]
  weighted average of \( Q(x) \) probability \( |\psi(x)|^2 \)
  recall doing the same classically for black body radiation!

- the same applies in other coefficients: \( \psi(x) = \sum_n c_n \phi_n(x) \)
  \( |c_n|^2 \) is the probability of being in the state \( \phi_n(x) \)
  let \( Q \) have the value \( q_n \) for the state \( \phi_n(x) \)

\[ \langle Q \rangle = \sum_n |c_n|^2 q_n = (c_1^* c_2^* \ldots) (q_1 q_2 \ldots) (c_1 c_2) = \langle \psi | Q | \psi \rangle \]
  again: a weighted average of different measurements:

- \( \hat{Q} \) is an operator: \( |\psi \rangle \rightarrow \hat{Q} |\psi \rangle \sim (c_1 q_2 \ldots) (c_1 c_2) = (q_1 c_1 \quad q_2 c_2) \)
  \( \hat{Q} |\psi_1 \rangle = q_1 |\psi_1 \rangle \) “stretches” \( |\psi_1 \rangle \) so that \( \langle Q \rangle = \langle \psi_1 | \hat{Q} | \psi_1 \rangle = q_1 \)
is just the "value of $Q$" for the state $|\psi\rangle$.

- reinterpret $\langle Q \rangle = \langle \psi | \hat{Q} | \psi \rangle$
in terms of stretches:
  $\hat{Q} |\phi_1\rangle = q_1 |\phi_1\rangle$
  $\hat{Q} |\phi_2\rangle = q_2 |\phi_2\rangle$
  $\hat{Q} |\psi\rangle = q_1 c_1 |\psi\rangle + q_2 c_2 |\psi\rangle$
  $\langle \psi | \hat{Q} | \psi \rangle = [c_1^* \langle \phi_1 | + c_2^* \langle \phi_2 |] \hat{Q} |\psi\rangle$
  $= q_1 |c_1|^2 \langle \phi_1 | \phi_1 \rangle + \ldots \langle \phi_1 | \phi_2 \rangle$
  $+ q_2 |c_2|^2 \langle \phi_2 | \phi_2 \rangle + \ldots \langle \phi_2 | \phi_1 \rangle$

- note: $|\phi_1\rangle \neq |\phi_2\rangle$ are special states for $\hat{Q}$
  - they are "eigenstates" or "determinate" states
  - they have a definite associated value $q_i$ of $\hat{Q}$
  - any other state is a superposition of these
  - they are the simplest basis to represent $|\psi\rangle$ for $\hat{Q}$
  - operators do not have to be diagonal, but measurements must be Hermitian.
  - then it is always possible to diagonalize them with a complete orthonormal set of eigenvectors

* Physical measurements: Expansion postulate
  $|\psi\rangle = c_1 |\phi_1\rangle + c_2 |\phi_2\rangle + \ldots$ is a superposition
  of states with different "values" of $\hat{Q}$
  - $|c_n|^2$ is the probability of measuring $q_n$
  - thus $\langle Q \rangle = \sum_n |c_n|^2 q_n$
  - after measurement, the state will be $|\phi_n\rangle$
    depending on which value was obtained,
    now $\hat{Q} |\phi_n\rangle = q_n |\phi_n\rangle$