L22-Expansion postulate

Friday, November 13, 2015 08:42

* Executive summary of measurement in Q.M.
  1) Observation is richer than Q(x,p) due to complementarity.
     It requires a Hermitian operator \( \hat{\Omega}(x_i, \hat{\phi}_n) \) on a state \( |\psi(x)\rangle \).
  2) Every observable has special "determinate states" \( |\phi_n\rangle \Rightarrow q_n \)
     These are eigenstates: \( \hat{\Omega} |\phi_n\rangle = q_n |\phi_n\rangle \).
  3) Any other state is a "superposition" of these (complex linear combo).
     \( c_n = \text{"probability amplitude";} |\psi\rangle \) is a complete set of amplitudes.
  4) Observation is an irreversible projection - collapses the state.
  5) Measurements with same eigenstates (commute) are compatible.
  6) Otherwise "rotation" from one basis to another is unitary.
     Involving projections (inner products) of each state.
  7) Dirac notation treats these operations in a unified framework.

* Recall: determinate states \# superpositions

\[
\langle Q \rangle = \langle \psi | \hat{Q} | \psi \rangle = \sum_n c_n^* \langle \phi_n | \hat{Q} | \phi_n \rangle = \sum_n |c_n|^2 q_n \quad \text{where} \quad \hat{Q} |\phi_n\rangle = q_n |\phi_n\rangle
\]

- Compare with original: \( \langle Q \rangle = \int dx |\psi(x)|^2 Q(x) \)
- What are the "\( c_n \)" or "\( \psi(x) \)"?
- Probability amplitude of "being at" \( q_n \) or \( x \)

\[
\langle Q^2 \rangle = \langle \psi | \hat{Q}^2 | \psi \rangle = \sum_n |c_n|^2 q_n^2
\]

\[
\Delta Q = \langle Q^2 \rangle - \langle Q \rangle^2 = \sum_n |c_n|^2 q_n^2 - \left( \sum_n |c_n|^2 q_n \right)^2
\]

\[
= 0 \quad \text{if and only if} \quad c_n = \delta_{n,j}
\]

- Every operator has a basis of determinate states.
(complete set of orthonormal vectors $|\phi_n\rangle$) with definite $q_n$

- these are the eigenvectors of $\hat{Q}$: $\hat{Q}|\phi_n\rangle = q_n|\phi_n\rangle$

  energy states: $\hat{H}|\psi_n\rangle = E_n|\psi_n\rangle$ Schrödinger's equation

  position: $\hat{x}|x'\rangle = x'|x\rangle$ Dirac $S$: $|x\rangle \sim \delta(x-x')$

  momentum: $\hat{p}|k\rangle = i\hbar k|k\rangle$ Plane waves $|k\rangle \sim e^{ikx}$

- so what are quantum mechanical states $|\Psi\rangle$?
  - a complete set of probability amplitudes (components)

* Dirac notation:

  orthonormality: $\langle \phi_n | \phi_m \rangle = \delta_{nm}$

  closure: $\sum_n \langle \phi_n | \phi_n \rangle = \sum_n P_n = I$

  $\hat{Q}|\phi_n\rangle = q_n|\phi_n\rangle$ MV = VD eigenvalue $q_n$V

  $\langle \phi_n | \hat{Q} | \phi_m \rangle = q_m \delta_{mn}$ V$^\dagger$MV = D diagonalization

  $\hat{Q} = \sum_n q_n |\phi_n\rangle \langle \phi_n|$ M = VDV$^\dagger$ spectral representation

  Note: orthonormality & closure are eigenvalues & spectrum of identity!

  $|\Psi\rangle = \sum_n \langle \phi_n | \Psi \rangle |\phi_n\rangle = \sum_n \langle \phi_n | U |\psi_n\rangle |\psi_n\rangle$ unitary