MEASUREMENT OF PARITY VIOLATION IN THERMAL NEUTRON CAPTURE ON A PROTON

BY

MIKAYEL DABAGHYAN

Master’s Degree, Yerevan State University, 1996

DISSERTATION

Submitted to the University of New Hampshire in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy in Physics

August 2007
This has been examined and approved.

Dissertation Director, F.W. Hersman, Professor of Physics.

S. Penttilä, Professor of Physics, Oak Ridge National Laboratory

D. Meredith, Professor of Physics

J. Calarco, Professor of Physics

P. Berglund, Assistant Professor of Physics

S. Covrig, Affiliate Assistant Professor of Physics

Date
ACKNOWLEDGMENTS

I would like to thank my adviser Bill Hersman for his guidance, support and stoic patience over my years as his graduate student. A special thank you goes to Seppo Penttila who has been my mentor for the past three years and has shown me what it means to be a scientist, taught me diligence and helped every step of the way.

I am thankful to David Bowman for insightful discussions and advice and scaring me into learning, Greg Mitchell for always keeping the door to his office open and the answers to my incessant questions ready, Michael Gericke for being my role model and helping me with everything, Scott Wilburn for his concise and meaningful input and helping discover my “special skills”, Michael Snow for making keeping up the excitement around the experiment and providing a brain to pick for physics, Gordon Jones for lending a helping hand when it is needed, Satyaranjan Santra and Gil Peralta for remembering to pick me up in the morning. Bernhard Lauss for being so damn nice, and Mike Mason for being a good friend and helping me enjoy life after work in New Mexico.

I would also like to express my deepest gratitude to Libertad Barron, Monisha Sharma,Americo Salas-Bacci, Christopher Crawford, Hongguo Zhu, Pil Neyo-Seo and Chad Gillis who made my life at the lab much much much easier.

My family and friends for believing in me and taking me seriously.

And last but not least, thank you to my beloved Iga for her unwavering love and support, for giving me courage and just for being her incredible self.
CONTENTS

ACKNOWLEDGMENTS ................................................................. iii

LIST OF TABLES ................................................................. vi

LIST OF FIGURES .............................................................. xiii

ABSTRACT ................................................................. xiv

CHAPTER

1 INTRODUCTION ................................................................. 1

2 Theoretical Background ............................................................ 7

2.1 Parity Symmetry ................................................................. 7
2.2 Hadronic-Weak Interaction. Nucleon Level .................................. 12
2.3 Compound Nuclei ................................................................. 24

3 The NPDGamma Experiment at LANSCE ........................................ 31

3.1 Flight Path 1FP12 and Neutron Guide ...................................... 36
3.2 Frame Defination Chopper of 1FP12 ......................................... 45
3.3 Neutron Beam Monitors ........................................................ 50
3.4 γ-Ray Detector Array ............................................................. 52
3.5 Commissioning the NPDGamma Apparatus with Nuclear Targets ........ 57
3.6 Magnetic Guide Field ............................................................ 59
3.7 Resonant Radio Frequency Spin Flipper ..................................... 67
3.8 Data Acquisition ................................................................. 80
3.9 Shielding for Low-Energy Neutrons ......................................... 82
3.10 Hydrogen Target ................................................................. 84

4 Polarimetry ................................................................. 91

4.1 Polarized $^3He$ ................................................................. 91
4.2 Nuclear Magnetic Resonance .................................................. 100
4.3 Polarizing neutron beams using $^3He$ spin-filters. Neutron beam polarizer and analyzer ............................................. 125
4.4 Curvature effect of the analyzer cell to the determination of the $^3He$ polarization 157
5 Data Analysis and Results
5.1 Parity Violating $\gamma$-ray Asymmetry from Detector Data
5.2 Systematics
5.3 The Geometric Mean approach
5.4 Results

6 Conclusion, Plans etc.

BIBLIOGRAPHY

APPENDICES
A. Polarizer Oven Design
B. Powder Target Container Design
C. The diagram of the DAQ logic.
LIST OF TABLES

2.1 Initial and final states with their spins, angular momenta and isospins. . . . . 21

3.1 RFSF efficiencies measured at on-axis and off-axis positions. . . . . . . . 77

4.1 Polarizer Cells Created and tested at NIST. The cells Boo Boo, Dino and Pebbles
were chosen as spin filters for NPDGamma experiments. . . . . . . . . . . 137

4.2 Fits using $cosh(bP_3)$ and $cosh(bP_3) \cdot (1 + a)$. . . . . . . . . . . 141

4.3 $^3He$ Thickness of the cell TS-11. . . . . . . . . . . . . . . . . . . . . . . 151

5.1 Table Caption . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 212
LIST OF FIGURES

1.1 Theoretical predictions and experimental results of parity violation measurements. 4

2.1 $\Delta S = 0$ part of the hadronic-weak interaction, an example of a non-leptonic weak process. ................................................. 13

2.2 Depiction of leptonic and semi-leptonic interactions. ........................................ 15

2.3 One meson exchange potential for the parity-violating $NN$ interaction. .... 16

2.4 The lowest order allowed electromagnetic transitions in the process of neutron formation - $n + p \rightarrow d + \gamma$. ........................................ 20

2.5 Gamma-Ray Asymmetry in radiative neutron capture. .......................... 25

3.1 Areal view of the LANSCE accelerator complex. ................................. 31

3.2 The measured 1FP12 moderator brightness. ........................................ 33

3.3 3D conceptual view of the $NP\gamma$ Experimental Setup. ....................... 35

3.4 Vacuum envelope and the exit window of the 1FP12 neutron guide. .......... 36

3.5 Flight path guide reflectivity. ......................................................... 37

3.6 Total neutron cross-sections for some polycrystalline materials. ?? ........ 39

3.7 Neutron transmission through a room temperature $Be$ block. .............. 41

3.8 Transmission signal after the Be target measured by Monitor 3. .............. 42

3.9 Monitor 2 measured neutron transmission after the Be target and its time-derivative. ................................................................. 43
3.10 Monitor signal with the chopper on and off. ................................. 47

3.11 A zoomed in part of the Fig. 10. time of flight frame around 34-50 ms. The step in the monitor signal is caused by very slow neutrons, two frames earlier.

The step is an neutron image of the moving chopper edge two frames earlier. 48

3.12 1FP12 frame defination chopper between two guide sections. .................. 49

3.13 1FP12 Neutron Beam Chopper housing between two guide sections. ........ 50

3.14 Neutron beam monitor - a parallel-plate $^3$He ion chamber. .................. 51

3.15 M2 signals when a $^6$Li loaded epoxy tile is blocking the neutron beam compared with pedestal signals taken before and after the $^6$Li measurement. The pedestal signal is mainly produced by electrical noise. .................. 53

3.16 The detector array with the radio frequency spin flipper mounted onto the up-stream end of the detector. ................................. 55

3.17 Distribution of $\gamma$-ray asymmetries measured from a natural boron target "Counting statistics". ................................. 56

3.18 List of mass number $A \sim 50$ nuclear targets measured during the 2005 run cycle. 58

3.19 Schematic view of the vessel containing powder targets (Ti, V, Sc, Co, Mn). 60

3.20 Four race track coils installed in the experimental cave around the experiment. 61

3.21 $B_y$ field component measured in the beam direction ($\vec{z}$) on different x- and y-positions ................................. 63

3.22 $B_x$ component of the field measured in the direction of the beam ($\vec{z}$) on different x- and y-positions. ................................. 64

3.23 $B_z$ field component measured in the direction of the beam ($\vec{z}$) on different x- and y-positions. ................................. 65
3.24 Field Gradients across the Polarizer Cell. ................................. 66
3.25 Spin Flipper installed inside the Guide Coils, visible is the aluminum can which
shields the experiment from the RF field of the flipper. ...................... 68
3.26 Spin Flipper control electronics. .................................................. 71
3.27 The RFSF current as a function of time of flight during a 50 ms long neutron
pulse. .................................................................................................. 72
3.28 Configurations used in the spin-flipper efficiency measurements. .... 75
3.29 Spin Flipper Efficiency plotted as a function of TOF. ..................... 76
3.30 Two electronic pedestals measured 1.5 h apart by M3. ..................... 78
3.31 Fourier transformed monitor 3 signal of Fig. xyz .......................... 79
3.32 Data Acquisition and Storage. ...................................................... 81
3.33 2" $^6$Li-doped collimator (note the radiation damage surrounding the aperture). 83
3.34 Ortho- and Para-hydrogen scattering cross sections. .................... 85
3.35 A schematic view of the liquid hydrogen target. ......................... 87
3.36 Transmission of neutrons through the LH$_2$ target after the ortho-hydrogen has
reached the equilibrium fractional concentration. When the energy reaches
values larger then the separation energy between two hydrogen states, the
transition significantly reduces. ....................................................... 88
3.37 The average percentage fraction of the para-HL$_2$ in the target during Phase 1
NPDGamma Production run. ............................................................. 90
4.1 Atomic Levels of $^{87}$Rb, $I = 3/2$ ................................................... 93
4.2 Emission scheme according to selection rules, in which the atom is walked to
the desired polarized state ...................................................... 95
4.3 Optical Pumping of the atomic levels of \( Rb \) and spin exchange during binary
collisions of the \( Rb \) atoms with \( ^3He \) nuclei. .......................... 97
4.4 Time-evolution of longitudinal and transverse components of the magnetization. 112
4.5 Net transverse magnetization vector immediately and time \( \tau \) after application
of RF pulse. ........................................................................... 114
4.6 Precession of the magnetization vector in the laboratory frame. ...................... 117
4.7 Sample free induction decay (FID) signal. ............................................. 118
4.8 Free Induction Decay signal (AFP) ..................................................... 121
4.9 Front view of the Polarizer. Drive coils are seen encircling the oven box. . . . 130
4.10 Attenuation of the neutron beam by two samples of GE-180. ......................... 132
4.11 Neutron Attenuation in four Si Wafers built into the polarizer oven. .......... 133
4.12 Figure Of Merit: (a) Un-polarized spherical transmission, (b) Polarized spherical
transmission ?? ....................................................................... 135
4.13 Neutron attenuation in the un-polarized \( ^3He \)-cell. ................................. 136
4.14 \( \text{Cosh}(n\sigma lP_3) \) vs. \((1+a)\cdot\text{Cosh}(n\sigma lP_3)\) fit. .......................... 139
4.15 \( ^3He \) polarization in 2005. ....................................................... 142
4.16 Neutron Beam polarization at 0.56 \( ^3He \) spin-filter polarization. ............. 143
4.17 Diagram of the NMR system of the Spin-Filter. ..................................... 144
4.18 Simulated neutron spectrum used in the optimization of the analyzer thickness. 146
4.19 Analyzer Thickness Figure of Merit. ................................................. 147
4.20 The Analyzer Cell (TS-12). ............................................................. 148
4.21 Experimental Setup. .............................................................. 149
4.22 Setup for the measurement of photon transmission through the sample glass plate. 151
4.23 Un-polarized cylindrical transmission. ...................................... 152
4.24 Photon transmission through sample glass plate. ....................... 153
4.25 Ratio of polarized and un-polarized cylindrical transmissions. .... 156
4.26 Beam passing through a curved cell. ........................................ 158
4.27 Transmission through an unpolarized (a) and a polarized (b) spherical $^3$He cells. 163
4.28 Schematics of the analyzer setup. .......................................... 164
4.29 Spectrum Of the Narrowed Laser before passing through the analyzer cell. . 165
4.30 Part of the broad-spectrum optics to polarize the analyzer cell. ....... 167
4.31 TS-12 pump-up curve. The negative slope is due to the negative sign in front
of the signal. ................................................................. 168
4.32 Analyzer AFP signal. ....................................................... 169
4.33 Diagram of the NMR system for the analyzer. Lock-In amplifier - Stanford
Research Systems, Model SR-830 DSP. Two Function Generators - SRS,
Model DS 345. RF Amplifier - KROHN-HITE, Model 7500. .............. 170
4.34 The full analyzer set-up with the electronics rack. .................... 171
4.35 Spin Flipper Amplitude optimization using the Beam Polarization Analyzer. . 172
4.36 Guide Field optimization using the Beam Polarization Analyzer. .... 173
4.37 Polarized cell transporter. ................................................ 174

5.1 Detector geometry with respect to the spin direction .................... 179
5.2 A depiction of the ideal case of a point source - point detector picture. However, in reality both, the source and the detector have finite sizes and the problem is more complicated. To estimate the "true" detector positions, Monte Carlo simulations were carried out. .............................................. 184

5.3 A sample data (Ring #3) describing the effective detector angles reconstructed from motion table measurements. ................................................................. 187

5.4 The effective detector angles calculated from the geometry factors for the case of hydrogen target. ................................................................. 191

5.5 Neutron depolarization vs energy ................................................................. 195

5.6 Beam depolarization in LH2 at two para-ortho ratios. ................................................................. 196

5.7 Detector # 47 signal when the target was In, empty Aluminum can with and without the gap, and pedestal as a function of time-of-flight. ....................... 200

5.8 Signal to Background ratio vs TOF and Detector Number. The four layers represent the four rings of the detector array. From the top - ring 3, 2, 1 and 0. ................................................................. 201

5.9 A histogram of multiplicative asymmetries measured with LED and Spin-Flipper on but beam off. ................................................................. 205

5.10 Raw hydrogen asymmetry measured versus effective detector pair angle. .... 209

5.11 Histogrammed sequence asymmetries for the $CCl_4$ target. ................................................................. 210

6.1 The side view of the analyzer oven. ................................................................. 219

6.2 The top view of the analyzer oven. ................................................................. 220

6.3 The side view of the drive coils, the optics stand and the oven containing the cell. 221
6.4 The Q measurements of the RF coil. ............................................. 221
6.5 The front plates of the target vessel support. .............................. 222
6.6 The powder target vessel design - side and top views. ................... 222
6.7 The Ti and Al targets in a powder and solid forms, respectively. The powder target material is loaded into an aluminum can and positioned in neutron beam inside the γ-ray detector. The solid targets were in a form of thin sheets loaded onto the holding rack. ............................................. 223
ABSTRACT

MEASUREMENT OF PARITY VIOLATION IN THERMAL NEUTRON CAPTURE ON A PROTON

by

Mikayel Dabaghyan

University of New Hampshire, AUGUST, 2007

The NPDGamma experiment is measuring the directional parity violating asymmetry in the emission of gamma rays from the capture of cold neutrons on protons. The asymmetry can be related in a straightforward way to effective couplings within an appropriate NN weak interaction theory, such as chiral perturbation based effective field theories.

Since this is a measurement within a two body system, the observables are calculable without uncertainties from few to many body (large nuclei) effects. The experiment consists of two phases. The first one, at the Los Alamos Neutron Science Center (LANSCE), has just been completed, providing a measurement of the asymmetry to an accuracy at the $10^{-7}$ level. Directional $\gamma$-ray asymmetries have been measured using a number of targets including liquid hydrogen and several medium-$A$ isotopes. The second phase of the experiment will commence at the Spallation Neutron Source at Oak Ridge, where it is currently being reassembled, to continue the measurement to an accuracy at the $10^{-8}$ level. In this work the results of the commissioning phases as well as the first production phase of the experiment are discussed.
CHAPTER 1
INTRODUCTION

The weak interaction between point-like quarks and leptons is well described in the electro-weak interaction framework within the Standard Model. Phenomenologically the weak interaction processes can be divided according to the level of the leptonic contributions

\[ H_{\text{eff}} = H_{\text{lep}} + H_{\text{semilep}} + H_{\text{hadronic}}. \]  

The hadronic sector can be divided into the \( \Delta S = 1, 2 \) and \( \Delta S = 0 \) parts, where \( S \) is the strangeness. This thesis focuses on the strangeness-conserving hadronic weak interaction.

A number of experiments have verified most of the weak interaction structure; i) in the leptonic sector - decays such as \( \mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e \) and \( \tau^- \rightarrow e^- + \nu_\tau + \bar{\nu}_e \) ii) in the semi-leptonic \( \Delta S = 0, 1 \) sector decay experiments such as \( n \rightarrow p + e^- + \bar{\nu}_e \) and \( \Lambda \rightarrow p + e^- + \bar{\nu}_e \) iii) in the hadronic \( \Delta S = 1 \) sector decay experiments such as \( \Lambda \rightarrow p + \pi^- \) and \( K^+ \rightarrow \pi^+ + \pi^0 \).

The modern standard model of electro-weak interaction is not quite complete especially the hadronic sector requires to be finalized. In the \( \Delta S = 1 \) hadronic part the dynamic origin of the \( \Delta I = \frac{1}{2} \) rule (where \( \Delta I \) is the change of the isotopic spin in the reaction) remains a mystery after decades of hard experimental effort. According to the isospin selection rules, non-leptonic decays of strange hadrons such as \( K \) and \( \Lambda \) satisfy the \( |\Delta I| = \frac{1}{2} \) rule. It is not clear why the \( |\Delta I| = \frac{3}{2} \) amplitude is suppressed as indicated by the experimental result favored over \( \Delta I = \frac{3}{2} \) such as given by the branching ratios of the decays.
\[ \Gamma(K_s^0 \rightarrow \pi^0 + \pi^0) \gg \Gamma(K^+ \rightarrow \pi^+ + \pi^0) \sim 600. \quad (1.2) \]

In the \( \Delta S = 0 \) non-leptonic interactions listed above, \( n + p \rightarrow n + p \) are not mentioned because it has been difficult to isolate the contribution of the tiny weak interaction from the dominating \( NN \) strong interaction in the \( n + p \rightarrow n + p \) processes. However, the weak interaction contributions in these hadronic weak interactions are detectable through sensitive measurements of parity-violating observables.

At low energies the hadronic weak processes can be described, by constructing the appropriate potential for the interaction. Several theoretical models have been developed to explain how low energy inter-nucleon processes take place. On the other hand a number of experiments are designed to substantiate these theoretical conclusions. However, currently not all experimental ground has been covered. Experiments that study the weak interaction between nucleons can offer more insight into the mechanisms by which hadrons interact. One such approach attributes the weak interaction to a process whereby the exchange is mediated by light mesons, such as \( \pi, \rho, \omega \) [3]. In particular, the \( (n, p) \) provides a way to probe the flavor-conserving neutral-current exchange between quarks, which has not yet been measured, as opposed to the strangeness changing non-leptonic channels of decay reactions. In addition, measurements of weak effects will at these energies will provide a tool to explore the low-energy limit of the strong interaction, and probe the low-energy non-perturbative aspects regime of QCD.

A measurement such as NPDGamma will give a quantitative assessment of the theory derived by Deplanques, Donahue and Holstein (DDH) [3] based on the SU(6) framework.
for the Weinberg-Salam electro-weak interaction model. According to their formulation, the term with the pion-nucleon coupling is the most sizable contribution to the Hamiltonian that describes the meson-exchange model in the neutron-proton interaction. Being the lightest, and hence the longest-range contribution, pion-exchange is the center focus of this experiment. It is all the more interesting, since it is dominated by neutral currents.

Theoretical approaches other than that taken by DDH have been made as well. Dubovik et al. [[11]] proposed an approach based on $SU(2)_L \times U(1) \times SU(3)_c$ model to calculated $H_\rho$, $H_\omega$ and $H_{\pi}^1$, coupling constants corresponding to the amplitude of the interaction mediated by the $\rho, \omega$ and $\pi$ mesons, respectively. The values for the first two constants agree with DDH best values, but the pion coupling constant is about 1/3 of the DDH prediction. Kaplan and Savage [[35]] and later Beane and Savage [[4]] used the effective field theory approach. In 1998 Henley [[18]] carried out the calculations using the QCD sum rules, while Meißer and Weigel based their predictions on SU(3) [[39]]. Figure [1.1] compares the results of these groups for $H_{\pi}^1$.

Weak forces possess a unique trait, parity non-conservation, that allows to discern weak effects among the overwhelming strong phenomena. NPDGamma makes use of the fact that in the process of radiative neutron capture on a proton, parity violation is manifested as the $\gamma$ – ray directional asymmetry, $A_{\gamma}$, with respect to the neutron spin. The DDH model then predicts a direct way to relate the dominant pion-nucleon coupling, from the experimentally measured quantity - $A_{\gamma}$.

In all, 6 meson couplings are included in this model, in which the weak potential is expanded in terms of the meson couplings as
Figure 1.1: Theoretical predictions and experimental results of parity violation measurements.
\[ V_{PV} = H^1_{\pi}V^1_{\pi} + H^0_{\rho}V^0_{\rho} + H^1_{\rho}V^1_{\rho} + H^2_{\omega}V^2_{\omega} + H^0_{\omega}V^0_{\omega} + H^1_{\omega}V^1_{\omega}, \]  

(1.3)

where the subscripts stand for the exchange meson and the superscript indicates change in isospin \(-\Delta I\). \(H^1_{\pi}\) is the coupling constant representing the pion, longest range interaction, contribution to the weak Hamiltonian in the DDH model.

Some of the constants have been measured in experiments over the past decades. In the late seventies and early eighties, several groups measured the circular polarization of photons emitted in transitions of excited \(^{18}\text{F}\) nuclei acquired consistent results for \(P_\gamma = (1.2 \pm 3.9) \times 10^{-4}[[13]]\). The value of \(H^1_{\pi}\) inferred from these measurements was quite low compared to that predicted by DDH, \(H^1_{\pi} = (0.7 \pm 2.0) \times 10^{-7}\).

A number or \(p + p\) scattering experiments were performed at different energies, using longitudinally polarized protons. In 1979 Nagle et al. reported a longitudinal asymmetry of \(A^p_{L} = -(1.7 \pm 0.8) \times 10^{-7}[[17]]\). Subsequent measurements by Balzer et al. and Yuan et al. yielded the values \(A^{pp}_{L} = -(2.31 \pm 0.89) \times 10^{-7}(45\text{ MeV})\) and \(A^{pp}_{L} = (2.4 \pm 1.1) \times 10^{-7}(800\text{ MeV})\) [[27], [29]]. The latest measurement was performed by a collaboration at TRIUMF in 2001. They reported \(A^{pp}_{L} = -(0.86 \pm 0.35) \times 10^{-7}\). Due to restrictions associated with reactions involving identical particles and CP-conservation, \(p + p\) reactions are insensitive to the pion channel, but are a measure of the \(\rho\)-couplings (see fig.).

Measurements of the nuclear anapole moment leading to atomic parity-violation in\(^{133}\text{Cs}\) [[15]] yielded a value of \(H^1_{\pi} = (2.26 \pm 0.5) \times 10^{-6}\). These results disagree with the \(^{18}\text{F}\) measurements as well as the measurements of the \(^{205}\text{Tl}\) anapole moment. As pointed out by Wilburn and Bowman [[48]], the former can be deemed to agree with the \(^{133}\text{Cs}\)
results, if considered at the edge of the DDH reasonable range. However overall, the discrepancy between these measurements has not yet been explained. Fig 1.1 depicts the current situation with the knowledge of the meson coupling constants for the DDH model.

In order to consolidate the theoretical models, as well as the experimental findings, experiments that isolate individual couplings and measure them with high precision are necessary.

NPDGamma is most sensitive to the weak pion exchange. It can be shown [[3]] that the $\gamma$-ray asymmetry measured in this experiment can be express via the meson couplings as

$$A_\gamma = -0.045H_\pi^1 + 0.001H_\rho^1 - 0.001H_\omega^1 - 0.002H_\rho'^1. \quad (1.4)$$

We expect to ultimately measure an asymmetry of $5 \times 10^{-8}$ with a 10% accuracy [[7]].
2.1.1 Conservation of Parity Symmetry
The transformation of parity, by definition, brings about a simultaneous reversal of the signs of coordinates:

\[ x \rightarrow -x, \quad y \rightarrow -y, \quad z \rightarrow -z \]  

(2.1)

or in spherical coordinate system

\[ r \rightarrow r, \quad \theta \rightarrow \pi - \theta, \quad \phi \rightarrow \pi + \phi \]  

(2.2)

A system subjected to the parity transformation may or may not exhibit behavior identical to that displayed by the original system. The former kind is said to be parity-even, while the latter is parity-odd. It can be seen from the transformation rules just shown (Eqn. 2.2), that for instance vectors of position \( \mathbf{r} \) and linear momentum \( \mathbf{p} \) are odd under parity transformation. Both are polar vectors. On the other hand, axial vectors, such as spin \( \mathbf{s} \) or angular momentum \( L \) are even under parity. Therefore for example the dot product \( \mathbf{s}_n \cdot \mathbf{k}_\gamma \) where \( \mathbf{s}_n \) is a neutron spin and \( \mathbf{k} \) is a linear momentum of a \( \gamma \)-ray, will change its sign under parity transformation, while \( \mathbf{s} \cdot (\mathbf{k}_n \times \mathbf{k}_\gamma) \) will conserve it. In general, one way to search for a parity-violating signal is by measuring a correlation which is odd under spatial inversion such as \( \mathbf{s}_n \cdot \mathbf{k}_\gamma \).
In quantum mechanics parity transformation is represented by the parity operator $P$. In this context, depending on whether or not parity the wave function is parity-even:

$$\hat{P}\psi(x, y, z) = \psi(-x, -y, -z) = \pm\psi(x, y, z), \quad (2.3)$$

where $\psi$ is the wave function of the given quantum-mechanical system. In terms of the Hamiltonian, which describes such a system, parity conservation ensures that the Hamiltonian $H$ commutes with $P$, the parity operator: $[H, \hat{P}] = 0$. Thus the parity operator of a quantum mechanical system satisfies $\hat{P}^2 = 1$ and has two eigenvalues: $\pm 1$.

Consider a simple Hamiltonian of a system of interacting particles,

$$H = -\sum \frac{\hbar^2}{2m} \left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right\} + V(r), \quad (2.4)$$

where $V(r)$ is the potential, that depends only on their relative positions. From the form of the Hamiltonian it is clear that both terms are even under the parity transformation - the first part only contains terms of $2^{nd}$ order in $x, y$ and $z$, while the second part, as mentioned, only depends on the relative coordinates, and is therefore parity even. The solution of this equation is naturally parity-even as well. The probability of finding a particle in the particular state should then be independent of the choice of the coordinates:

$$|\psi(x, y, z)|^2 = |\psi(-x, -y, -z)|^2, \quad (2.5)$$

or in spherical coordinates,

$$|\psi(r, \theta, \phi)|^2 = |\psi(r, \pi - \theta, \pi + \phi)|^2. \quad (2.6)$$

Thus, for instance, the probability of a particle’s emission at angles $\theta$ and $\pi - \theta$ with respect to some preferred direction should be equal in a PC process. In other words, the
expansion of the amplitude

\[ f(\theta) = |\psi(r, \theta, \phi)|^2 = a + b \cos \theta + c \cos^2 \theta + \ldots \]  

(2.7)

should not contain terms of odd powers: \( b = 0 \). Such is the case for electromagnetic and strong interactions.

The wave function of a system consisting of two particles, \( A \) and \( B \) can be expressed as

\[ \Psi_{A+B} = \psi_A \psi_B \chi_A \chi_B, \]  

(2.8)

where \( \psi_A \) and \( \psi_B \) are the intrinsic wave functions of the constituents, and \( \chi_A, \chi_B \) are terms describing their relative motion. The overall parity of the system can then be calculated from

\[ \hat{P} \Psi_{A+B} = \hat{P} \psi_A \hat{P} \psi_B \hat{P} \chi_A \hat{P} \chi_B = P_{A+B} = P_A P_B P_{A+} P_{B+} \]  

(2.9)

where \( P_A \) and \( P_B \) are the intrinsic parities of the particles \( A \) and \( B \) respectively. Thus parity is multiplicative. By expressing \( P_{\chi_{A,B}} \) via spherical harmonics, it can be shown that the parity of \( \chi_{A,B} \) is determined from \( P_{\chi_{A,B}} = (-1)^l \), where \( l \) is the relative angular momentum. This is best demonstrated for example by solving the problem of a hydrogen atom, where the spherical symmetry of the potential can be used and the variables can be separated.

According to the premise of the theoretical description of inter-nucleonic weak interaction which will be discussed in more detail later, the gamma-rays borne in the process of neutron capture on protons, will have an angular distribution with a term linear in \( \cos \theta \), where \( \theta \) is the angle between the linear momentum of the emitted photon and the spin of the neutron. In other words \( \gamma \)-rays emission will be directionally preferential towards the
spin of the neutron and therefore parity odd:

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi} (1 + A_\gamma \cos \theta) \tag{2.10}$$

where \(\frac{d\sigma}{d\Omega}\) is the differential cross-section and \(A_\gamma\) is the directional asymmetry.

### 2.1.2 Parity Non-Conservation in Weak Interactions

The phenomenon of parity violation in weak interactions had to be given serious consideration in the light of experiments which observed the decay of the \(K - meson\). The apparent conclusions from the observations of the group lead by Powell \([14]\) lead to the so called \(\tau - \theta\) puzzle: it was noticed, that in two instances, the the otherwise indistinguishable particles (later unified under the name of \(K - meson\)) could decay through two different channels;

\[ K^0 \rightarrow \pi^+ + \pi^- \tag{2.11} \]

and

\[ K^0 \rightarrow \pi^+ + \pi^- + \pi^0 \tag{2.12} \]

If one assumes that parity was conserved in these processes, as well as angular momenta and charge, it would appear that one is dealing with two distinct, though almost identical particles - \(\tau^+\) and \(\theta^+\) mesons. Whereas the the \(\theta^+\) meson decayed into two pions, with overall parity \(++\) the \(\tau^+\) decayed into three pions with total parity \(+-\). Since all mesons are parity eigenstates, particles decaying through these different mechanisms must also be different. Since pions consist of a quark-anti-quark pair, with respective intrinsic parities of 1 and -1, and since the relative angular momentum \(L = 0\) for these light mesons, the parity of the meson will be \(P_\pi = -(-1)^L = -1\). In 1956 Lee and Yang \([36]\) pointed out that the weak interaction which is involved in the process of the kaon decay, was previously
assumed to conserve parity by analogy with the strong and the electromagnetic forces. If
on the other hand, parity violation is associated with the weak forces, then the experiments
such this will make sense - \( \tau^+ \) and \( \theta^+ \) may in fact be the same particle.

Shortly thereafter, Madam Wu’s experiment, [[16]] confirmed this supposition. They
measured the angular distribution \( W(\theta) \) of the electrons, which in their experiments were
the product of the \( \beta - \text{decay} \) of polarized \( ^{60}\text{Co} \).

\[
W(\theta) = 1 + A(\vec{J} \cdot \vec{k}_e), \tag{2.13}
\]

where \( \vec{J} \) is the spin of the \( ^{60}\text{Co} \) nucleus, \( \vec{k}_e \) is the electron’s momentum and \( A \) is the
asymmetry and \( \theta \) is the angle between the momentum and the spin. Their product is the
parity odd term representing the PNC effect. In a series of careful measurements, Wu’s
group confirmed a presence of a non-zero asymmetry and hence of parity violation in the
weak interaction. In an independent measurement of the \( \pi \) and \( \mu \) decays at the Columbia
University cyclotron, Leon Lederman et al. confirmed Wu’s conclusions.

Measurements of PNC asymmetries ensued in the following years: in 1964, Abov et al.
[[1]] performed an experiment in which polarized neutrons captured on \( ^{113}\text{Cd} \) nuclei and
measured a \( \gamma - \text{ray} \) asymmetry of \((-3.7 \pm 0.9) \times 10^{-6}\). Later, in 1996 Lobashev et al. [[37]]
measured circular polarization of \( \gamma - \text{rays} \) emitted by the decaying polarized nuclei of \( ^{181}\text{Ta} \).

2.1.3 Hadronic-Weak Interaction
Nuclear studies of neutron \( \beta - \text{decays} \) have significantly
contributed to establish the \( V - A \) nature of the weak interaction and the conserved vector
current hypothesis that are the corner stones of the modern electro- weak standard model.
Research of weak interaction carried out today is much a search for weaknesses in the
standard model and violation of low-energy symmetries that may indicate a type of physics
in the expanded standard model.

The hadronic weak interaction at low energies has been studied in a series of difficult parity-violating experiments for the last 20 years. Nevertheless, there remain deep and unresolved questions. Though $\Delta S = 0$ parity violating interactions are simple at the quark level, experiments involve strongly interacting hadrons which makes very difficult to connect models and experimental signals.

2.2 Hadronic-Weak Interaction. Nucleon Level

In order to arrive at an analytical solution of the problem of the hadronic-weak interaction at low energies, one needs to consider the most simple process, involving the least number of particles, wave functions, transitions etc. Such is the case under the study of NPDGamma in which a neutron is captured by a proton and a bound state (deuteron) is formed by emitting a $\gamma$-ray. In this case the two body problem can be explicitly solved by considering individual matrix elements of the few states involved in the reaction.

At low energies, the nucleon interaction range is on the order of 1fm, comparable to the nucleon size. On a quark-quark level the weak interaction is well described by exchanges of gauge bosons $W^\pm$ and $Z^0$. But this conventional mechanism cannot be applied to the range of nucleon-nucleon interactions, since at 80.4 GeV and 91.2 GeV respectively the $W^\pm$ and $Z^0$ bosons are too massive to mediate the interaction between the nucleons. Over time $\Delta t$ of the nucleon-nucleon interaction the virtual exchange-particle of mass $m$ is formed. According to the uncertainty relation,

$$\Delta t \Delta E \geq \hbar,$$  \hspace{1cm} (2.14)

12
and

\[ m = \frac{\Delta E}{c^2} \implies \frac{\hbar}{c^2 \Delta t} \]  

(2.15)

which means that the distance traveled by the exchange quantum during the exchange is

\[ a = c \Delta t = \frac{c \hbar}{\Delta E} = \frac{\hbar}{mc} \]  

(2.16)

Hence the ranges of the gauge bosons are on the order of 0.002 \( fm \).

In the hadronic-weak interaction model, studied in this work, the heavy boson emitted by the nucleonic quark at the weak, parity-violating vertex, converts into the lightest meson, \( \pi (140 MeV) \), which then strongly couples at the opposite, parity-conserving vertex. Since the process includes two vertices, one that conserves parity and one that does not, the "overall" parity is not conserved (see Fig. 2.1).

Figure 2.1: \( \Delta S = 0 \) part of the hadronic-weak interaction, an example of a non-leptonic weak process.
2.2.1 Structure of the Nucleon-Nucleon weak interaction

The effective weak Hamiltonian for the $NN$ weak interaction can be written as a point interaction of two currents,

$$H_W = \frac{G_F}{\sqrt{2}} \left\{ J_W^a J_W^a + J_W^b J_W^b + J_Z^a J_Z^a \right\},$$

where $G_F$ is the Fermi constant, $J_Z$ and $J_w$ represent the neutral and charged currents, respectively.

Although the weak currents are coupled via the exchange of the intermediate charged and neutral bosons, in the low-energy weak processes the interaction can be considered at the quark level as a local four-fermion interaction leading to the weak current-current Hamiltonian.

Charged currents $J_W$ have $\Delta I = \frac{1}{2}, 1$ components. If $\Delta I = \frac{1}{2}$, the total isospin change due to $J_W J_W^a$ will be $\Delta I_{tot} = 1$. This term mixes $u$ and $s$ quarks and but this is Cabibbo suppressed (by $\sin^2 \theta_c$) with $\theta_c$ being the Cabibbo angle. For the case of $\Delta I = 1$, $\Delta I_{tot}$ can assume values of 0, 1, and 2. Since the Hamiltonian is Hermitian, the $\Delta I = 1$ components from $J_W J_W^a$ and $J_W J_W^b$ come with opposite signs, so they cancel out. Therefore only $\Delta I_{tot} = 0, 2$ remain. This means that the main contribution to the Hamiltonian is from the isoscalar ($\Delta I = 0$) and isovector ($\Delta I = 1$) neutral currents are responsible for the $\Delta I = 1$ channels.

The ordinary parity conserving low energy $NN$ interaction $V_{NN}$ is represented in terms of a sum of single ($\pi-, \rho-, \omega$-meson), or multiple meson exchanges. It is expected that the parity-violating interaction $V_{weak}$ can be also represented by the meson exchange except that one of the meson-nucleon vertices has to be weak, parity violating, while the other is strong, parity conserving (see Fig. 2.1).
Exchanges mediated by $\rho$ and $\omega$ are associated with $\Delta I = 0, 2$ processes. On the other hand, CP-violation [Barton’s theorem] forbids the exchange of neutral spinless mesons. This precludes $\pi^0, \eta$ and $\sigma$ mesons from being considered in this context.

![Diagram of lepton interactions](image)

Figure 2.2: Depiction of leptonic (a) and semi-leptonic (b) interactions.

Thus, NPDGamma is sensitive to the strangeness-conserving, $\Delta I = 1$ channel, that involves an exchange of the $Z^0$ boson and $\pi^\pm$ meson at the weak and the strong vertices, respectively. The momentum transfers of quarks in the nucleon are less than the QCD scale of $1\, GeV/c$, the quarks in this regime are permanently confined and thus quark-quark interactions can be observed through the $NN$ weak interactions in which they are involved.
At these energies quark-quark weak amplitude scale is

\[ \frac{g_W^2 m^2_\pi}{g_{\pi,NN}^2 M_W^2} \approx 10^{-6}, \]

where \( g_W \) and \( g_\pi \) are the weak boson and pion (strong) propagators, respectively. The weak processes, however, are overshadowed by the strong interactions, which possess much larger amplitudes. This creates a significant difficulty in regard to measuring weak effects. Under these circumstances parity violation helps to separate weak and strong effects, since only weak interactions violate parity.

\[ \text{Weak } NN \text{ Potential. The pseudo-scalar terms containing inner products involving spins and coordinates will, as discussed, violate parity. The contribution of each light meson to} \]

Figure 2.3: One meson exchange potential for the parity-violating \( NN \) interaction. One of the vertices is weak including the \( W \) and \( Z \) exchange, and one is strong.
the total weak potential, $V_{\text{weak}}$ is marked by the corresponding coupling constant, and is estimated in the DDH model [DDH] (see. section).

$$V_{\text{weak}} = \sum_\mu \sum_{\Delta I} H_{\mu}^{\Delta I} V_{\mu}^{\Delta I},$$  \hspace{1cm} (2.17)

where $H_{\mu}^{\Delta I}$ is the coupling constant that describes the contribution of the meson type $\mu = \pi, \rho, \omega, \ldots$, and $V_{\mu}^{\Delta I}$ is the corresponding potential. The selection of mesons included in the description of the DDH model 1.3 is limited by their Compton wavelength of $\sim 800 \text{MeV}$ taking into consideration the repulsion of nucleons at short ranges. Due to the relatively small magnitude of the weak potential, it is treated as a perturbation of the strong Hamiltonian.

$$H = H_{\text{strong}} + V_{\text{weak}}$$  \hspace{1cm} (2.18)

Then the solution of the Schrodinger equation with this Hamiltonian is a wave function

$$\psi' = \psi + \epsilon \phi,$$  \hspace{1cm} (2.19)

where $\psi$ is the eigenstate of the parity-conserving strong Hamiltonian, and $\phi$ is the parity-odd state and

$$\epsilon = \frac{\langle \phi | V_{\text{weak}} | \psi \rangle}{\Delta E}$$  \hspace{1cm} (2.20)

is the coefficient which quantifies the PV admixture into the resulting wave function, $\Delta E$ is the energy difference between the two states.

The pion is the lightest meson, therefore, as discussed, it is responsible for the longest range of the interaction and thus makes the largest contribution to the potential. The $V_{\text{weak}}$ can be derived from the Lippmann-Schwinger equation for the T-matrix for two particle
system with a Hamiltonian

\[ H = H_0 + V, \]  

where the potential \( V \) contains both parity conserving (\( V_{PC} \)) and parity non-conserving (\( V_{PNC} \)) terms \([45]\)

\[ T = V + VGT \]  

with the general solution of the form

\[ T = V + VGV, \]  

where \( G \) is the Green’s function of the interaction.

\[ G = \frac{1}{E - H_0 \pm i\epsilon}. \]  

Here \( E \) is the eigenvalue of the free Hamiltonian \( H_0 \), and the imaginary term in the denominator ensures that there is no singularity. The T-matrix elements between the initial \((\phi)\) and final \((\psi)\) states can be written as

\[ < \psi | T | \phi > = < \psi | V_{PC} \frac{1}{E - H_0} V_{PNC} | \phi > + < \psi | V_{PNC} \frac{1}{E - H_0} V_{PC} | \phi >. \]  

Which demonstrates the mixed \( PC \) and \( PNC \) of the weak potential. The final expression for the weak potential is obtained by substituting \( V_{PNC} \) and \( V_{PC} \) calculated in the Standard Model framework \([3]\), into 2.25 and Fourier transforming the result, in order to extract the potential from the matrix element. As specified, we are particularly interested in the pion contribution to the weak potential, which now can be written explicitly as:

\[ V_\pi^1 = \frac{i}{m} \left[ \vec{I}_1 \times \vec{I}_2 \right]_{\pm} (\vec{\sigma}_1 + \vec{\sigma}_2) \left[ \vec{p}, \frac{e^{-m_\pi r}}{4\pi r} \right], \]  

(2.26)
where \( m \) is the nucleon mass, \( \bar{I} \) is the isospin, \( \sigma \) and \( \vec{p} \) are the spin and momentum, \( \vec{r} \) is the relative coordinate. The term that contains the exponent comes from the Yukawa potential and contains the information about the meson’s interaction range. The term containing inner products of the \( \sigma \) and \( \vec{r} \) terms contributes to the parity violating effects.

2.2.2 Parity Violationg Electromagnetic Transitions in NPDGammaNPDGamma measures the correlation between the directions of the neutron spin and the propagation of the \( \gamma \)-ray radiated in the process of the deuteron’s formation.

A bound state the deuteron, is the product of the \( n + p \to d + \gamma \) reaction. It is formed when the initial, unbound state formed by the neutron and the proton transitions to the final bound state, by emitting a 2.2 MeV \( \gamma \)-ray. An emission of a photon is described by a plane wave, which then can be represented as the multipole radiation, by expanding the expression into terms according to the angular momentum, \( l \), carried away by the \( \gamma - quantum \).

In the present work we only consider the lowest order components of the multipole radiation - the dipole electric \( E1 \) and magnetic \( M1 \) transitions, for the contributions from the higher order transitions are negligible. Emission probability for the transition between the states \( \psi \) and \( \phi \) can be roughly estimated from

\[
\omega_{if} = \frac{2\pi}{\hbar} |< \psi | H' | \phi >|^2 \frac{dn}{dE} \propto \left( \frac{R}{\omega} \right)^{2l} \tag{2.27}
\]

where \( \frac{dn}{dE} = \frac{E^2 V d\Omega}{(2\pi \hbar c)^3} \) is the density of the final states and \( \omega \) is the frequency of the emitted \( \gamma \)-ray. Hence, when \( l \) increases by 1, the transition probability is reduced by 4 orders of magnitude. It should be noted that in general the intensities of the \( ML \) transitions are \((d/\mu)^2 \approx 10^2 - 10^3\) times smaller than \( EL \), where \( d \) and \( \mu \) are the electric and magnetic
moments of the given nucleus.

Figure 2.4: The lowest order allowed elestromagnetic transitions in the process of deuteron formation - $n + p \rightarrow d + \gamma$.

Looking at the origin of the dipole moments one can see how they behave under the parity transformation. E.g. the electric dipole transition contains terms proportional to the electric dipole moment, which exhibit properties of polar vectors. On the other hand the dipole magnetic transition contains terms magnetic dipole moment - an axial vector. Therefore the parity of the corresponding transitions is determined from $P_{E,J} = (-1)^J$ and $P_{M,J} = (-1)^{J+1}$, for the electric and magnetic terms respectively.

As mentioned, we limit the number of possible electro-magnetic transitions from the excited states, by considering only the lowest-order multipoles. However, we can further reduce the number of candidates to be considered, by employing selection rules pertaining to the angular momenta and isospins involved in the reaction.

Empirically, the bound state is formed by a neutron-proton pair with parallel spins,
with total spin $S = 1$ relative angular momentum $L = 0$. The excited state decays to the bound state, denoted as $^3S_1$, from the two main initial S-wave states $^1S_0$ and $^3S_1$ (we ignore the 4% admixture of the $^3D_1$ state due to which the end result changes negligibly). However, the weak interaction introduces additional states of opposite parity and angular momentum $L = 1$ such as 2.19 into both the initial and final states (see Fig. 2.4). The initial and final states that result from this mixing are superpositions of the allowed “pure” states described in detail below (see table 2.1).

The $^3S_1$ and $^3P_1$ mixed state comes about due to the weak $\Delta I = 1$ contribution, attributed to the pion, while the $^3S_1$ mixture with $^1P_1$ and that of $^1S_0$ with $^3P_0$ are created by the $\Delta I = 0$, 2 component of the weak potential, and is attributed to the $\rho$-meson [6]. The mixing of the S-wave states with P-wave states can in principle occur in both the initial and the final states. Each of the states however has to comply with the selection rule applicable to the quantum numbers of a two-nucleon system: $L + S + I = 2n + 1$, where $n$ is an integer.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$I$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$</td>
<td>initial\rangle$</td>
<td>$</td>
<td>^1S_0, 1\rangle$</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>final\rangle$</td>
<td>$-$</td>
<td>$</td>
<td>^3S_1, 0\rangle$</td>
</tr>
</tbody>
</table>

Table 2.1: Initial and final states with their spins, angular momenta and isospins.

The main channel by which the neutron-proton pair arrives at the final (bound) state is the $M1$ transition, induced by the strong interaction. Additional electric dipole $E1$ transi-
tions appear from the admixed states. Imposing additional constraints on the possible combinations of initial and final states allows to narrow down the criteria in selecting the states and transitions that participate in the reaction. For example, in mirror nuclei with \( N = Z \), \( E1 \) transitions with \( \Delta I = 0 \) are absent therefore matrix elements \(<^3 P_1,1|E1|^1S_0,1 >\), \(<^3 S_1,0|E1|^1P_1,0 >\) and \(<^1 P_1,0|E1|^3S_1,0 >\) are excluded. Transitions between identical states such as \(<^3 S_1|M1|^3S_1 >\) are not allowed through \( M1 \), since after the dipole operator has acted on the initial state it will be orthogonal to the final, so that the matrix element vanishes. The resulting transitions can be seen in fig. 2.4. Note, that states in the admixture possess the same total angular momentum thus asymmetry cannot originate in transitions from \( J = 0 \) states, \(^1S_0\) and \(^3P_0\). The interference of the \( E1 \) from \(^3S_1\) and \(^3P_1\), and \( M1 \) transitions from \(^1S_0\) ultimately contribute to \( A_\gamma \). Each of these transitions change the isospin by \( 1 : \Delta I = 1 \).

The rate of the radiation can be calculated from the matrix element according to Fermi’s Golden rule (2.27) where the Hamiltonian is

\[
H_{EM} = - \int d^3x \mathbf{j}(x) \cdot \mathbf{A}(x,t),
\]

(2.28)

where \( \mathbf{j}(x) \) is the current density, and \( \mathbf{A}(x,t) \) is the quantized field of the emitted photon. In terms of plane waves the vector potential can be written as

\[
\mathbf{A}(x,t) = \sqrt{\frac{\hbar c^2}{(2\pi)^3}} \sum_{j,m,\lambda} \frac{(-1)^{l-J_i+J_f}}{\sqrt{2l+1}} \times \\
\times \int \frac{k^2dk}{\sqrt{2\omega_k}} \left[ b_{jm\lambda} e^{-i\omega_k t} f_{jm\lambda} - b^\dagger_{jm\lambda} e^{i\omega_k t} f_{jm\lambda} \right],
\]

(2.29)

(2.30)

where

\[
f_{jm\lambda} = \int \mathbf{e}_{k\lambda} e^{-ikr} D_{m\lambda}^{(j)}(\hat{k}) d\hat{k},
\]

(2.31)
$b_{jm\lambda}$ and $b_{jm\lambda}^\dagger$ are the photon destruction and creation operators respectively (they are required in the quantum-mechanical picture of the electro-magnetic field), $D_{m\lambda}^{(j)}$ is the Wigner d-function, $\lambda = \pm 1$ describes the helicity state, and $e_{k\lambda}$ is the unit helicity vector. The d-functions generalize the potential with respect to the direction of the $\gamma$–emission by rotating $k$. Without this, the photon is assumed to propagate along $\hat{z}$.

On the other hand a multipole expansion can be used to separate the electric and magnetic parts of the vector potential:

$$A_m^l = (M_m^l(\bar{r}) - \mu E_m^l), \quad (2.32)$$

where

$$M_m^l = \frac{1}{\sqrt{l(l+1)}} (r \times \nabla) [j(kr)Y_{lm}(\hat{r})] \quad (2.33)$$

and

$$E_m^l = \frac{1}{\sqrt{l(l+1)}} \left( k r j_l Y_{lm} + \frac{1}{k} \nabla \left[ Y_{lm} \frac{d}{dr} (r j_l) \right] \right) \quad (2.34)$$

where $j_l(kr)$ are spherical Bessel functions, and $Y_{lm}(\theta, \phi)$ are the spherical harmonics. Thus the Hamiltonian can be expanded into the multipole terms.

At this point, one can separate the angular distribution part of the matrix element by employing the Wigner-Eckart theorem. One has only to use the lowest order transitions, while observing the selection rules that apply to the helicity and angular momentum of the photon. The resulting expression will then include the reduced matrix elements. The matrix elements vanish unless

$$|J_i - J_f| \leq l \leq |J_i + J_f| \quad (2.35)$$
and

\[ \lambda = M_f - M_i, \]  

(2.36)

where \( J_{i,f} \) and \( M_{i,f} \) are the angular momenta and spins of the initial and final states respectively. Plugging the remaining terms of Eqn. 2.29 into 2.27 one can obtain

\[ \omega_{i,f} = \frac{k}{4\pi} \left( 1 - 2\sqrt{2} \frac{| < E1 > |}{| < M1 > |} \cos \theta \right) | < M1 > |^2 d\Omega_k, \]  

(2.37)

hence the asymmetry of the angular distribution of the photons emitted in the reaction

\[ A_\gamma \propto Re \epsilon \frac{<^3 P_1 | E1 | ^3 S_1 >}{<^3 S_1 | M1 | ^1 S_0 >} = -2\sqrt{2} \frac{< E1 >}{< M1 >}, \]  

(2.38)

where \( < E1 > \) and \( < M1 > \) are the expectation values of the electric and magnetic dipole operators, which is related to \( f_\pi^1 \) as

\[ A_\gamma \approx -0.107 \cdot f_\pi^1. \]  

(2.39)

\[ 2.3 \text{ Compound Nuclei} \]

2.3.1 Nuclear Enhancement of PNC Effects In the previous section is we mentioned that the weak contribution to the Hamiltonian is considered a perturbation due to its small size compared to the strong channel. The observables associated with the weak interaction are therefore are harder to detect experimentally. Spurious asymmetries and backgrounds make the task even harder which emphasizes the importance of accumulating abundant statistics and eliminating potential systematic effects in NPDGamma. On the other hand NPDGamma we are dealing with a relatively simple two-body system, for which the matrix
Figure 2.5: Gamma-Ray Asymmetry in radiative neutron capture. The cartoon depicts an angular asymmetry in the $n + p \rightarrow d + \gamma$ experiment.

elements and wave-functions for the initial and final states can be calculated explicitly. Because of that, it is possible to make the connection between the theoretical model behind the process and measured asymmetries.

Conversely, when dealing with heavier, more complicated nuclei, one has to consider the great multitude of the quantum states ($\sim 10^6$) involved in the nucleus before and after the neutron is absorbed, and be able to calculated the matrix elements behind all possible $\gamma$-transitions, which makes direct inferences about the weak potential etc. virtually impossible. However in the presence of a large number of nuclear states with small energy-spacing, $\delta E$ parity-violating effects can be dramatically augmented. Such was the case with the $^{18}F$ measurements (see ??). This amplification is thought to happen through mechanisms known as **kinematic and dynamic** enhancements [[44]]. In this case, although due to the complexity of the nuclear structure direct analytical connections are difficult to
make, some important characteristics of the weak interaction can studied by using a “statistical” approach, in which all matrix elements are assumed to be random statistically independent variables, in order to obtain the RMS of the asymmetry, $A_\gamma$.

Dynamic and Kinematic enhancements  

The PNC asymmetry for the case of compound nuclei, as derived by Sushkov and Flambaum can be expressed as

$$A \propto \frac{2 < \psi_s | H_{ew} | \psi_p >}{E_s - E_p} \sqrt{\frac{\Gamma^n_s}{\Gamma^n_p}}$$

where $E_{s,p}$ - Energies of the $S$ and $P$ resonances. $\Gamma^n_{s,p}$ - width of these resonances [[44]], or in terms of Eqn.(2.20)

$$A \propto 2\varepsilon \cdot \sqrt{\frac{\Gamma^n_s}{\Gamma^n_p}}$$

Dynamic enhancement arises due to the rapid, near exponential, decrease of the spacing $D$ between levels the same spin and different parity, as the excitation energy increases. As can be seen from eqn. 2.40 the PV effect increases with the increasing matrix element and a decreasing difference of the energies of the $S$ and $P$ resonances. The denominator is inverse-proportional to the number of excited states, $N$ participating in the transitions, which dynamically enhances the observed asymmetry. On the other hand the value of the compound matrix element, compared to its single-particle counterpart is suppressed by a factor of $\frac{1}{\sqrt{N}}$ [[28]]. Therefore the ratio of in front of the square root grows with $\sqrt{N}$.

Typically the width of the $S$-resonances exceeds $\Gamma_p$, therefore $\sqrt{\frac{\Gamma^n_s}{\Gamma^n_p}} > 1$, resulting in the so-called kinematical enhancement. Under these circumstances the probability of the long-living state mixing into the neighboring one before the decay is increased.
The overall scale of the enhancement, typically on the order of $10^5$ but is dependent on the particular structure of the nucleus in question. However it should be noted that the dynamic enhancement does not strongly depend on the excitation energy, therefore the experiments can be performed at different energy ranges with respect to the neutron threshold. This is not the case with the kinematical enhancement, which is prominent at low neutron energies, where the capture is likely to occur in the $P$ and $S$ resonances.

### 2.3.2 PNC asymmetry. Statistical Approach

We alluded to the complexity of explicit calculations of parity-violating observables in medium and heavy nuclei due to the number of states involved. The manifold of the states, can however be used to the study’s advantage, by taking a statistical approach, in which each of the individual matrix elements that describe the $\gamma$–transitions are considered independent random variables.

As mentioned before at low energies the main contribution to the parity violating asymmetry is the interference of the $E1$ and $M1$ dipole transitions. The NPDGamma experimental setup measures the asymmetry between the rates counted in pairs of detectors, in current mode. Such asymmetry can be written as

\[
A_\gamma = 2\text{Re} \left\{ \varepsilon \cdot \frac{\sum_{J_f} \langle J_f^p | E1 | J_i^p \rangle \langle J_i^p | M1 | J_f^p \rangle}{\sum_{J_f} (| \langle J_f^p | M1 | J_i^p \rangle |^2 + | \langle J_f^p | E1 | J_i^p \rangle |^2)} E_{\gamma,i,f}^4 \xi F(J_T, J_i) \right\},
\]

where the subscripts $i$ and $f$ denote the initial and final compound states, and $p$ and $p'$ in the superscripts are their respective parities. $F(J_T, J_i)$ is the angular momentum coupling factor [V.V. Flambaum, O.P. Sushkov, Nucl. Phys. A.435, 352, 1985] and
with $S_n$ being the neutron separation energy. This factor $\xi$ accounts for the fact that the detectors in current mode measure all $\gamma$’s, independently of whether or not they originated in a $PV$ transition. The number of states is determined from [46]

$$\rho(E) = \frac{A}{E + 1MeV} e^{\alpha \sqrt{E}},$$

where $E$ is the energy of the compound nucleus, after the $\gamma$–ray is emitted.

Now, by definition the RMS of the numerator is calculated as the expectation value of its square.

$$4 \left( \sum_{J_f} |\langle J_f^p | E1 | J_f' \rangle|^2 \langle |J_f^p| |M1| J_f^p > E_{\gamma,i,f}^4 \right)^2$$

We can use our assumption that the matrix elements comprising the expression are random variables, to write

$$4 \sum_{J_f} \langle |\langle J_f^p | E1 | J_f' \rangle|^2 \langle |J_f^p| |M1| J_f^p > E_{\gamma,i,f}^8 \rangle$$

At this point we point out that

$$\langle | J_f^p | E1 | J_f > |^2 \rangle = \frac{\Gamma_{E1}}{2\pi\rho_f(S_n)}$$

and

$$\langle | J_i^p | M1 | J_f > |^2 \rangle = \frac{\Gamma_{M1}}{2\pi\rho_f(S_n)}$$

where $\Gamma_{E1}$ and $\Gamma_{M1}$ are the electric and dipole transition rates, and $S_n$ is the neutron
separation energy. Using these substitutions and replacing sums with integrals we arrive
at the final expression for the numerator’s RMS.

\[ RMS_{num} = \frac{\Gamma_{E1}\Gamma_{M1}}{\pi^2 \rho_f^2(S_n)} \int_0^{S_n} E_{\gamma}^8 \rho_f(E_{\gamma}) dE_{\gamma} \]  

(2.49)

In similar fashion, for the denominator we have

\[
\left\langle \left( \sum_{J_f} \left( |J_f^p| M1 |J_i^p| \right)^2 + |J_f^p| E1 |J_i^p'\right)^4 \right) \right\rangle 
\]

\[ = \left( \int_0^{S_n} E_{\gamma}^4 \frac{\Gamma_{E1}}{2\pi \rho_f(S_n)} \rho_f(E_{\gamma}) dE_{\gamma} \right)^2 + \left( \int_0^{S_n} E_{\gamma}^4 \frac{\Gamma_{M1}}{2\pi \rho_f(S_n)} \rho_f(E_{\gamma}) dE_{\gamma} \right)^2 \]  

(2.50)

Hence

\[ RMS_{den} = \frac{\Gamma_{E1}^2 + \Gamma_{M1}^2}{4\pi^2 \rho_f^2(S_n)} \left( \int_0^{S_n} E_{\gamma}^4 \rho_f(E_{\gamma}) dE_{\gamma} \right)^2 \]  

(2.51)

and the RMS of the asymmetry distribution becomes

\[ A_{\gamma}^{RMS} = 2\epsilon \cdot F(J_T, J_i) \xi \sqrt{ \frac{\Gamma_{E1}\Gamma_{M1}}{\Gamma_{E1}^2 + \Gamma_{M1}^2} \int_0^{S_n} E_{\gamma}^8 \rho_f(E_{\gamma}) dE_{\gamma} / \left( \int_0^{S_n} E_{\gamma}^4 \rho_f(E_{\gamma}) dE_{\gamma} \right)^2} \]  

(2.52)

2.3.3 Weak Spreading WidthEqn. (1.18) shows that in the two-nucleon \( \bar{n} + p \rightarrow d + \gamma \)
reaction the \( \gamma \)-asymmetry \( A_{\gamma} \) has a simple expression where parity violating component \( \epsilon \)
is given by Eqn. (1.6). In the neutron capture by proton the parity violating potential
mixes the initial or final states with same spin but opposite parity and then the matrix
element is divided by the energy difference of the states.

In the compound nuclei the \( \gamma \)-asymmetry is product of complicated initial and final
compound states as shown by expression Eqn. (1.20). The weak interaction quantity \( \epsilon \) that
carries on the weak interaction is also formed by the weak mixing matrix element between the compound nuclear states and now divided by the average distance between compound nuclear levels $D$.

The quantity $\epsilon$ can be also expressed with the hadronic weak spreading width $\Gamma_W = 1.8^{+0.4}_{-0.3} \times 10^{-7}$ eV [[20]] which is expected to be nearly constant as a function of mass number $A$. The use of the weak spreading width removes most of the level density effects.

$$\epsilon^2 = \frac{\Gamma_W}{2\pi \rho_i D^2} \approx \frac{\Gamma_W}{2\pi D},$$

(2.53)

where $\rho$ is the density of compound states and $D$ is a single-particle level spacing.

All the parameters for the calculation of the RMS of the $\gamma$-asymmetry of Eqn. (no number??) for a specific compound nuclei can be now performed as demonstrated in Ref. [[23]].

A more interesting task is to use Eqn. (2.52) and in this thesis measured $A^{RMS}_\gamma$ for an extraction of the variance of the weak matrix element $M_J^2$. The individual weak matrix elements are assumed to be mean-zero random variables.

$$\Gamma_W = 2\pi \frac{M_J^2}{D_J},$$

(2.54)

where $D_J$ is the averaged spacing for levels with spin $J$. In order to calculate the weak matrix elements for a given nucleus from measured asymmetries, one needs to incorporate spectroscopic information.
CHAPTER 3

The NPDGamma Experiment at LANSCE

The NPDGamma experiment took place in Experimental Room 2 (ER2) of the Lujan Neutron Scattering Center at the Los Alamos Neutron Science Center (LANSCE), in Los Alamos, NM. Figure 3.1 shows an areal view of the LANSCE accelerator complex.

Figure 3.1: Areal view of the LANSCE accelerator complex. In the foreground is the 800 m long proton linac. Before the end station - a large building at the end of the linac - \( p^- \) beam is deflected to the Proton Storage Ring (PSR) located next to the Lujan building indicated by arrow.

Low-energy neutrons are produced by spallation process in the LANSCE short-pulse
spallation source. First, protons $p^-$ are accelerated by the linear accelerator to 800 MeV in 625$\mu$s-long macro pulses which are then accumulated in the proton storage ring (PSR) and compressed into triangular 250 ns wide in base pulses before being released at a frequency of 20 Hz towards the spallation production target. The 800-MeV protons interact with the tungsten target and through the spallation process produce neutrons in the MeV energy range. These neutrons are moderated by water or cold $H_2$ moderators down from epithermal to cold neutron energies. Total of 16 flight paths are viewing the moderators. The neutron guide of the flight path 12 (1FP12) views a novel upper tier cold hydrogen moderator that is operated in back-scattering geometry. The LANSCE spallation source consists of two cylindrical tungsten targets, each $\approx 10$ cm in diameter and $\approx 7.5$ cm and 27 cm long with a 14-cm gap between them. The moderators are either around the gap or below the lower tungsten target. In the moderators the neutrons undergo several elastic and inelastic scatterings and some of the neutrons are absorbed by hydrogen. After the moderator, the neutrons have a close to the Maxwellian energy distribution with the most probable kinetic energy of

$$E_n = \frac{3}{2}k_BT,$$

(3.1)

where $k_B$ is the Boltzman constant and $T$ should be the moderator temperature if the neutrons are fully thermalized. From Fig. 3.2 the temperature of 30-40 K is obtained for the maximum of the neutron distribution. This means that the neutrons are not at equilibrium when they exit the moderator. Using the measured moderator brightness of Fig. ??, the dimensions of the 1FP12 neutron guide, and average delivered proton current on the spallation target of $\approx 100\mu A$, one finds that about $10^{12}$ neutrons per second will
enter the guide. Normally the brightness is calculated by neutron transport simulations but in the case of 1FP12 the NPDGamma collaboration measured the brightness since it is a quantity that defines a limit to the statistical sensitivity of the experiment.

Figure 3.2: The measured 1FP12 moderator brightness. The measurement was performed by the NPDGamma collaboration [we need to give a reference here Pil et al.!!!]

Located about 20 m from the spallation source is the radiological shielding housing of the NPDGamma experiment. Figure ?? shows a 3D schematic view of the setup of the NPDGamma apparatus at the end of the neutron guide. The components of the experiment and their functions are in the order in which the neutrons interact with them:

- Beam monitor #1 - used to normalize the neutron beam for the experiment,
- $^3$He polarizer - used to polarize the neutron beam,
- beam monitor #2 - with monitor #1 used to measure the beam polarization,
- spin flipper - used for frequent reversal of the beam polarization
- LH2 target - neutron capture by protons take place in the target,
- γ-d detector array - detects 2.2-MeV γ-rays from the neutron capture reaction,
- analyzer - a polarized ³He cell to analyze beam polarization,
- beam monitor #3 - together with monitor #2 and analyzer is used to measure beam polarization after the LH2 target,
- guide field coils - the experiment is immersed to a homogeneous 10-Gauss magnetic field which is used by polarizer and the spin flipper. The field also maintains the direction of the beam polarization in the experiment.

One of the main components of the apparatus is the CsI detector array that covers a solid angle of about 3π of the γ-rays produced when the neutrons are captured in the target in the center of the detector array.

Since the experiment measures a parity-violating asymmetry in γ-ray yields relative to the direction of the neutron spin,

$$\frac{d\sigma}{d\Omega} = (1 + A_\gamma \cos \theta_{k_\gamma} \cdot \vec{\sigma}_n)$$

(3.2)

the neutrons incident on the target have to be polarized. Here $\vec{k}_\gamma$ is the γ-ray momentum and $\vec{\sigma}_n$ is the neutron spin. This is accomplished by the neutron beam polarizer, where one of the two neutron spin states is filtered out by the polarized ³He gas contained in a glass cell. The orientation of the neutron spin is defined by the vertical uniform static magnetic field $B_0 (\approx 10 \text{ G})$, created by four “race track coils” mounted symmetrically around beam in such a way that they encircle the entire setup. Three beam monitors measure the neutron flux in transmission as a function of time of flight. The monitors are used to deduce the
beam polarization, track beam fluctuations, and study the performance of the spin flipper.

Since parity violating $\gamma$-asymmetries are very small, in the range of $10^{-7}$, systematic effects that can produce false asymmetries for the experiment, have to be identified and controlled below the statistical limit of the experiment that is defined mainly by the neutron beam intensity.

![Figure 3.3: 3D conceptual view of the NPD$\gamma$ Experimental Setup.](image)

To control some of the systematic effects, the Radio Frequency Spin Flipper (RFSF), located between the polarizer and the target, is used for the frequent reversal of the neutron spin. By using a specific 8-step spin sequence, the first and second order effects in the detector system, that could introduce a false asymmetry in the measurement, are canceled.

Data acquisition and analysis are governed by a network of electronics and computers
organized into a local network.

The experimental cave serves as a radiological shield for the personnel in ER2, a magnetic return yoke for the 10-Gauss magnetic field, a magnetic shield against outside static fields, and as a Faraday cage for the experiment. Most of the communication between the experiment in the cave and outside world took place through fiber optics.

3.1 Flight Path 1FP12 and Neutron Guide

Figure 3.4: Vacuum envelope and the exit window of the 1FP12 neutron guide.

The function of the neutron guide is to transport neutrons from the moderator to the experiment without losses and changes in phase space. The 1FP12 guide is 20 m long, has a $m = 3$ super-mirror (SM) coating, and a cross-sectional area of $9.5 \times 9.5\text{cm}^2$. When operated the guide is under vacuum of less than $10^{-4}$ bar, see Fig.3.4. The $m = 3$ super-mirror surface has the critical reflection angle ($\theta_c$) three times larger than that of
$^{51}Ni$-coated surfaces: $\theta_c(m = 3) = 3 \times \theta_{^{51}Ni_c}$. During the commissioning of the guide system in 2003 and 2004 the phase space of the neutrons from the guide was studied by measuring beam profiles and waveforms as a function of neutron energy in order to verify the performance of the guide. The reflectivity at the end of the 9 m long guide section was found to be consistent with the data provided by the manufacturer for a 50 cm long guide element. Figure 3.5 shows a typical measured reflectivity curve of 1FP12 SM guide element using 4.27 Å neutrons as a function of $m = \theta_c/\theta_{^{51}Ni_c}$.

![Figure 3.5: Reflectivity of a 50 cm long section of the 1FP12 neutron guide as a function of the glancing angle using 4.27 Å neutrons.](image)

3.1.1 Measurement of the Flight Path Length

One of the main advantages of a pulsed neutron source is the availability of accurate time-of-flight information. Coupled with the
accurate knowledge of the length of the flight-path, a precision determination of the neutron energies is possible, which is a prerequisite for many of fundamental physics experiments. Since a direct measurement of the flight-path length is not possible, we must resort to alternative methods. One of these methods is based on diffraction, when neutrons are transmitted through a crystal they are scattered the neutrons coherently and elastically according to Bragg’s diffraction law:

$$n\lambda = 2d \sin \theta.$$ 

Here $n$ is the diffraction order parameter, $\lambda$ is the neutron wavelength, $d$ is the lattice spacing (or the distance between the lattice planes), and $\theta$ is the incident angle with respect to the given crystal plane. When the neutron wavelength satisfies the Bragg condition, the total cross-section has a discontinuity and as a result the diffracted beam shows characteristic peaks whereas the transmitted neutron spectrum contains sharp edges (Bragg edges) (see Fig. 3.7).

The edges occur for a given $hkl$ combination, where $h$, $k$, and $l$ are the Miller indices that characterize the lattice. The Bragg angle increases with the wavelength until the “critical angle” is achieved at $2\theta = 180^\circ$. Beyond this angle no scattering can occur from this particular set of $hkl$ indices. This results in an increased transmission, until the conditions arise appropriate to the next $hkl$ set.

The total scattering cross-section responsible for the Bragg edges can be calculated by integrating the expression for the cross-section for a single primitive cell over all sets of lattice planes with a $d$ – spacing smaller than $\lambda/2$. The discontinuity in the cross-section
Figure 3.6: Total neutron cross-sections for some polycrystalline materials.
causes the jump in the intensity at the particular neutron wavelength.

In a process of determining the length of the flight path, 1FP12, i.e. the distance between the surface of the moderator and the end of the neutron guide, using the time of flight (TOF) information contained in the transmitted neutron spectrum, it is imperative to accurately determine the TOF position of the given Bragg edge. For some pure crystals the wavelengths of their Bragg edges are accurately enough known from neutron scattering experiments. For instance, Be has three edges at 3.483 Å, 3.597 Å, and 3.98 Å. Using these data and the neutron wavelength

\[ \lambda = \frac{h}{mv} = \frac{ht}{mL}, \]  

(3.3)

where \( h = 6.63 \times 10^{-34} \ m^2kg/s \) is the Planck’s constant, \( L \) is the flight path length, \( t \) is the time of flight, and \( m = 1.675 \times 10^{27} \ kg \) is the neutron mass, we can extract the flight path length \( L \), assuming that we know the time-of-flight \( t \).

In the measurement the neutron beam was transmitted through a 10 cm long room temperature Be block. Two beam monitors positioned downstream of the Be target measure the neutron TOF spectra [8], p.19. In the NPDGamma experiment the DAQ sampling scheme is set in such a way that each neutron 50 ms long macro-pulse is divided into 100 bins, so that each time bin is 0.4 ms long. In order to achieve greater precision in the flight path length measurement, we reconfigured the data acquisition so that the macro-pulse consisted of 2500 time bins. Hence increasing the time-resolution by a factor of 25 to give \( \Delta t = 16 \mu s \), which corresponds to spacial resolution of \( \Delta L = 1.6 \) cm for 5.23 meV neutrons.

There are a few simple ways to describe the Bragg peaks such as using a Dirac \( \delta \)-
Figure 3.7: Neutron transmission through a room temperature Be block. Note: the peak around 22\textit{ms} is due to Bragg scattering in Al (at 4.7Å) that is present in abundance throughout the experimental setup. The red and blue curves correspond to the signal taken without the Beryllium target.
function or a Gaussian. The transmission data, containing the Bragg edges can be then obtained by integrating over the diffraction pattern.

Figure 3.8: Transmission signal after the Be target measured by Monitor 3.

The vertical edge with clearly defined start and end points which one would expect to see in an ideal case, are not seen in a measured signal. Rather the signal edges have a small slope and the endpoints are rounded, washing out a straightforward way to find the edge parameters (see Figs. 3.8, 3.9).
Figure 3.9: Monitor 2 measured neutron transmission after the Be target and its time-derivative.
The rounding of the edges and the slope in the transmission spectra is mainly produced by three processes: 1) the response time constant of the detector electronics; 2) a washout of the time-resolution due to the moderation processes (250 μs); and 3) a thermal motion of atoms in the crystal (contribution to the elastic incoherent, the so-called “Doppler Broadening”). The first one is defined by the time constant of the preamplifier (6 μs) and by the time constant of the filter giving the total time constant of ≈ 100 μs [30].

Because of the moderation process the neutrons with the same kinetic energy will not leave the moderator at the same time. Instead they will have a TOF distribution meaning that they arrive at the detector at slightly different times, causing a broadening of the edges.

We located the Bragg edges from the measured transmission spectrum by selecting a linear region on the corresponding slope and choosing its center as the TOF for the edge (see Fig. 3.8). The values obtained in this way were later verified by taking the time-derivative of the transmission data and locating the peaks (see Fig. 3.9). Both methods yielded consistent results which also agree with direct length measurement during the construction phase of the flight path.

For each monitor (M2 and M3) spectra where the Bragg edges were observed, three TOF ranges were chosen to provide a reasonable linear fit. Then an average and a standard deviation were calculated for a time of flight for the corresponding Bragg edge. The results for the three edges in each monitor signal were then combined to produce the final flight path length:

\[ L_0 = 21.10 \pm 0.03 \, m \]  

(3.4)
\[ L_1 = 21.11 \pm 0.03 \, m \]  
\[ L_2 = 21.49 \pm 0.03 \, m \]  
\[ L_3 = 22.81 \pm 0.04 \, m, \]

where \( L_0 \) is the distance from the moderator to the end of the guide, \( L_1, L_2, \) and \( L_3 \) are the lengths up to the centers of \( M_1, M_2 \) and \( M_3 \), respectively, (first, second and third downstream beam monitors). The systematic errors are on the order of \( \leq 100 \, \mu s \).

Hence neutrons reaching the target inside the \( \gamma \)-ray detector array (\( \sim 22.3 \, m \) from moderator) at time of flight of \( \sim 25 \, ms \) (peak signal), the spacial resolution of \( \leq 4 \, cm \) will correspond to TOF-resolution of

\[ \Delta t = \frac{4 \times 10^{-2}}{22/25 \times 10^{-3}} = 45 \, \mu s \]

or energy resolution of

\[ \Delta E = 0.013 \, meV \]

which is significantly higher than the limit set by the NPDGamma DAQ sampling rate: \( \Delta \text{TOF}=0.4 \, ms \).

### 3.2 Frame Definition Chopper of 1FP12

As described above, neutron macro pulses are produced at the rate of 20 Hz, i.e. the macro pulse or time-of-flight window is 50 ms long. After the moderator the neutrons enter the guide with the acceptance angle defined by the supper-mirror coating \( (m=3) \). If unobstructed, the slow part of the neutron spectrum does not reach the experiment before...
the next neutron pulse is coming out from the moderator due to the length of the flight path. The fast neutrons overlap time-wise with the slow neutrons from the previous pulse leading to an admixture of neutrons with different energies, and thus diffuse the critical knowledge of the neutron energy. In order to eliminate these slow neutrons, 1FP12 utilizes a frame definition chopper which is located at 9.38 m from the surface of the moderator and includes two blades rotating independently at 1200 rpm. Each of the blades is 102.4 cm in diameter and covers 4.38 rad. The area of the blades interacting with neutrons are plasma coated with a thick layer of Gd$_2$O$_3$ which was measured to be black (no neutrons getting through) for neutron energies up to 30 meV due to the Gd enormous neutron absorption cross-section. The chopper is used to absorb the slow neutrons at the tail of the time-of-flight spectrum when either one or both of the blades cover the beam. Since the flight path is about 21 m long and the neutron time of flight frame is 50 ms long, the slowest neutrons that reach the end of the guide in each pulse have an energy of about 1 meV.

The effect of the chopper to the neutron wave form is illustrated in Figs. (3.10) and (3.11).

With a single frame definition chopper all slow neutrons in the beam cannot be removed. In the NPDGamma experiment where the experiment is about 21 m from the moderator, a few neutrons from the \((n - 2)\)-th frame (where \(n\) is the current frame) can still leak into the current frame. This is apparent in the “chopped” part of the monitor signal shown in Fig.(??), the weak step in the transmission signal is the neutron image of the opening chopper edge for 0.1 meV neutrons.

The frame overlap chopper housing between two neutron guide sections is seen in
Figure 3.10: Monitor signal when the chopper is on and phased to T0 (open circles) and monitor signal when the chopper is parked so that the beam is not blocked (squares). The two plots are not normalized.
Figure 3.11: A zoomed in part of the Fig. 10. time of flight frame around 34-50 ms. The step in the monitor signal is caused by very slow neutrons, two frames earlier. The step is an neutron image of the moving chopper edge two frames earlier.

Fig.(3.12). Both of the chopper blades have their own motor which allows them to be operated independently.

The chopper aperture is fully open 4 \textit{ms} after \(T_0\) and then the chopper starts eclipsing the beam at 27 ms. As can be seen from the shape of the monitor signal, the number of neutrons at high energy side of the neutron spectrum (TOF<10 ms) is low relative to neutron number between 10 and 30 ms, in addition, their neutron absorption cross-section is inversely proportional to the neutron’s velocity, the high energy neutrons result in poorer statistics measured by the \(\gamma\)-ray detectors. Therefore, a DAQ timing scheme was implemented, where the chopper phase with respect to \(T_0\) is not changed, but the acquisition of the data commences 10 ms after \(T_0\) enters the electronics. At the same time the chopped part of the spectrum was lengthened, providing a total of \(\simeq 15\text{ms}\) long TOF window to study the backgrounds, while the neutrons were isolated from the experiment by the chopper.
Figure 3.12: 1FP12 frame definition chopper between two guide sections. Each chopper blade has its own motor
Figure 3.13: 1FP12 Neutron Beam Chopper housing between two guide sections.

3.3 Neutron Beam Monitors

Three commercial neutron beam monitors were used in the experiment to measure transmitted beam intensities. The first monitor, \( M_1 \), is located at the downstream end of the neutron guide. \( M_1 \) measures the intensity of the beam, before it interacts with components in the experiment, therefore, \( M_1 \) is used to normalize the neutron beam for the experiment. The next monitor, \( M_2 \) was located downstream of the polarizer thus giving the intensity of the beam transmitted through the polarizer cell. Monitors \( M_1 \) and \( M_2 \) are used to measure the beam polarization in relative beam intensity measurements. The ”back-monitor”, \( M_3 \), is a thick monitor absorbing most of the beam. It was used to measure the beam intensity transmitted through the target. \( M_3 \) is used to measure the beam polarization passing through the target, the efficiency of the spin-flipper, and then
the otho-para ratio of the LH$_2$ target [Libertad tech note] (see text below).

The internal structure of the monitors is nearly identical. The monitors are parallel-plate ion chambers with an active area of 12x12 cm$^2$, large enough to cover the beam area of the 1FP12 guide of 10x10 cm$^2$. The monitors consist of three 0.5 mm thick Al plates encased inside the Al housing (grounded). The two outer plates are connected to $-3\, kV$ HV power supplies ($-300\, V$ for $M3$). The middle plate collects the charge and is connected with a short BNC cable to a pre-amplifier, where the current signal is converted to voltage. The ion chambers are DC-coupled current mode detectors with a response time of less than 0.1 ms. They are designed to be stable, linear, insensitive to $\gamma$-rays, and to produce minimal background.

![Figure 3.14: Neutron beam monitor - a parallel-plate $^3$He ion chamber.](image)

The monitors are filled with a mixture of $^3He$, $^4He$ and $N_2$ (see appendix Table[?]). The amount of $^3He$ in the first two monitors is small so that only about 4% of the incident beam is absorbed. In the case of $M3$ however, the $^3He$ thickness is large enough so that a significant fraction of the beam is absorbed.

The neutrons in the monitor are detected through the capture reaction,
\[ n + ^3He \rightarrow T + p + 765keV, \]  

(3.10)

where the absorption cross section has the $1/v$-dependence. Note that the final total energy after the capture is independent on the initial energy of the neutron.

The $N_2$ gas in the gas mixture decreases the recombination time of the produced ions thus making the signal a little faster. One of the requirements for the monitor is that it needs to be insensitive to $\gamma$-rays. To measure $\gamma$-ray sensitivity the neutron beam was blocked by a $^6Li$-loaded epoxy plate, which absorbed all neutrons but did not produce an effect on the $\gamma$-ray flux. Results of the measurement with $M2$ are shown in Fig. (3.15).

### 3.4 $\gamma$-Ray Detector Array

The central component of the experiment is the $\gamma$-ray detector array made up of 48 thallium doped cesium iodine crystals. The $\gamma$-rays from the neutron capture reactions produce scintillation light in the crystals mainly through Compton scattering ionization. The excited $Cs$ and $I$ atoms decay to the atomic ground state by emitting $540\,nm$ photons which then are detected by the vacuum photo-diodes (VPD) attached to the crystals.

The detectors are arranged to cover a solid angle of $\sim 3\pi$ for the $\gamma$-rays. The 48 detectors are grouped in four annular sets with 12 detectors per ring and each ring perpendicular to beam and centered on the beam axis, see Fig. (3.16). Collimation and neutron shielding around the detector array ensure that most of the detector signal is produced by the target. In the data analysis each detector can be used as a single detector but typically detectors on the opposite side of the target are paired up, when determining $\gamma$-asymmetries.

A single detector is formed by two optically coupled blocks of CsI(Tl) crystals each with
Figure 3.15: M2 signals when a $^6$Li loaded epoxy tile is blocking the neutron beam compared with pedestal signals taken before and after the $^6$Li measurement. The pedestal signal is mainly produced by electrical noise.
an equal volume of $14.7 \times 14.7 \times 7.35 \text{ cm}^3$. The surface of the combined crystal is treated to be a diffuse reflector for light, and the detector crystal is encased in a hermetically sealed aluminum housing. The size of the detector was defined by interaction length of 2.23-MeV $\gamma$-ray; the design assumes that 90% of the energy of the 2.23 MeV $\gamma$-rays is absorbed by the crystal. The scintillation light is transmitted to VPD through a 7.6 cm diameter $K^+$ glass window in the housing. To keep light losses minimal, optical grease is used between the window and the VPD. To minimize electrical noise such as that caused by ground loops, 90 V batteries are used to bias the VPDs. This bias voltage is supplied to the VPD by two 45 V batteries mounted on top of the pre-amplifier housing of the VPD. In addition, each detector has two light emitting diodes (LED) for diagnostics.

Due to high $\gamma$-ray rates (>100 MHz) and the length of the scintillation light pulse in the CsI(Tl) crystal (up to $\sim 1\mu$s) the detectors have to be operated in current mode. The photo cathode of a VPD converts the light into a charge which is subsequently converted to voltage and amplified by low-noise (20 fA/\sqrt{\text{Hz}}) solid-state pre amplifiers [[?]] before the signal is read by 16-bit ADC in the data acquisition system. The time constant of the VPD signal is limited by 100 ms specially constructed filters in the preamplifier circuit. Another reason for using the VPDs is their insensitivity to magnetic field changes. A change in magnetic field can affect not only the VPDs but also the gain of the detector system. This kind of effect on the signal is called “multiplicative noise” as opposed to ”additive noise” which is an addition of any spurious signal, such as electronic pick-up, to the detector signal. Systematic effects from both multiplicative and additive noise must be well below the $5 \times 10^{-9}$ level [[24]]. An experiment running in current mode is possible only if the total noise level of the detectors is significantly below the signal level. The
statistical noise (Johnson noise) of the amplifier and environmental noise such as 60 Hz ground loop noise, added to the detector signal by the detector electronics must be small compared to the signal shot noise in order to maintain γ-ray counting statistics. The noise level must be small enough to enable measurements of beam-off systematic effects such as pedestals, multiplicative, and additive noise contributions in a reasonable amount of time. An additional requirement is that any pickup of environmental noise cannot be correlated with the neutron spin state at the < 10⁻⁸ level.

Figure 3.16: The detector array with the radio frequency spin flipper mounted onto the upstream end of the detector.

3.4.1 γ-ray Counting Statistics
A statistical error in a counting experiment follows the Poisson statistics where the statistical error is given by the standard deviation which is a root-mean-square (RMS) of the deviations, \( \sigma = 1/\sqrt{N} \) where \( N \) is number of counts. In an experiment operated in current mode, the statistical performance of the experiment is not
as straightforward to define, the statistical uncertainty is defined as a shot noise caused by $\gamma$-rays interacting with the detectors. The standard deviation for the shot noise is given by

$$\sigma_s = \sqrt{2qIf_B},$$

(3.11)

where $q$ is the charge created on the photo-cathode of the VPD by each $\gamma$-ray, $I$ is the photo-current and $f_B$ is the bandwidth of the preamplifier.

Figure 3.17: Distribution of $\gamma$-ray asymmetries measured from a natural boron target "Counting statistics". Solid line is a Gaussian fit to the data. As a comparison a distribution of asymmetries for pedestal runs (electrical noise) is shown. The two histograms indicate that the instrumental noise will not have effect to the asymmetry measurements.

Different error (or noise) sources will increase the RMS value of this counting statistics. Since the CsI crystals do not fully stop the 2.2 MeV $\gamma$-rays there is a fluctuation of the energy loss of $\gamma$-rays in the crystals ([31], [24]). This fluctuation will add about 7\% to the
RMS of the distribution according to a simulation. Other instrumental errors will do same if they are of a significant level.

The RMS value of the shot noise was verified with a measurement where the RMS width of the detector signal was measured using a target containing $^{10}$B (20% by weight in natural boron). The n-$^{10}$B capture is known to produce a single $\sim 478$ keV $\gamma$-ray with the 93.7% branching ratio in the reaction:

$$n + ^{10}B \rightarrow ^{4}He + ^{7}Li + \gamma(478keV) + 2.312MeV$$  \hspace{1cm} (3.12)

Together with other known parameters such as aperture, number of neutrons per pulse, the solid angle of the detector, backgrounds, beam losses before the target, and depolarization in target, one can estimate an expected RMS width of a signal when the pulse counting is consistent with Poisson statistics. Of course, since the measurements are conducted in current mode, the appropriate conversion from current has to be made. The RMS width measured with the Boron target had to be corrected for noise and background contributions which have to be significantly smaller in order to isolate the counting statistics. A result of the Boron measurement showed a good agreement with the "beam-on" RMS with that expected from 3.12.

3.5 Commissioning the NPDGamma Apparatus with Nuclear Targets

Due to the interaction of the neutron beam with other materials than the target during the experiment, measurements had to be performed to check that these materials do not produce $\gamma$-ray significant asymmetries that would affect the NPDGamma result. Therefore, we measured $A_{\gamma}$ on Al [[31]], Cu, In, Pb, SST, Li, and B [[31]] to the level of sensitivity
Figure 3.18: List of mass number $A \sim 50$ nuclear targets measured during the 2005 run cycle. The main criteria for selecting the targets are listed for each target; high capture cross section, small scattering cross section, and small incoherent cross section.
that is less than $A_\gamma$ on hydrogen. Based on well known large asymmetry, a $CCl_4$ target was used to verify the performance of the apparatus.

In addition $A_\gamma$ was measured on nuclear targets around mass $A = 50$ to set the upper limit for the PV effects on these nuclei. Measurements were performed on $CCl_4$, $Co_3O_4$, $Sc_2O_3$, $Ti$, $V$, $Mn$, $Cu$ and $In$. These targets were chosen based on their large neutron absorption cross section to provide sufficient statistics for the measurement. Also as discussed in the Theory section close level spacing near the threshold is conducive to the creation of admixed states and hence a parity violating asymmetry. The small incoherent cross sections reduce the chance of spin-flip scattering. In general, one of the criteria in selecting the target materials was that the scattering cross section be as small as possible compared to the radiative absorption cross section. This will ensure that the neutrons scattered from the target, that, as discussed above, can be a source of background $\gamma$-rays upon capture in the surrounding elements of the apparatus, are negligible in comparison to the fraction of those captured in the target. The measurements on In were repeated in order to reduce the error on the asymmetry measurement.

The targets used were in liquid ($CCl_4$), powder ($Co_3O_4$, $Sc_2O_3$, $Ti$, $V$ and $Mn$) and solid ($Al$, $Cu$, $In$) forms as shown in Fig. (6.7) of the appendix.

3.6 Magnetic Guide Field

The experiment was immersed into a 10-Gauss vertical homogeneous magnetic field produced by a coil system consisting of four race track shape coils in the double Helmholtz condition around the beam and a number of shim coils, see Fig. (3.20).

The function of the guide field is to
Figure 3.19: Schematic view of the vessel containing powder targets (Ti, V, Sc, Co, Mn).

The targets are loaded into the cylindrical can and packed by the plunger seen in the drawing.
- preserve the direction of the neutron spin from the polarizer to the analyzer cell,
- to be homogeneous enough for the operation of the \( ^3 \)He polarizer and RF spin flipper,
- to be homogeneous enough between the spin flipper and the target to avoid the Stern-Gerlach up-down steering. In order to avoid field inhomogeneities all components of the apparatus were required to be non-magnetic. Several shim coils were mounted to correct the direction of the field. The field is required to be vertical with the accuracy of ~ 1° which was achieved with the shim coils that also help to minimize field inhomogeneities created by the asymmetric and little magnetized cave walls.

Figure 3.20: Four race track coils installed in the experimental cave around the experiment.

When a polarized neutron moves in a magnetic field, loss of polarization has to be considered. If the rate of change of the magnetic field direction seen by a moving neutron is significantly slower than the Larmor frequency of the spin in the static magnetic field,
then the projection of the spin on the field direction is conserved and the spin follows the
direction of the magnetic field adiabatically. Since the cold neutron energy is low and the
magnetic field is homogeneous, the depolarization due to this mechanism is insignificant.

Another possible source of the systematic effect related to the magnetic field is the
Stern-Gerlach up-down steering; a neutron moving in a static magnetic field is deflected
by gradients of the field. Especially dangerous is the steering of the beam in the up-down
direction which would change the distribution of neutrons respect to the detector. This
would change the solid angle and lead to a false asymmetry. The neutron spin ($\mu_n$) moving
in a magnetic field $B$ experiences a force $\vec{F} = \mu_n \nabla B$. If the NPDGamma guide field has a
gradient $\partial B/\partial y$, then an up-down steering (Stern-Gerlach steering) of the neutrons occurs.
Therefore, the field homogeneity requirement of less than 1 $mGauss/cm$ was required in a
beam volume between the spin flipper and the LH2 target.

In summary, in the polarizer volume, the field gradients were measured to be

$$\frac{dB_x}{dx} < 0.5 \frac{mG}{cm}, \quad \frac{dB_y}{dx} < 0.5 \frac{mG}{cm}, \quad \frac{dB_z}{dx} < 6 \frac{mG}{cm}$$

the latter value is caused by slightly magnetic neutron guide vacuum tube. Along the
beam direction the gradients were

$$\frac{dB_x}{dz} < 1 \frac{mG}{cm}, \quad \frac{dB_y}{dz} < 0.4 \frac{mG}{cm}, \quad \frac{dB_z}{dz} < 1 \frac{mG}{cm}.$$ 

The $B$-field characteristics indeed met the specifications of the experiment [[7]].
Figure 3.21: $B_y$ field component measured in the beam direction ($\hat{z}$) on different x- and y-positions
Figure 3.22: $B_x$ component of the field measured in the direction of the beam ($\hat{z}$) on different x- and y-positions.
Figure 3.23: $B_z$ field component measured in the direction of the beam ($z$) on different x- and y-positions.
Figure 3.24: Field Gradients across the Polarizer Cell.
3.7 Resonant Radio Frequency Spin Flipper

Measurement of the $\gamma$-ray asymmetry implies a measurement of $\gamma$-ray yields in the detectors relative to the neutron spin direction which is defined by the direction of the static magnetic field, $B$. Ideally, if we would have 48 identical detectors with same efficiency and with same solid angle, then from the signals of the opposite detectors the up-down asymmetry could be determined. In reality, it is not possible to match the gains of the detectors or their solid angles within precision of $10^{-9}$. Therefore, the experiment frequently switches the beam polarization direction between the $\uparrow$- and $\downarrow$-spin states using the Radio Frequency Spin Flipper (RFSF). Detector signals corresponding to the $\uparrow$- and $\downarrow$-spin states, may also change with beam fluctuations. The function of the RFSF is to reverse the spin direction of the beam, thereby canceling out main systematic effects, which could otherwise produce a false $\gamma$-asymmetry. Rather than using a simple $\uparrow$ and $\downarrow$ combination of the spin states, the NPDGamma experiment employs a "sequence" of eight spin states $\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow$ or its compliment which cancels systematic effects up to the second order. Also the frequent reversal of the spin direction takes care of slow changes in the detector efficiencies caused by temperature drifts, activation of the crystals, or other changes in detector environment.

3.7.1 Principles of the Operation of the Spin Flipper

In the laboratory reference frame the spin of the neutron moving in the static magnetic field $B_0\hat{z}$ rotates about the $+\hat{z}$-axis with the angular velocity of $\omega_L$ determined by $B_0$. Since the adiabaticity requirement is met the $z$-component of the spin is a constant of the motion. When the neutron enters the spin flipper, in addition of the static field the neutron sees the RF field of the spin
Figure 3.25: Spin Flipper installed inside the Guide Coils, visible is the aluminum can which shields the experiment from the RF field of the flipper.
flipper. The RF field with amplitude $B_1$ can be viewed in the laboratory reference frame as a rotating field about the $z$-axis with the same angular frequency $\omega_L$ if tuned to the resonance. As discussed in the polarimetry section, the oscillating field can be considered to be formed by two field components rotating in opposite directions; one at the resonance frequency and one at twice the resonant frequency. The second component, being very far from resonance has negligible effect on the neutron spins. In the rotating ($\omega_L$) reference frame about the $\hat{z}$-axis, the RF field cancels the effect of the static field and left is only a constant field component $B_1$ which will torque the neutron spin causing it to precess about $+\hat{y}$-axis. The relation between these quantities was discussed in chapter Polarimetry. The rate at which the spin will incline away from the $\hat{z}$-axis is given by $\omega_{tip} = \gamma_n B_1$, where

$$\gamma_n = 1.83 \times 10^8 H z/T$$

is the neutron gyromagnetic ratio. How large is the tipping angle depends on the time $\Delta t$ that the neutron spends in the RF field which is determined by the neutron velocity.

$$\theta = \Delta t \gamma_n B_1,$$  \hspace{1cm} (3.13)

where $\theta$ is the "tip"-angle. For the purposes of this experiment $\theta$ has to be equal (or a multiple of) $180^\circ$. A neutron moving with velocity $v_n$, spends time $\Delta t = \frac{d}{v_n}$ inside the spin flipper, where $d$ is the length of the coil. Since the neutron velocity is inversely proportional to the measured time-of-flight of the neutron, then

$$\Delta t = \frac{d}{v_n} = \frac{d}{L t_{tot}}.$$  \hspace{1cm} (3.14)
Therefore the amplitude $B_1$ has to be selected for each neutron velocity, so that the spin of the neutron is rotated by exactly $180^\circ$. Since the $B_1$ is not constant on the neutron trajectory inside the spin flipper, therefore it is rather the integral of $B_1(r, z)$ that has to satisfy the equation.

$$\int B_1(r, z)dz = \frac{\pi L}{\gamma \tau_{t_{\text{tof}}}}$$

(3.15)

where $L$ is the length of the flight path, and $t_{\text{tof}}$ is the time of flight.

### 3.7.2 Spin Flipper Control Electronics

The Spin Flipper consists of a coil, wound concentrically around the axis of the neutron beam. The coil is encased in an $Al$ cylinder with 5 mm thick walls. The neutron windows at the ends of the cylinder are 0.5 mm-thick $Al$ plates. Considering the $\sim 0.5$ mm skin depth of aluminum at 29kHz, the aluminum housing constitutes an efficient RF shielding preventing the RF-field coupling into the detector electronics. Since the RF field of the spin flipper is correlated with the neutron spin state, the coupling of the RF power to the detector signal would create a false asymmetry. This is one of the most dangerous instrumental source of false asymmetries, and has therefore be possible easily to be verified during experiments.

Figure 3.26 shows the control electronics of the spin flipper. The spin flipper has two states; RFSF on and RFSF off. In the on-state the AC current is flowing in the coil, in the off-state the current is directed to a dummy load which has about same resistive load as the coil. The AC current in the spin flipper coil is a sine wave driven by a generator (FG2 in Fig. 3.26) with frequency of $f = 2\pi/\omega_L$. The sine wave is modulated by an exponentially decaying voltage signal (from FG1) which is phased with the start of the neutron pulse $T_0$.
given by the facility. The amplitude of the sine wave and the ramp is tuned to match the neutron energy so that the each neutron spin will be rotated by 180 degree. Figure 3.27 shows the RFSF current signal during the neutron pulse. As discussed above, the shape of the modulated envelope is related to $\frac{1}{t_{TOF}}$ which ensures that all the spins with different energies in neutron pulse are reversed. The current in the spin flipper coil is measured by ADC and then multiplied by the time of flight resulting a constant. This result is used on-line to monitor the tuning and the phase of the RFSF current during an experiment.

![Spin Flipper control electronics diagram](image)

Figure 3.26: Spin Flipper control electronics.

The 29 kHz sine wave corresponding to the neutron resonant frequency in a static field of 10G is generated by function generator FG2 and modulated by the $\frac{1}{t_{TOF}}$ shape by FG1, see Fig (3.27). The amplitudes of the two signals have to be matched. This is achieved
by constantly feeding back the amplitude of the rectified sinusoidal signal to the difference amplifier with the help of a pick-off transformer. The difference amplifier, connected the input of FG2 adjusts the amplitude of the signal accordingly. A capacitor connected in series with the RFSF coil cancels out the imaginary part of the impedance of the resonant circuit on resonance. The $1/t_{tof}$ shape of the signal is ensured by multiplying voltage and current waveforms by the time-of-flight. The $\uparrow$ and $\downarrow$ states of the neutron spins are determined by the switch-box, which depending on whether the spin-state is up or down directs the current to the Spin-Flipper coil or the equivalent resistive "dummy" load [26].

Figure 3.27: The RFSF current as a function of time of flight during a 50 ms long neutron pulse.
3.7.3 Measurement of the Spin-Flip Efficiency

During the first commissioning run in 2004, the method for measuring the RFSF efficiency involved a protocol in which the knowledge of absolute values of the $^3$He polarizations in both the polarizer and the analyzer was crucial. By definition, the spin-flip efficiency is the ratio of the number of $\uparrow$-state ($\downarrow$-state) neutrons before the RFSF to the number of $\downarrow$ ($\uparrow$)-neutrons after the RFSR. The total number of $\uparrow$-state ($\downarrow$-) neutrons after the RFSF consists of the spins which were reversed by the spin-flipper and the spins which were already $\uparrow$($\downarrow$) and were not affected by the RFSF.

\[
N_{p,\uparrow}^{on} = \epsilon N_{p,\downarrow}^{off} + (1 - \epsilon)N_{p,\uparrow}^{off}
\]

and

\[
N_{p,\downarrow}^{on} = \epsilon N_{p,\uparrow}^{off} + (1 - \epsilon)N_{p,\downarrow}^{off}
\]

where $\epsilon$ is the spin-flip efficiency, the ”on/off” superscript denotes the corresponding state of the RFSF and the ”$p$” subscript indicates that the neutrons have only been transmitted through the $^3$He in the polarizer. Propagating these neutrons further down the beam line through the analyzer, and separating the states where the RFSF is on and off, one obtains for the number of $\uparrow$-state and $\downarrow$-state neutrons at $M3$ respectively.

\[
N_{a}^{off} = N_{p,\downarrow}^{off} e^{-\sigma\alpha(1-Q)}
\]

\[
N_{a}^{on} = N_{p,\uparrow}^{on} e^{-\sigma\alpha(1-Q)}
\]

Then explicit expressions can be written for the total numbers of measured neutrons for the RFSF ON and OFF, and then the efficiency can be extracted by solving the equation
\[
\frac{N_a^{on}}{N_a^{off}} = 1 - \epsilon \left[ 1 + \frac{\cosh \sigma (nLP + mlQ)}{\cosh \sigma (nLP - mlQ)} \right],
\] (3.20)

where \(N_a^{on/\text{off}}\) are the total numbers of neutrons transmitted through the analyzer with the RFSF turned ON/OFF, \(nLP\) and \(mlQ\) are the number density, length of \(^3\text{He}\), and \(^3\text{He}\) polarization in the polarizer and the analyzer, respectively. The curve obtained from the ratio of the measured transmissions, \(\frac{N_a^{on}}{N_a^{off}}\) can then be fit to the function in Eqn. (3.20) to obtain \(\epsilon\). As already mentioned, this method requires knowledge of the polarization of the analyzer and polarizer. Section about the Analyzer describes the procedure in which these parameters were determined for the analyzer.

In 2005 modifications were made to the NMR system of the polarizer allowing reversal of the \(^3\text{He}\) polarization in the polarizer by means of Adiabatic Fast Passage (AFP) (see NMR section) by sweeping the RF frequency across the resonance. This created an opportunity to utilize a different approach to measure the RFSF efficiency. In this method, the underlying idea is the assumption, that to a good precision the spin reversal of the \(^3\text{He}\) by AFP and thus also the beam polarization is 100\% efficient. This point was verified by measuring the \(^3\text{He}\) polarization of the polarizer using the neutron transmissions, and was found to be \((\approx 40\%)\) and unchanged over the course of the measurements.

The optimum current of 18.53 A for the 0 \(G\) static magnetic field \(B_0\) and the RFSF current of 0.750 mA for the amplitude \(B_1\) were determined by searching the maximum spin-flip efficiency.

The spin-flipper efficiency measurements used different configurations of the polarizer, the analyzer, the spin-flipper, and the beam monitors, see fig. 3.28.
Figure 3.28: Configurations used in the spin-flipper efficiency measurements.

Figure (3.28) shows different arrangements for the spin-flip efficiency measurements:

1. (a) The spin-flipper is OFF, but the polarization directions of the two \( ^3He \) cells are anti-parallel and transmission through the spin flipper is \( T_{\uparrow\downarrow}^{\text{off}} \).

(b) Polarization directions of the polarizer and the analyzer are parallel, the spin-flipper is turned off, the polarization of the beam is defined by the polarizer and is unchanged after the neutrons pass the RFSF. The transmission with this configuration is \( T_{\uparrow\uparrow}^{\text{off}} \).

(c) The polarization directions of the polarizer and the analyzer are again parallel, but the spin-flipper is ON. The transmission is \( T_{\uparrow\uparrow}^{\text{on}} \).

It can be shown, that from the following expression the efficiency can be calculated to be
\[
\frac{1 + \epsilon}{2} = \frac{T^{\text{off}}_{\uparrow\uparrow} - T^{\text{on}}_{\uparrow\uparrow}}{T^{\text{off}}_{\downarrow\downarrow} - T^{\text{off}}_{\downarrow\downarrow}},
\]  

(3.21)

where \(\epsilon\) is the spin-flipper efficiency. The effectiveness of this method lies in the fact that only relative quantities such as transmissions measured directly are used to determined the efficiency of the spin-flipper. These transmissions are in principle energy dependent, but the efficiency constructed from Eqn. (3.21) is expected to be energy-independent, since the amplitude of the AC current supplied to the RFSF coil is designed to flip all neutron spins within the energy range of interest.

Figure 3.29: Spin Flipper Efficiency plotted as a function of TOF.

The curve corresponding to Eqn. (3.21) was fit to a constant in order to extract \(\epsilon\). The
structure of the curve is strongly effected by the electronic pedestals of the monitors. In order to reduce their effect the pedestals were subtracted from each signal. The electronic pedestal signals is thought having two components a ”DC”-component or the offset of the waveform due to slow drifts, and an ”AC”-component, visible as the structure in a monitor (or detector) signal taken in the absence of neutrons.

The RFSF efficiencies were measured both on and off the beam axis; 3.3 cm to left, right and above the beam center. Results are given in Table 3.1.

<table>
<thead>
<tr>
<th>Analyzer Position</th>
<th>RFSF Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center of the RFSF</td>
<td>0.997 ± 0.001</td>
</tr>
<tr>
<td>3.3 cm beam left</td>
<td>0.941 ± 0.001</td>
</tr>
<tr>
<td>3.3 cm beam right</td>
<td>0.938 ± 0.001</td>
</tr>
<tr>
<td>3.3 cm beam up</td>
<td>0.997 ± 0.001</td>
</tr>
</tbody>
</table>

Table 3.1: RFSF efficiencies measured at on-axis and off-axis positions.

Uncertainties and systematic effects in the RFSF efficiencies

The flipping-angle is fluctuating around 180° caused by

- the RF field of the RFSF is not uniform,
- the neutrons with the same energy have a time distribution when they exit the moderator,
- beam divergence,
- the variation of the flight path length, and
- finally, the kinetic energy of the neutron is affected if there is a difference in the strength of the static magnetic fields in the entrance and exit of the RFSF.
All these effects lower the spin-flip efficiency $\epsilon$ on $10^{-4}$ level \[26\].

![Pedestals](image)

Figure 3.30: Two electronic pedestals measured 1.5 h apart by M3.

The electronic pedestals in the neutron monitors closely preserved their shape from run to run over a few hours, although the amplitudes of the peaks corresponding to different times of flight fluctuated on the level of few percent. This is indication of the beam correlated noise in the beam monitor. Therefore, the “AC”-shape can be removed from the measured signal by subtracting the shape of the pedestal run taken immediately before or after the beam-on run.

The origin of the monitor pedestal noise was studied by making a fast Fourier transformation of the measured pedestal signal, see Fig. (3.31). The fundamental frequency is obtained to be 60 Hz and some of the harmonics have large amplitudes like 180 Hz.
In order to extract the DC part of the pedestal signal, the shape was integrated over a TOF range corresponding to a period of 180 Hz oscillations. The integral of the periodic part of the signal vanishes, leaving the DC offset. This offset is then subtracted from the signal. The effect of proton beam fluctuations was removed by normalizing the signals involved in the three transmissions to the corresponding signals in monitor $M_1$. The errors included in Table 3.1 are purely statistical, the error is estimated as the RMS width of the signal distribution for each time bin. The runs used for the efficiency measurements typically contained 1000 neutron pulses.
3.8 Data Acquisition

3.8.1 Sampling Scheme

In order to exploit the full dynamic range of the 16-bit ADCs (±10 V) used to read the detector signals, a method was chosen, where a "sum" signal is constructed for each ring by summing up the 12 signals of a ring and calculating their average. This way the signals are well above the digitization noise of the digitizers. At the same time, a difference signal corresponding to a given detector is created by subtracting the "sum" signal from the signal of the given detector. The "sum" and "difference" signals are later recombined during the analysis procedure, in order to reconstruct the original signal for each detector. Both sum and difference signals are scaled by a factor of 3 after passing through the high-pass Bessel filters, which removes the high-frequency part of the noise band. The difference signals are further amplified by a factor of 10. The error associated with the conversion into the integers is the order of $2^{-16}$.

A neutron macro pulse comes every 50 ms (20 Hz) together with a trigger signal $T_0$ from the Accelerator Facility. The DAQ samples the pre-amplifiers for the next 40 ms after each $T_0$. Every 50-ms time interval is divided into 100 time bins each 0.4 ms each. Since the sums are sampled at 62.5 kHz, while the differences at 50 kHz, the corresponding ADC count is a sum over 25 and 20 samples, respectively, and recorded as a single number for each of the 100 time bins. As a result each time bin contains effectively 20 and 25 samples from the difference and sum amplifiers. Then, in order to reconstruct the detector signal the following calculation takes place in the analysis stage. For a given ring then, the $i^{th}$ detector signal is reconstructed as follows:
The voltage signals from the monitors is stored in 100 time bins of 0.4 ms long.

![Diagram of Data Acquisition and Storage](image)

The three VME computers (365 MHz PC) are governed by the DAQ software written in C and PERL. They collect parts of the data from the modules connected to the measuring devices and transfer the data to the main storage computer (HAZEL). VME1 contains information about proton current. VME2 handles the data from the difference amplifiers for the 48 detector channels. VME3 has the sum-amplifier data as well as date from the beam monitors (M1, M2, and M3) and RFSF waveforms.

Since the data storage and computers are located outside of the experimental cave, they are connected to the VME computers via fiber-optic Ethernet.
Other stand alone PCs used in the experiment include a computer running LabView software to monitor and regulate the guide field, also connected to the Ethernet using fiber-optics, a PC controlling the polarizer and operating the NMR via an IGOR Pro procedure. An additional PC was used to change the position of the detector table for the determination of the solid angles of the individual detectors.

The data were stored on a 4TB RAID array (Redundant Array of Independent Disks), which was later complemented by another 4TB of storage. As a precaution, each run was duplicated in an "archive" disk, connected to the secondary computer "FIVER", used as the main analysis computer, so as not to interfere with the data-taking processes running on the main computer, HAZEL. The DAQ is required to handle data rates larger than 700 kbytes per second. A detailed diagram of the DAQ logic can be found in the appendix.[ref]

3.9 Shielding for Low-Energy Neutrons

Due to the fact that $^6\text{Li}$ has a very large neutron absorption cross-section with the $1/v$ energy dependence, $^6\text{Li}$-doped epoxy shielding is used in the experiment.

An advantage of use of $^6\text{Li}$-doped shielding as opposed to the $^{10}\text{B}$-doped polyethylene, is that the reaction products of the $n +^6\text{Li}$ capture do not include $\gamma$-rays while those of $n +^{10}\text{B}$ do:

$$n +^{10}\text{B} \rightarrow \alpha +^7\text{Li} + \gamma_{478 \text{ keV}} + 2.78 \text{ MeV}$$ (3.23)

and
\[ n + ^{6}Li \rightarrow \alpha + ^{3}H + 4.76 \text{ MeV}. \] (3.24)

The epoxy doped with enriched $^6Li$ was cast at the Oak Ridge National Laboratory, into various 1 cm thick shield plates. Some of these pieces were later fitted with proper circular openings for the beam collimators. Figure 3.33 shows one of the collimator pieces that was used after M2. Neutron capture on $^6Li$ and consequent radiation effect by the decay products can be seen in the plates used in the beam, like a collimator plate of Fig. (3.33). These radiation effects are also produced by $\gamma$-ray Compton scattering.

Figure 3.33: 2" $^6Li$-doped collimator (note the radiation damage surrounding the aperture).

$\gamma$-ray yield in the detector largely depends on the radiative neutron capture cross-
section, target thickness, and the incident neutron flux. Excessive $\gamma$-ray yield results in saturated detector channels. This were mitigated by reducing the gains of the detector pre-amplifiers or by reducing the neutron flux by shrinking the size of the beam aperture. Since the pre-amplifier gains can only be electronically changed by $\pm 20\%$, the reduced beam collimation had to be used. However, a change in the collimation had an effect on the polarization; hydrogenous constituents of the epoxy possess a significant scattering cross-section. Neutrons reflecting back from the epoxy shield will contribute to the monitor signal, and thus could affect, for instance, on the determination of the beam polarization.

3.10 Hydrogen Target.

The capture of cold neutrons in hydrogen has a small cross section compared to the scattering process in ortho- and para-hydrogen, hence most of the neutrons incident on the LH$_2$ target will scatter at least once before being captured, thereby making the spin dependence of the scattering a fundamental factor. Since the spin of the ground state of the para-hydrogen is zero, and the energy difference between the ground and excited states is 14.7 meV, only the capture and the coherent scattering are allowed in the interaction of cold neutrons ($E<14.7$ meV) with para-H$_2$. On the other hand, the interaction of the neutrons with the non-zero spin ortho-H$_2$ molecule can lead to both, coherent and incoherent scattering. However, the cross section for the latter is more than 50 times greater than that of the former ($\sigma_{\text{incoh}} = 20.052 \pm 0.014$ $b$ and $\sigma_{\text{coh}} = 0.439 \pm 0.003$ $b$ for incoherent and coherent scattering respectively for the neutrons with $v = 2200$ $m/s$ [[40]])

Due to this huge cross section of the spin-reversing collisions (Fig. 3.34), it is vital to keep the concentration of ortho-hydrogen to the minimum.
3.10.1 Ortho-Para Ratio. For the NPDGamma experiment the target is an aluminum cylinder containing 16L of LH$_2$ kept at 17K. At this temperature the concentration of ortho-hydrogen is limited to 0.02% (Eqn. 3.25) \[[38]\]

$$f_e = \frac{\sum_{J=1,3,5,...} d_o (J) e^{-\frac{BJ(J+1)}{kT}}}{\sum_{J=1,3,5,...} d_o (J) e^{-\frac{BJ(J+1)}{kT}} + \sum_{J=0,2,4,...} d_p (J) e^{-\frac{BJ(J+1)}{kT}}}$$ \( (3.25) \)

where $J$ is the rotational quantum state, $d_{o,p} (J)$ is the degeneracy of the para $(2J + 1)$ and ortho $(3(2J + 1))$ states, and $B = 7.35$ meV is the total rotational constant for the hydrogen molecule.

The time necessary to reach the equilibrium fraction (Eqn.(3.25)) at a determined temperature depends on the natural ortho-to-para conversion rate $K_n$, which for LH$_2$ is $(11.4 \pm 2.7) \times 10^{-3}$ $h^{-1}$ \[[12]\]. It has been shown \[[33]\] that the natural ortho-para conversion
time is
\[
t = -\frac{1 - f_e}{K_n f_e} \ln \left[ \frac{f_i (f_o - f_e)}{f_o (f_i - f_e)} \right],
\]  
(3.26)

where \( f_{i,e} \) are the initial \( (f_i = f_o|t=0) \) and equilibrium \( (f_e = f_o|t=\infty) \) fractions of the ortho-H\(_2\). Then for the liquid hydrogen at 17K with normal initial fraction \( f_i = 0.75 \) and a conversion rate of \( 12.7 \times 10^{-3} \text{ h}^{-1} \), it will take over 30 years to double the equilibrium fraction of ortho-H\(_2\). However it is possible to catalyze this process through the interaction of hydrogen with paramagnetic surfaces. This method accelerates the process greatly and makes the production of the necessary quantities of "enriched" para-H\(_2\) feasible. For the NPDGamma liquid hydrogen target, two \( FeO_2 \) ortho-to-para converters (OPC) were used (see Fig. (3.35)).

**Neutron Transmission through LH\(_2\)** As mentioned before, there are 3 monitors registering the flux along the beam line: from the data of the 1\(^{st} \) two monitors (M1 and M2) the beam polarization is measured, while using the data from M2 and M3 the transmission through the LH\(_2\) target is studied. Since the scattering cross section in LH\(_2\) is strongly dependent on the fractional concentration of the two species, the neutron transmission through the hydrogen can be used to monitor the ortho-para fraction in the target.

The ratio of the signals from M3 and M2 can be written as
\[
\frac{S_{M3, \text{ full}}}{S_{M2, \text{ full}}} = \frac{K_3}{K_2} T_{\text{other}} T_{H_2},
\]  
(3.27)

where \( S_{M2, \text{ full}} (S_{M3, \text{ full}}) \) is the signal from M2 (M3) when the target is full, \( K_2 \) (\( K_3 \)) - gain of M2 (M3), and \( T_{\text{other}} \) and \( T_{H_2} \) are transmissions through RF spin flipper, target vessel, air (included all together in "other" and hydrogen, respectively. Similarly, in the
Figure 3.35: A schematic view of the liquid hydrogen target. It is an aluminum cylinder containing 16L of LH$_2$ kept at 17K. Seen on the right is the neutron window, with the actual target located just behind. There are 2 ortho-to-para converters (OPC) installed to accelerate the ortho-para conversion which can be seen in the figure. OPC#1 is located in the fill line and promotes conversion before the hydrogen is introduced, and OPC#2 is installed in the recirculation loop.
case of the emptied target vessel one obtains

\[
\frac{S_{M3, \text{empty}}}{S_{M2, \text{empty}}} = \frac{K_3}{K_2} T_{\text{other}},
\]  

(3.28)

resulting in

\[
T_{H_2} = \frac{S_{M3, \text{full}}/S_{M2, \text{full}}}{S_{M3, \text{empty}}/S_{M2, \text{empty}}},
\]  

(3.29)

A typical set from such analysis is presented in Fig. (3.36).

![Figure 3.36: Transmission of neutrons through the LH$_2$ target after the ortho-hydrogen has reached the equilibrium fractional concentration. When the energy reaches values larger then the separation energy between two hydrogen states, the transition significantly reduces.](image)

3.10.2 Average Fraction of Para-hydrogen

The phase 1 of the NPDGamma production runs was planned to take place at LANL. It took place in 2006, in two separate periods:
August - September and November - December. The Fig. 3.37 shows the average fraction of the para-hydrogen during these periods. The target temperature and pressure information are recorded by the monitor target program every 4 sec. The average temperature of the liquid hydrogen was slightly lower during the 2nd period of 2006 run (16.7K during August-September and 15.2K during the November-December runs), resulting in higher para-hydrogen fraction (99.8 vs. 99.9%).
Figure 3.37: The average percentage fraction of the para-HL$_2$ in the target during Phase 1 NPDGamma Production run.
4.1 Polarized $^3$He

4.1.1 Optical Pumping of Rubidium

The NPDGamma experiment selected the $^3$He neutron spin filter method to polarize cold neutrons. The advantages of the $^3$He polarizer are that it effectively uses the neutron phase space determined by the 1FP12 neutron guide and that the $^3$He polarizer can operate in the same 10 G magnetic field that is used by the spin flipper. These features allowed the design of the experiment to be very compact which in turn decreased the loss of neutrons caused by divergence of the beam in this statistically limited experiment. The $^3$He polarizer also offers a possibility for a reversal of the beam polarization without any changes in the static magnetic field. The AFP of the $^3$He polarization in the spin filter will reverse the polarization of the beam. This is also an important feature when controlling systematic effects in the experiment. On the other hand, to cover the cross-section of the full beam the size of the polarizer cell had to have a size never before used. It has also been exposed to the neutron beam for longer time than in any other previous experiments.

In the $^3$He polarizer the unpolarized neutrons interact with the polarized $^3$He gas which is contained in a glass cell. Since the large spin dependent absorption cross section the neutrons with their spin opposite to the $^3$He polarization will be absorbed and the parallel spin state transmitted through the cell. The $^3$He is polarized in the cell by optically polarized Rb atoms.
The polarization of the $Rb$ atoms in the cell is defined by the spin state of its single valence electron. Due to a nearly spherical symmetry of the Rb atom and a single valence electron, the $Rb$ atom can be treated similarly to the hydrogen atom. In the presence of a weak magnetic field, the otherwise degenerate ground state $5S_{1/2}$ and the excited state $5P_{1/2}$ split, by virtue of Zeeman Effect, into $5S_{1/2}, m = +1/2, 5S_{1/2}, m = -1/2$, and $5P_{1/2}, m = +1/2, 5P_{1/2}, m = -1/2$ sub-levels, here $m$ gives size of the spin component. Suppose that we impose an external magnetic field $\vec{B} = \hat{z}B_0$ parallel to the $z$-axis. This field will exert a torque on the atomic magnetic moment of the Rb atom, and the energy associated with the interaction between the magnetic moment and the external field will be

$$E = g_F \mu_B \vec{B} \cdot \vec{F} = g_F \mu_B B_z F_z$$  \hspace{1cm} (4.1)$$

where $\mu_B = e\hbar/2me = 9.274 \times 10^{-24} J \cdot T^{-1}$ is the Bohr magneton and $g_F$ is the dimensionless Lande $g$ factor. Since the total angular momentum vector is quantized, so is the energy:

$$E_m = g_F \mu_B B_m F.$$  \hspace{1cm} (4.2)$$

Here

$$g_F = g_J \frac{F(F + 1) + J(J + 1) - I(I + 1)}{2F(F + 1)}$$  \hspace{1cm} (4.3)$$

and

$$g_J = 1 + \frac{J(J + 1) + S(S + 1) - L(L + 1)}{2J(J + 1)},$$  \hspace{1cm} (4.4)$$

where $I$ is the nuclear spin, $S$ and $L$ are the spin and orbital angular momentum of the electron that add up to its total angular momentum, $J = L + S$, and $F = J + I$ is the total
angular momentum of the atom. The electron spin interacts with its angular momentum, which creates additional energy levels according to the spin’s orientation relative to \( L \).

![Figure 4.1: Atomic Levels of \(^{87}\text{Rb}, I = 3/2\)](image)

This leads to the fine splitting of atomic levels as indicated in Fig. 4.1. Similarly, but on a much smaller scale, the interaction between the electronic and nuclear magnetic moments results in the splitting of the energy levels into the hyperfine levels. In the presence of a weak external magnetic field the spin of the nucleus and that of the electron contribute to enhancement of the energy of the system through the Zeeman interaction.

Quantum mechanically these interactions are described by the following Hamiltonian

\[
H = A \mathbf{I} \cdot \mathbf{S} + g_s \mu_B S_z B_z - \frac{\mu_I}{I} I_z B_z, \tag{4.5}
\]

where \( A \) is the dipole coupling constant, \( \mu_B \) is the Bohr magneton, \( g_s = 2.00232 \), the gyromagnetic ratio of the electron. The first term represents the hyperfine interaction,
while the second and the third stand for the electron and nuclear Zeeman energy; coupling of the nuclear and electron spins with the external magnetic field.

Circularly polarized photons of resonant frequency $\omega$ and positive helicity, $\sigma_+$, from a laser light, can be absorbed by the $Rb$ atoms. They excite the single valence electron of the $Rb$ atom from the ground state $5S_{1/2}, m = -1/2$ into the excited state $5P_{1/2}, m = +1/2$. The atom relaxes back to the ground state by emitting a photon. The probability of decay to the $+1/2$ and $-1/2$ sub-levels of the ground state is defined by the corresponding Clebsch-Gordon coefficients equal to $2/3$ and $1/3$ respectively. $^3He$ as a buffer gas collisionally mixes the excited sub-levels. The energy is transferred to gas molecule independently of the angular momentum of the excited state. This equilibrates the two excited sub-levels as far as the de-excitation is concerned. As a result the atom de-excites to the ground state sub-levels with equal probability, $1/2$. The lower sub-level however is still being pumped with the laser, so that the population of the desired level, $5S_{1/2}, m = +1/2$ is continuously replenished. Through this mechanism the atomic polarization of the $Rb$ is built, or “optically pumped”. The process is depicted schematically in fig. 4.2

Each circularly polarized photon absorbed adds one unit of angular of angular momentum in the direction of the axis of the light beam to the system of $Rb$ atoms. Applying the usual angular momentum selection rules $\Delta m = \pm 1$, one can trace the evolution of the atomic sub-levels. Atoms in the singlet Zeeman sub-level of the ground state with the highest angular momentum projection cannot be excited to a higher angular momentum level. Thus, in the absence of overwhelming relaxation effects, a surplus of atoms in this sub-level gradually accumulates, producing a net macroscopic magnetic moment. This condition can be detected by the resulting increased transmission of the pumping light. In the
Figure 4.2: Emission scheme according to selection rules, in which the atom is walked to the desired polarized state

The presence of various relaxation effects the evolution of the $Rb$ polarization can be described by considering

$$\frac{d\rho_{\pm1/2}}{dt} = \pm \left[ \frac{\Gamma_{SD}}{2} + \gamma_{opt}(\vec{r}) \right] \rho_{-1/2} + \frac{\Gamma_{SD}}{2} \rho_{+1/2},$$

(4.6)

where $\rho_{\pm1/2}$ are the population densities, normalized so that $\rho_{+1/2} + \rho_{-1/2} = 1$, $\Gamma_{SD}$ is the spin destruction rate, and $\gamma_{opt}$ is the rate of photon absorption per atom:

$$\gamma_{opt}(\vec{r}) = \int \Phi^+(\nu, \vec{r})\sigma(\nu)d\nu,$$

(4.7)

where $\Phi^+(\nu, \rho)$ is the flux of circularly polarized photons at frequency $\nu$, $\sigma(\nu)$ is the absorption cross-section for linearly polarized light.

The polarization builds as the gap in populations of the $\frac{1}{2}$ and $-\frac{1}{2}$ states grows. By rewriting 4.6 in terms of $P_{Rb}$,

$$\frac{dP_{Rb}}{dt} = \gamma_{opt}(1 - P_{Rb}) - \Gamma_{SD}P_{Rb},$$

(4.8)

from which one obtains
\[ P_{Rb}(t) = \frac{\gamma_{opt}(\vec{r})}{\gamma_{opt}(\vec{r}) + \Gamma_{SD}} \left[ 1 - e^{-(\gamma_{opt}(\vec{r}) + \Gamma_{SD})t} \right], \]  

which at long \( t \) becomes the steady state solution

\[ P_{Rb}(r) = \frac{\gamma_{opt}(\vec{r})}{\gamma_{opt}(\vec{r}) + \Gamma_{SD}}, \]  

where, taking into account the various contribution to the spin destruction, \( \Gamma_{SD} \) is given by

\[ \Gamma_{SD} = k_{Rb-Rb} + k_{Rb-N_a} + k_{Rb-wall} + k_{Rb-He}. \]  

The first term encompasses the relaxations due to the \( Rb \) itself, which is largely determined by the binary \( Rb-Rb \) collisions. During such events the rubidium atoms can exchange their electrons’ spins, thereby conserving the total spin of the two electrons, or, in an interaction of the spins with the relative momentum of the \( Rb \) atoms, lose the electron spin to \( N \). The fourth term consists of two parts, one of which describes the spin-exchange between the \( Rb \) atom and the \( ^3He \) nucleus.

The photons created in the process of de-excitation possess the right wavelength in order to be absorbed but are of undetermined polarization, and are therefore capable of exciting an electron out of the \( 5S_{1/2}, m = +1/2 \) state, and reducing the total polarization. The high density of the \( Rb \) vapor, in turn, effectively shortens the mean free path of such photons and increases the likelihood of a depolarizing re-absorption, called “radiation trapping”. Introducing a certain amount of \( N_2 \) gas into the mixture helps to avoid radiative de-excitation from \( P \) to \( S \) states. \( N_2 \) molecules possess many vibrational and rotational
degrees of freedom and can absorb the excess $Rb$ energy by colliding with its atoms, thus obviating the radiative channel [S.Appelt, A. Ben-Amar Baranga, C.J. Erickson, M.V. Romalis, A.R. Young, and W. Happer, Phys. Rev. A 58, 1412 (1998)]. The natural lifetime of the excited states of the $Rb$ atoms is reduced by the presence of nitrogen from $\sim 28\text{ns}$ to $\sim 1\text{ns}$ [Corney, Atomic and Laser Spectroscopy, 1997]. By adding $\sim 100$ torr of $N_2$ into the cell, the effect of radiation trapping can be effectively removed.

Figure 4.3: Optical Pumping of the atomic levels of $Rb$ and spin exchange during binary collisions of the $Rb$ atoms with $^3He$ nuclei.

4.1.2 Spin Exchange

In the ground state the two proton spins in the $^3He$ nucleus are anti-aligned so that they effectively cancel each other out. Therefore the spin of the nucleus overall is dictated by the spin of the neutron.

The spin of the $Rb$ atoms can be transferred to the $^3He$ nuclei through spin exchange (SE), where via binary collisions between the $Rb$ atoms and $^3He$ nuclei the former, with
a spin projection $m = +1/2$ transfers spin $+1$ to the latter, with $m = -1/2$. Another occasion to transfer the electron’s spin presents itself when the alkali atom and the nucleus of the noble gas form a short-lived van der Waals molecule. However unlike the case with $^{129}\text{Xe}$, such molecules with $^3\text{He}$ break up so quickly, that binary collisions remain the dominant spin-exchange channel. The period of the binary collisions is on the order of $\sim 10^{-12}$s. The hyperfine interaction, on the other hand, is much slower order of $\sim 10^{-9}$ s. This fact, coupled with the collisional broadening of the hyperfine levels, to the point where they become practically unresolved, greatly reduces the probability of spin loss to the hyperfine levels. At typical total pressures of the noble and buffer gasses, the pump-up rate is $\sim 10^6$, which can be reduced by up to an order of magnitude due to the spin loss to the hyperfine interaction.

During a binary collision, the $\text{Rb}$’s valence electron has a small chance to penetrate the $^3\text{He}$ nucleus and exchange its spin. However the spin of the electron can also be transferred to the relative angular momentum of its rotation with respect to the noble gas’ nucleus or through the coupling of the electron spin with that of the nucleus itself. These possibilities are included in the Hamiltonian 4.12, where the third term is largely responsible for the spin-exchange, and describes the Fermi-contact interaction. Most of the angular momentum carried by the $\text{Rb}$’s electrons is however lost in the process of the spin-rotation interaction, in which the spins interact with the relative angular momentum of the two interacting atoms.

$$H_{SE} = \gamma \mathbf{N} \cdot \mathbf{S} + \alpha \mathbf{K} \cdot \mathbf{S} + A \cdot \mathbf{K} \cdot \mathbf{S},$$

(4.12)
where \( A = \frac{32\pi}{3} \mu_B \mu_I \delta^3(r) \), with \( \delta^3(r) \) being the probability of the \( Rb \) electron being at the position of the \( ^3He \) nucleus. Given a constant probability of spin exchange the rate at which it occurs can be increased by raising the \( Rb \) vapor density, while keeping the \( Rb \) highly polarized. However past certain concentration the laser radiation will be absorbed in the initial, opaque volumes of the \( Rb \) thus obscuring rest of the atoms from the laser. The compromise is achieved by balancing the \( Rb \) vapor density through varying the temperature, given constant laser power. Usually \( Rb \) concentrations of \( \sim 10^{14} \text{atoms/cm}^3 \) are used. If all other conditions of the pump-up are optimal, the efficiency of spin-exchange will be dictated by the relaxation rate, which has to be relatively low compared to that of SE. Similarly to the case of optical pumping, discussed above, for the \(^3He\) polarization we can construct the equation by using

\[
\frac{d\rho_{He}}{dt} = \left[ \frac{\Gamma_{He}}{2} + \gamma_{SE}\rho_{Rb,1/2} \right] \rho_{He,-1/2} - \left[ \frac{\Gamma_{He}}{2} + \gamma_{SE}\rho_{Rb,-1/2} \right] \rho_{He,1/2},
\]

(4.13)

where, \( \gamma_{SE} \) is the spin-exchange rate, \( \rho_{Rb,\pm1/2} \) and \( \rho_{He,\pm1/2} \) are the normalized population densities for the states \( m = \pm1/2 \) of the \( Rb \) and \( ^3He \) respectively, and \( \Gamma_{He} \)is the total relaxation rate of \( He \). Substituting

\[
P_{He} = (P_{Rb} - P_{He})\gamma_{SE} - \Gamma_{He}P_{He},
\]

(4.14)

we obtain the steady state solution

\[
P_{He} = \frac{P_{Rb}}{\gamma_{SE} + \Gamma_{He}} \gamma_{SE},
\]

(4.15)

where \( P_{He} \)is determined from eqn. 4.10.

In addition to the spin-spin and spin-orbital interactions, additional mechanisms such
as the presence of paramagnetic impurities, wall collisions and field inhomogeneities contribute to the relaxation of $^3$He polarization. The wall relaxation rate is determined by several factors such as the micro-fissures on the glass, which increases the surface area thus increasing the number of depolarizing collisions, but is currently understood in detail. Coating the glass with $Cs$ has been shown to prolong the relaxation in $^3$He cells by tens of hours[[32]]. On the other hand, between collisions, relaxation can occur due to field inhomogeneities, causing spins to evolve in a non-adiabatic fashion. The rate in this case is $\Gamma_{\nabla B} \sim \chi^2 \tau$, over time $\tau$ the atoms experience a field rotating at $\chi \sim v \frac{\nabla B_\perp}{\omega}$

$$\Gamma_{\nabla B} = D \frac{\nabla B_\perp^2}{B^2},$$  \hspace{1cm} (4.16)

where $D$ is a diffusion constant, and $\nabla B_\perp$ is the field gradient in the transverse direction. Typically $0.1 - 3\%$ per cm is sufficient to maintain reasonable polarization. $O_2$ is the most important paramagnetic impurity, whose rate constant $0.45s^{-1}/amagat$, measured by [Saam et al.] at 1.4 Tesla, displayed a temperature dependence $\sim 1/\sqrt{T}$.

The relaxation rates were measured by several groups. Their values are summarized in the table below.

4.2 Nuclear Magnetic Resonance

4.2.1 Quantum-Mechanical Approach

Formalism using the evolution operators, or the spinors.

The Hamiltonian In the absence of a spin the Hamiltonian of a particle in an electromagnetic field can be written as
\[ H_0 = \frac{1}{2m} \left( \vec{p} - \frac{e}{c} \vec{A} \right)^2 + e\Phi, \]  

(4.17)

where \( \vec{A} \) is the vector potential, \( \Phi \) is the scalar potential and \( e \) and \( m \) are the electron’s charge and mass respectively. We can use the expression for the Lorentz force in conjunction with the identities

\[ \vec{v} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{v} \cdot \vec{A}) - (\vec{v} \cdot \vec{\nabla})\vec{A} \]  

(4.18)

and

\[ \frac{dA}{dt} = \frac{\partial \vec{A}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{A} \]  

(4.19)

\[ \vec{F} = e \left( \vec{E} + \frac{\vec{v}}{c \times \vec{B}} \right) = e \left( -\nabla \phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t} + \frac{1}{c} \vec{v} \times (\nabla \times \vec{A}) \right) \]

\[ = e \left[ -\nabla \phi + \frac{1}{c} \nabla (\vec{v} \cdot \vec{A}) - \frac{1}{c} \frac{dA}{dt} \right] \]  

(4.20)

If we now rewrite \( \frac{dA}{dt} \) as \( \frac{dA}{dt} = \nabla_v (\vec{A} \cdot \vec{v}) \) then \( \vec{F} \) assumes the shape of a generalized force:

\[ F_j = -\frac{\partial}{\partial j} \left( e\phi - \frac{e}{c} \vec{v} \cdot \vec{A} \right) + \frac{d}{dt} \frac{\partial}{\partial v_j} \left( e\phi - \frac{e}{c} \vec{v} \cdot \vec{A} \right), \]  

(4.21)

where \( j = x, y, z \) and the term within each pair of parentheses is the generalized potential.

The Lagrangian is then

\[ L = T - U = \frac{1}{2} - e\phi + \frac{e}{c} \vec{v} \cdot \vec{A}. \]  

(4.22)
Finally the Hamiltonian is

\[
H_0 = \sum_i p_i q_i - L = \frac{1}{2m} \left( \vec{p} - \frac{e}{c} \vec{A} \right)^2 + e\phi.
\]  

(4.23)

However by adding a spin we introduce an additional term, describing its interaction with the magnetic field. As it is the case with any magnetic moment, \( \vec{\mu} \), the potential energy \( U \) of its interaction with a magnetic field \( \vec{B} \) is \([34]\)

\[
U = -\vec{\mu} \cdot \vec{B}
\]  

(4.24)

So the Hamiltonian becomes

\[
H = H_0 + \vec{\mu} \cdot \vec{B}
\]  

(4.25)

Now if we substitute \( B_0 \) and the expression for \( p \), keeping in mind that \( \vec{A} \) and \( \vec{\nabla} \) do not necessarily anti-commute, we find

\[
H = \frac{1}{2m} \left( \frac{\hbar}{i} \nabla - \frac{e}{c} \vec{A} \right) + e\Phi + \mu \hat{\sigma} \vec{B} = -\frac{\hbar^2}{2m} \Delta + \frac{ie\hbar}{mc} \vec{A} \nabla + \frac{ie\hbar}{2mc} \vec{\nabla} \vec{A} + \frac{e^2}{2mc^2} \vec{A}^2 + e\Phi
\]  

(4.26)

and choosing the Coulomb gauge, where \( \vec{\nabla} \cdot \vec{A} = 0 \), we obtain

\[
H = H_0 - \frac{e}{mc} \vec{A} \cdot \dot{\vec{p}} + \frac{e^2}{2mc^2} \vec{A}^2
\]  

(4.27)

The last two terms represent the influence of the magnetic field.
We are interested in the case of spin \( \frac{1}{2} \) particles, which includes the \( ^3He \) nuclei. The spin operators corresponding to this case have 2 eigenvalues (and eigenstates \( \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) and \( \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \), and therefore the Pauli matrices will be two-dimensional:

\[
\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

so that \( \sigma_z^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1 \)

and using \([\sigma_i, \sigma_j]_+ = 2\delta_{ij}\)

\[
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = 1, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = 1
\]

We can now write the wave-function in terms of the spinors

\[
\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \psi_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \psi_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \psi_1 \chi_+ + \psi_2 \chi_-,
\]

where \( \psi_{1,2} = \psi_{1,2}(\vec{r}, t) \) is a function of time and position.

In order to describe the motion of the spin when it is in a constant magnetic field we need to calculate its expectation value.

\[
\langle \vec{S} \rangle = \frac{\hbar}{2} \chi^+ \sigma \chi,
\]

where \( \chi = \begin{pmatrix} \chi_+ \\ \chi_- \end{pmatrix} \) is the time-dependent spin function.
Now we need to obtain $\chi$ which can be done by solving the part of the Pauli equation involving magnetization.

$$i\hbar \frac{\partial \chi}{\partial t} = \mu_B \sigma B_z \chi,$$  \hspace{1cm} (4.30)

since the field is aligned with the $\hat{z}$-axis.

$$\chi = a_0 \chi_+ + b_0 \chi_- = \chi(t = 0). \hspace{1cm} (4.31)$$

If we now make the substitutions

$$a_0 = e^{-i\gamma} \cos \frac{\theta}{2} \hspace{1cm} (4.32)$$

and

$$b_0 = e^{-i\delta} \sin \frac{\theta}{2} \hspace{1cm} (4.33)$$

then for the spins expectation value we will obtain

$$\langle S \rangle = \begin{pmatrix} \cos(2\omega_L t + \delta - \gamma) \sin \theta \\ \sin(2\omega_L t + \delta - \gamma) \sin \theta \\ \cos \theta \end{pmatrix} \hspace{1cm} (4.34)$$

which describes a precessing motion.

4.2.2 Semi-Classical Approach

Each $^3He$ nucleus is comprised of two protons and a single neutron, so that its total spin is $I = 1/2$ with $z$-projection $m = \pm \frac{1}{2}$, the latter $(-\frac{1}{2})$
corresponding to the the lower energy state. In the presence of an applied magnetic field \( B_0 \) along some direction, e.g. parallel to \( \hat{z} \) the energy of the state \( m \) :

\[
E_m = \gamma \hbar m B_0. \tag{4.35}
\]

The difference in energy between the two states is then equal to \( \Delta E = \hbar \omega \), where \( \omega = -\gamma B_0 \), is the Larmor frequency, and \( \gamma \) is the \textit{gyromagnetic ratio} of the nucleus in question. For helium and hydrogen the gyromagnetic ratios are \( \gamma_{^3He} = 2.04 \times 10^8 \ [\text{rad} \text{s}^{-1} \text{T}^{-1}] \) and \( \gamma_{^1H} = 2.67 \times 10^8 \ [\text{rad} \text{s}^{-1} \text{T}^{-1}] \) respectively.

As known from quantum mechanics, an angular momentum \( I \) measured in units of \( \hbar \) assumes quantized values of \( \pm \hbar / 2 \) along some preferred axis. In general the corresponding magnetic moment \( \mu \) of the particle with such angular momentum is related to \( I \) by

\[
\vec{\mu} = \gamma \hbar \vec{I}, \tag{4.36}
\]

with a gyromagnetic ratio \( \gamma = \frac{g e}{2 m_p} \), where \( g \) is the Lande’ factor \( e \) is the charge of an electron and \( m_p \) is the mass of a proton.

When placed in a magnetic field this magnetic moment will experience a torque

\[
\vec{N} = \vec{\mu} \times \vec{B} = \hbar \frac{d\vec{I}}{dt} \tag{4.37}
\]
or using eqn. 4.36,

\[
\gamma \vec{\mu} \times \vec{B} = \frac{d\vec{\mu}}{dt} \tag{4.38}
\]
It is easy to show that as a result of this torque the magnetic moment will start precessing about the direction of the magnetic field $B$. In order to do this let’s switch to a coordinate system that rotates with some angular velocity $\bar{\omega}$. The time derivative of the magnetization vector in the lab-system can now be expressed using it’s counterpart in the rotating system:

$$\frac{d\bar{\mu}}{dt} = \frac{\partial \bar{\mu}}{\partial t} + \bar{\omega} \times \bar{\mu},$$

where $\frac{\partial \bar{\mu}}{\partial t}$ is the time derivative of magnetization in the rotating frame. Then using

$$\frac{\partial \bar{\mu}}{\partial t} = \frac{d\bar{\mu}}{dt} + \bar{\mu} \times \bar{\omega} = \gamma \bar{\mu} \times \bar{B} + \bar{\mu} \times \bar{\omega} = \gamma \bar{\mu} \times (\bar{B} + \frac{\bar{\omega}}{\gamma})$$

(4.40)

So if the rotating frame rotates with such angular velocity that

$$\bar{\omega} = -\gamma \bar{B}, \text{ then } \frac{\partial \bar{\mu}}{\partial t} = 0.$$  

(4.41)

Viewed from the laboratory frame this looks as though the magnetization vector precesses about $\bar{B}$. This fact is heavily exploited in NMR techniques as will be demonstrated below.

We derived the equation of motion of the magnetic moment corresponding for a single spin. However dealing with an ensemble of such spins it is more convenient to define and use a so-called net magnetization, $\bar{M}$. 

106
\[
\tilde{M} = \tilde{\mu}NP,
\]

where \(\tilde{\mu}\) is the individual magnetic moment of each particle in the ensemble, in our case \(^3\text{He}\), \(N\) is the number of \(^3\text{He}\) nuclei and \(P\) is the polarization of the sample defined as,

\[
P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}.
\]

When a field \(B_0\) is applied in some direction, say \(\mathbf{z}\) in our case, the individual \(^3\text{He}\) spins begin precessing around it. The \(xy\)-plane projection of each spin will rotate around the \(z\) \text{-axis} with the same angular velocity, provided the field is uniform, but with different phases, so that the total \(xy\) projection is zero. The \(z\)-projection, \(M_z\), however will have a non-zero value due to "equilibrium polarization". This quantity is determined from the Boltzmann distribution:

\[
M = \sum_i N\gamma_i \hbar \sum_{m=\pm I} m e^{\gamma_i h m B_0 / kT} \sum_{m=-I} e^{\gamma_i h m B_0 / kT}
\]

\[
= N\mu \tanh \frac{\mu B}{kT} = N\mu \tanh \frac{\hbar \omega}{kT}
\]

So for a thermal sample the polarization can be calculated from

\[
P_{\text{thermal}} = \tanh \left( \frac{\hbar \omega}{2kT} \right)
\]
One can see from 4.45 that in order to increase thermal polarization, the temperature has to be kept low, while maintaining high magnetic fields. Using this "brute force" method of polarizing a sample, typically, in magnetic fields of the order of $\sim 10^4 Gauss$ at room temperatures, proton polarizations of $\sim 10^{-5}$ can be achieved. This approach is widely used in Magnetic Resonance Imaging [MRI] in medicine.

In the previous section we described methods (spin-exchange optical pumping), whereby $^3He$ samples can be polarized to much greater values ($\times 10^5$). We will now describe NMR techniques that allow detecting the net magnetization in such samples.

4.2.3 Relaxation

The equations of time evolution for spins and magnetizations up to this point implied their perpetual and unhindered motion in the transverse plane. However, in reality several damping mechanisms exist that need to be considered in the description of the system. They cause the so-called longitudinal and transverse relaxations. The former refers to the processes in which the longitudinal component of the thermal magnetization returns to its equilibrium value and orientation, while the latter describes the diminishing of the transverse counterpart as a result of the de-phasing of the transverse components of individual spins with respect to each other. Such mechanisms include magnetic field inhomogeneity, temperature, presence of paramagnetic impurities etc.

Paramagnetic impurities create time-varying fields in local spots within the sample in such a way that different small areas will experience "random" fields and therefore promote the de-phasing of the transverse magnetization, $M_{xy}$. Oxygen and some metals are among materials, which create such conditions. The characteristic times for these relaxation kinds are usually denoted by $T_1$ and $T_2^*$. The star in $T_2^*$ emphasizes the fact that this quantity is
accounts for the external factors as well as intrinsic relaxations effects.

**Longitudinal Relaxation** As mentioned before \( z \)-components of contributing spins add up to a net magnetization along \( \hat{z} \). In other words, \( M_0 = M^0_z \propto n_{up} - n_{down} \), where \( n_{up} \) and \( n_{down} \) are populations of "spin-up" and "spin-down" states respectively. When energy is delivered to the nuclei by means of an RF pulse the balance between "up" and "down" spins is tipped towards populating the "spin-down" states. During the relaxation process on the other hand the corresponding populations will tend to return to their equilibrium values - \( n_{up}^0 \) and \( n_{down}^0 \). Obviously in order for one of these populations to increase, the other will have to decrease. In other words

\[
\frac{dn_{up}}{dt} = \nu(n_{down} - n_{up}), \quad (4.46)
\]

where \( \nu \) is the rate of change of populations. Similarly

\[
\frac{dn_{down}}{dt} = \nu(n_{up} - n_{down}), \quad (4.47)
\]

These equations would however imply that at equilibrium the \( n_{up} = n_{down} \), which is obviously not true. To resolve the situation we need to introduce a term in our equation that would indicate the equilibrium value for each state, \( n_{up}^0 \) and \( n_{down}^0 \):

\[
\frac{dn_{up}}{dt} = \nu(n_{down} - n_{down}^0 - n_{up} + n_{up}^0), \quad (4.48)
\]

and
\[ \frac{dn_{\text{down}}}{dt} = \nu(n_{\text{up}} - n_{\text{up}}^0 - n_{\text{down}} + n_{\text{down}}^0), \quad (4.49) \]

Now we can express the rate of change of the magnetization:

\[ \frac{d\vec{M}}{dt} \propto \frac{d(n_{\text{up}} - n_{\text{down}})}{dt} = \nu[(n_{\text{down}} - n_{\text{down}}^0 - n_{\text{up}} + n_{\text{up}}^0) - (n_{\text{up}} - n_{\text{up}}^0 - n_{\text{down}} + n_{\text{down}}^0)] \]

\[ = 2\nu(n_{\text{up}}^0 - n_{\text{down}}^0 - n_{\text{up}} + n_{\text{down}}) = 2\nu(M_z^0 - M_z), \quad (4.50) \]

where \( M_z^0 = n_{\text{up}}^0 - n_{\text{down}}^0 \). We now have a differential equation whose solution describes the evolution of \( M_z(t) \).

\[ \frac{dM_z(t)}{dt} = 2\nu[M_z^0 - M_z(t)] \quad (4.51) \]

It is evident from eqn. 4.51 that the rate of change of the magnetization is proportional to its deviation from the equilibrium value. Integrating this equation yields

\[ M_z(t) = [M_z(0) - M_z^0] e^{2\nu t} + M_z^0 = M_z^0 \left[ 1 - \left( 1 + \frac{M_z(0)}{M_z^0} \right) e^{-\frac{t}{T_1}} \right], \quad (4.52) \]

where \( \nu = -\frac{1}{T_1} \) is negative, and \( T_1 \) is the longitudinal relaxation time.

The processes that cause the spin-flips that re-populate \( n_{\text{up}} \) and \( n_{\text{down}} \) where nuclei lose energy to molecular motion, are referred to as lattice in this context. The type of relaxation is hence called spin – lattice relaxation.

The atomic electrons of paramagnetic species mixed in to the gas in question posses large magnetic moments and therefore are able to contribute to the relaxation. Motion of
molecules with non-zero net magnetic moment will also create oscillating magnetic fields, which if close to $\omega_0$ in frequency units will cause "spin-flips" and promote decay of $M_z$.

The magnetic moments of two nearby nuclei create a magnetic field mutually experienced by both. The magnitude of this interaction depends on the distance between the nuclei and the direction of the vector connecting them with respect to $\vec{B}_0$. Two nuclei within say a rotating molecule would experience a change in the magnetic field as the aforementioned direction changes. Change in the distance plays an important role as the strength of the interaction is proportional to $\frac{1}{r^3}$ where $r$ is the distance between the nuclei, so it deteriorates very rapidly with distance. For the same reason two nuclei within the same molecule have a greater influence on relaxation then if they are parts of different molecules, much farther apart. The magnitude of this kind of relaxation is proportional to the square of the coupling. And as the coupling itself is proportional to $\frac{\gamma^2}{r^3}$ the relaxation is proportional to $\frac{\gamma^4}{r^6}$. Although exchanging energy between the two nuclei itself will not contribute to the relaxation, the transfer of energy to the molecular motion, or the "lattice" will in turn create oscillating magnetic fields, promoting relaxation.

Transverse Relaxation Looking back at the definition of the net magnetization as essentially the sum of individual spins comprising the system we can consider the transverse aspect of relaxation. The essential causes for the transverse relaxation are as follows

- $T_1$ processes that flip the spins and destroy spin coherence (so that $T_2 \leq T_1$)
- Slow-moving molecules create magnetic fields in the $\hat{z}$-direction, thereby de-phasing the local spins.
Figure 4.4: Time-evolution of longitudinal (a) and transverse (b) components of the magnetization.
• Different fields experienced by nuclei, depending on the electron cloud’s shielding factor. This depends on the molecular environment.

• Gradients in the magnetic field \((B_0)\)

• Difference in magnetic field susceptibility of the polarized medium.

While the second and third type of processes contributing to the transverse relaxation are commonly referred to as “\(T_2\)-processes” the fourth gives rise to \(T'_2\) processes.

After the system has been excited most of the individual spins will be aligned in such a way that the net transverse magnetization \(M_{xy}\) is non-zero. It will remain that way as long as the transverse projections of the precessing spins are coherent, i.e. they move in phase. De-phasing of the spins will cause deterioration of \(M_{xy}\).

One way to de-phase the spins is to introduce the same type of impurities, which will create local oscillating magnetic fields close to \(\omega_0\) and cause ”spin-flips” in some of the nuclei. Magnetic field gradients across the sample are capable of de-phasing the spins in the transverse plane. That way the Larmor frequency will be dictated by the magnetic field value in the vicinity of the given nucleus, so that if the spins were initially coherent they will gradually acquire different phases thereby reducing \(M_{xy}\). In case of heterogeneous samples the difference in magnetic susceptibility will also create this kind of effect. The overall characteristic transverse relaxation parameter can be expressed as

\[
\frac{1}{T^*_2} = \frac{1}{T_2'} + \frac{1}{T_2''} + \gamma \Delta B_0,
\]
Figure 4.5: (a) Depiction of "excited" state of transverse magnetization at time $t = 0$ after the application of the RF pulse. (b) Somewhat "relaxed" magnetization at time $t$ after the RF pulse application.
where \( \frac{1}{T_2} \) and \( \frac{1}{T_1} \) represent "internal relaxation mechanisms" such as paramagnetic impurities and molecular vibrations/rotations, and those inflicted by the magnetic field gradients.

As a consequence mechanisms causing longitudinal relaxation and therefore contributing to \( 1/T_1 \) will also always contribute to transverse relaxation, i.e. shorten \( T_2 \). Therefore

\[
T_2 \leq T_1.
\]

To summarize, time evolution of net-magnetization can be characterized by the Bloch equations:

\[
\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} = \frac{M_z \hat{i} + M_y \hat{j}}{T_2} + \frac{M_z - M_0}{T_1} \hat{k},
\]

where \( M_0 \) is the longitudinal magnetization at equilibrium.

4.2.4 Detection of the NMR Signal

During Phase Sensitive Detection (PSD) the signal from the \(^3\)He is not of the same frequency as the reference frequency coming from the internal oscillator. Thus after the PSD the oscillations occur at \( \omega_{FID} = \omega_0 - \omega_{ref} \).

Free Induction Decay

In order to detect a signal created by the moving magnetization vector one needs to create conditions in which its projection onto the \( xy \)-plane is relatively large. In that case we if we position a coil so that it is wound around for instance the \( x \)-axis, in a coordinate system in which \( \vec{B}_0 \) is parallel to \( \hat{z} \) we will detect the rotation of \( \vec{M} \) as it "cuts" through the coil. However in the presence of only \( B_0 \), \( \vec{M} \) lacks a component in
the transverse plane with \( z \). In order to project \( \vec{M} \) onto the \( xy \) plane one would need to tip it away from the \( z \)-axis. This can be accomplished by applying a relatively small magnetic field \( \vec{B}_1 \), normal to the main field’s direction. This will create a torque necessary to induce a precession of the magnetization vector towards the transverse plain. The angular velocity of this precession will be \( \vec{\omega}_1 = -\gamma \vec{B}_1 \).

However once tipped from the \( z \)-axis the magnetization will try to precess around it with angular velocity \( \vec{\omega} = -\gamma B_0 \), in which case applying a constant \( \vec{B}_1 \) will result in a more complicated motion of the magnetization vector. For example in the instance where \( \vec{M} \) makes some angle with \( z \) and lies in the positive quadrant of the \( zy \)-plane \( \vec{B}_1 \) will tend to tip it towards the \( y \)-axis. But when the vector finds itself in the negative quadrant of the \( zy \) plane \( \vec{B}_1 \) will make it precess in the opposite direction. In fact \( \vec{B}_1 \) will not be effective at precessing the magnetization into the \( xy \)-plane unless its own vector rotates about \( z \) with nearly the same angular velocity as the magnetization vector. This condition is referred to as the resonance condition. In that case the magnetization vector is will not move with respect to \( \vec{B}_1 \), so that the only torque and therefore precession in this new coordinate system rotating with angular velocity \( \vec{\omega} \) is that inflicted by \( \vec{B}_1 \), also constant in the rotating system.

Hence the applied magnetic field should be of the form

\[
\vec{B}_1 = 2B_1 \cos \omega t \cdot \hat{x} = B_1 [\cos \omega t \hat{x}]
\]  

(4.54)

Now to quantify this phenomenon we will call the angle by which the magnetization
Figure 4.6: Precession of the magnetization vector in the laboratory frame.
is tipped from the $z$-axis - the tip angle. If the tip angle is equal to $\theta$ at some point in time, then $\vec{M}$'s projection onto $xy$ will be equal to $M_{xy} = |\vec{M}| \sin \theta = M \sin \theta$. The $x$ and $y$ projections then will be respectively

$$M_x = M \sin \theta \cos \omega t$$

$$M_y = M \sin \theta \sin \omega t,$$

where $\omega t$ is the angle that $M_{xy}$ makes with the $x$ axis.

**Adiabatic Fast Passage** A different approach to the detection of magnetization of polarized samples is called the Adiabatic Fast Passage (AFP). In this method the spins are tipped (or flipped, referring to $180^\circ$ tip angle) by sweeping the holding field $B_0$ through a range of
values, during which process $B_1$, the oscillating (RF) field is turned on.

As mentioned before, a magnetic moment $\vec{\mu}$ placed in a magnetic field $\vec{B}$ experiences a torque

$$\frac{d\vec{\mu}}{dt} = \gamma (\vec{\mu} \times \vec{B})$$ (4.57)

due to which it will start precessing about the direction of $\vec{B}$

$$\vec{B}_1 = 2B_1 \cos \omega t \cdot \hat{x} = B_1 (\cos \omega t + \sin \omega t)\hat{x} + B_1 (\cos \omega t - \sin \omega t)\hat{y}$$ (4.58)

which represents two vectors of magnitude $B_1$ rotating in opposite directions. In a frame rotating around $\hat{z}$ with angular frequency $\omega$ one of these components will remain static while the other will rotate in the opposite direction at $2\omega$. This can be done using a transformation:

$$\begin{align*}
\hat{x}' &= \cos \omega t \hat{x} - \sin \omega t \hat{y} \\
\hat{y}' &= \sin \omega t \hat{x} + \cos \omega t \hat{y} \\
\hat{z}' &= \hat{z}
\end{align*}$$ (4.59)

$\hat{z}' = \hat{z}$ The second component, rotating at twice the frequency in the opposite direction is so far of resonance that it will not affect the magnetization’s direction, and can therefore be omitted. In this rotating frame the field is

$$\vec{B}' = B_0 \hat{z}' + B_1 \hat{x}'$$ (4.60)
If we use this field in 4.38 in the rotating frame the effective field in the \( \hat{z}' \)-direction will be
\[
B_0 \hat{z} - \frac{\omega}{\gamma} \text{ and that in the } \hat{x}' \text{-direction } - B_1 \hat{x}'
\]

\[
\frac{\partial \vec{\mu}}{\partial t} = \gamma \mu \times \left[ \left( B_0 - \frac{\omega}{\gamma} \right) \hat{z} + B_1 \hat{x} \right] = \gamma (\vec{\mu} \times \vec{B}_{eff})
\] (4.61)

This becomes the rotating-frame equivalent of the original equation. In this frame the magnetization will be precessing around the effective field \( \vec{B}_{eff} \).

In the AFP configuration \( B_1 \) is small, i.e. \(|B_1| \ll |B_0|\). Initially the holding field is set far off resonance. During AFP its value is “swept” from \(-B_0^0\) to \(+B_0^0\) (and typically back) through resonance. Then if the magnitude of the holding field reaches the value of \( B_0 = \left| \frac{\omega}{\gamma} \right| \) where the effective field becomes \( B_{eff} = B_1 \hat{x} \), the magnetization vector \( \vec{M} \) precesses with an angular frequency \( \omega_1 = \omega_{frame} \). This is known as the “resonance condition”.

Under this condition the magnetization precesses about \( \hat{x}' \), which in turn rotates around \( \hat{z}' \).

If during this precession the angle between the magnetization \( \vec{M} \) and the field \( \vec{B}_{eff} \) is small, then the former will effectively follow the latter. This, once \( \vec{B}_{eff} \) is in the transverse plane, will create the desired signal in the pick-up coils positioned in a plane perpendicular to \( xy \). This constraint is the essence of the adiabatic condition. In order for it to be satisfied \( \vec{M} \)'s precession must occur at a much higher angular frequency than that of \( B_{eff}' \)'s rotation around \( \hat{z} \). As \( \vec{B}_0 \) passes through resonance \( B_{eff} \) starts tipping toward \( \hat{x}' \) in the rotating frame, while \( \vec{M} \) is left behind. However the angle between \( \vec{B}_{eff} \) and \( \vec{M} \) is small
as long as the magnetization has time to “swing” around the effective field. This way the magnetization follows $\tilde{B}_{eff}$, which starts off parallel to $\hat{z}$ and ends up anti-parallel to it, i.e. flipped.

As the field tips in the rotating frame it makes an angle

As the field tips in the rotating frame it makes an angle
\[ \theta = \tan^{-1} \frac{B_1}{(B_0 - \omega/\gamma)} \]  

(4.62)

with the z-axis. Hence the rate at which \( \vec{B}_{eff} \) tips towards the \( xy \)-plane is

\[ \frac{d\theta}{dt} = \frac{\vec{B}_1}{\vec{B}_{eff}^2} \frac{d\vec{B}_0}{dt} \]  

(4.63)

whereas the rate of \( \vec{M} \)'s precession, 

\[ \frac{d\phi}{dt} = \gamma B_{eff} \]  

(4.64)

so the condition

\[ \frac{d\theta}{dt} \ll \frac{d\phi}{dt} \]  

(4.65)

becomes

\[ \frac{\vec{B}_1}{\vec{B}_{eff}^2} \frac{d\vec{B}_0}{dt} \ll \gamma B_{eff} \Rightarrow \gamma B_{eff}^3 \frac{dB_0}{dt} \]  

(4.66)

Bearing in mind that \( \min(B_{eff}) = |B_1| \) the inequality is simplified to

\[ \frac{dB_0}{dt} \ll \gamma B_1^2 \]  

(4.67)

The angle by which the magnetization \( \vec{M} \) rotates about \( \vec{B}_{eff} = \vec{B}_1^{resonance} \) is

\[ \phi = \gamma B_{eff} \tau, \]  

(4.68)
where

\[ \tau = \frac{B_1}{dB_0/dt} \quad (4.69) \]

Transverse relaxation will be most prominent at resonance where the de-phasing of individual spins is more intense. In addition, since the \( T_1 > T_2 \), the latter becomes the upper limit on the duration of an AFP sweep.

\[ \tau \ll T_2 \Rightarrow \frac{B_1}{T_2} \ll \frac{dB_0}{dt}. \quad (4.70) \]

Finally we obtain the conditions for the Adiabatic Fast Passage

\[ \frac{B_1}{T_2} \ll \frac{dB_0}{dt} \ll \gamma B_1^2. \quad (4.71) \]

It should be noted that the relaxation times associated purely with field inhomogeneities (in rotating frame)

\[ \frac{1}{T} \approx D \frac{\Delta B_z}{B_z^2}, \quad (4.72) \]

where \( D \) is the \(^3\)He self-diffusion constant \((D = 1.8 \pm 0.2 \text{ cm}^2 \text{s}^{-1} \text{ at } 1 \text{ bar and } 20^\circ \text{C})\). [[19]]

Thus, in the lab frame at resonance the bulk magnetization rotates in the \( xy \) plane. In NMR experiments the pick-up coils are set perpendicular to \( xy \) so that as the magnetization vector “cuts” through the coils it induces an EMF measurable with electronic devices. The signal picked up by the coils is proportional to the magnitude of \( M \)’s transverse component, \( M_T \) and \( dM_T/dt \) [?] 

\[ M_t = \mu N P \hat{B}_{eff} \cdot \hat{x}' = \mu N P \frac{(B_0 - \omega/\gamma)\hat{z}' + B_x\hat{x}'}{\sqrt{(B_0 - \omega/\gamma)^2 + B_x^2}} \quad (4.73) \]
\[ S(t) = S_0 \frac{B_x \hat{x}'}{\sqrt{(B_0 - \omega/\gamma)^2 - B_x^2}}, \]

Similarly the signal

\[ S(t) = S_0 \frac{B_x \hat{x}'}{\sqrt{(B_0 - \omega/\gamma)^2 - B_x^2}}, \tag{4.74} \]

where \( S_0 \) is the peak signal. Accordingly the signal produced by AFP is expected to be of the shape

\[ S(t) = A_1 + A_2 t + \frac{A_3}{\sqrt{(t - A_4)^2 + A_5^2}}, \tag{4.75} \]

a Lorentzian function, where \( A_5 \) is the width, the ratio \( A_3/A_5 \) is the amplitude, \( A_4 \) is the center and \( A_2 \) and \( A_1 \) are the slope and the intercept of the linear background.

Under normal conditions (with an appropriate lock-in time constant and in the absence of large magnetic field inhomogeneities) the signals in the two Lock-In channels will be

\[ S_x(t) = -k_{He} M_T(t) G_{coil} G_{pre} G_{He} \cos \phi \]
\[ S_y(t) = -k_{He} M_T(t) G_{coil} G_{pre} G_{He} \sin \phi, \tag{4.76} \]

where \( k_{He} \) is a constant depending on the frequency response of the lock-in and the cell geometry, \( M_T(t) \) is the transverse polarization, \( G_{coil} \) is the pick-up gain, \( G_{pre} \) is the preamplifier gain, \( G_{He} \) is the lock-in gain and \( \phi \) is the phase of the phase between the input signal
and that of the lock-in reference.

\[ S(t) = S_0 e^{i\phi} e^{i\Delta\omega e^{-t/T}} , \]  

(4.77)

where \( \phi \) is the absolute receiver phase and \( \Delta\omega = \omega_0 - \omega_r \) is the difference between the reference and the detected frequency. The peak signal is

\[ S_{He} = \mu N P k_{He} G_{coil} G_{pre} G_{H} \]  

(4.78)

4.3 Polarizing neutron beams using \(^3\)He spin-filters. Neutron beam polarizer and analyzer

4.3.1 Polarizer

Spin-dependent Absorption Cross-section  The \(^3\)He spin-filter neutron beam polarizer is based on 1) a very large spin-dependent neutron absorption cross-section on \(^3\)He, \( \sigma_0 = (5333 \pm 7) \text{ b} \) at 23.5 \( \text{meV} \) (corresponding to the neutron velocity of 2200 m/s) [2] and 2) that a large quantities of \(^3\)He can be polarized. The large absorption cross-section is produced by a broad \( J^r = 0^+ \) excited \(^4\)He state 650 \( \text{keV} \) below the \( n + ^3\)He threshold. This cross-section is spin-dependent owing to the fact that the resonance is open only to the \( J = 0^+ \) channel. [2]. The absorption cross section follows the \( 1/v \) neutron energy dependence. In the \( n ^3\)He reaction elastic scattering is also possible but with a much smaller probability (the constant scattering cross-section is only \( \sim 3\)b).

Because of the total spin of the \(^4\)He state \( J = 0 \), the neutrons with spins anti-parallel to the \(^3\)He nuclear spin are absorbed. Thus, the polarized \(^3\)He functions as a spin filter; one neutron spin state is absorbed and other spin state transmitted through assuming that
\(^{3}\)He polarization is unity. Since the absorption cross section depends on the neutron energy, the beam polarization depends on the thickness of the \(^{3}\)He and neutron energy.

In the \(^{3}\)He spin filter the polarized \(^{3}\)He is contained in a glass cell at 4.9 atm pressure made from the GE-type glass. The cell has also a small amount of \(N_2\) gas and \(Rb\) metal. The rubidium atoms are optically polarized as described in section ???. The \(N_2\) gas serves as a buffer gas.

If \(\sigma_p\) and \(\sigma_{re}\) are defined as a spin-dependent and spin-independent cross-sections, then the two neutron spin state cross sections - “spin-parallel” and spin “anti-parallel” - are

\[
\sigma_{\pm} = \sigma_{re} \mp P_3 \sigma_p, \tag{4.79}
\]

where \(P_3\) is the \(^{3}\)He polarization. For \(^{3}\)He \(\sigma_{re}\) and \(P_3 \sigma_p\) are measured to be almost equal: \(\sigma_p = 1.010 \sigma_{re}\). [41] which means that the cross-section for the neutrons with spins parallel to the \(^{3}\)He spins is \(\sigma_+ = 0\) and with anti-parallel spins \(\sigma_- = 2 \sigma_{re}\). Transmissions of the two neutron spin state through polarized \(^{3}\)He are

\[
T_{\pm} = e^{-n\sigma l(1 \mp P_3)}, \tag{4.80}
\]

where \(n\) is the number density of \(^{3}\)He atoms and \(l\) is the thickness of the \(^{3}\)He gas. Today, the technology has been developed to produce large volume \(^{3}\)He cells that can match the neutron phase space defined by neutron guide. The other advantages of the \(^{3}\)He spin filter are that it does not produce \(\gamma\)-rays, supports straight neutron optics, offers an alternative method to reverse the beam polarization with AFP, allows a compact design of experiments [10].
According to equation ?? transmission of neutrons through un-polarized $^3He$ is

$$T_0 = e^{-n\sigma l} \quad (4.81)$$

Then the transmission of an un-polarized beam through polarized $^3He$ is

$$T_n = e^{-n\sigma l} \cosh(n\sigma lP_3)$$

and using 4.81

$$T_n = T_0 \cosh(n\sigma lP_3) \quad (4.82)$$

The neutron beam polarization on the other hand is

$$P_n = \frac{T_+ - T_-}{T_+ + T_-} = \tanh(n\sigma lP_3). \quad (4.83)$$

To obtain the beam polarization, in this way the $^3He$ polarization and thickness and the neutron energy have to be known. Oh the other hand For the beam polarization we also have

$$P_n = \sqrt{1 - \left(\frac{T_0}{T_n}\right)^2} \quad (4.84)$$

This expression shows that there is a parametric relation between the beam polarization and transmissions which allows the neutron beam polarization to be measured without any knowledge of the $^3He$ polarization or its thickness. Only the ratio of the polarized and the un-polarized transmissions need be measured. The transmissions and thus beam polarization depends on the neutron energy.
Polarizer Oven  The $^3$He spin filter consists of a $^3$He cell, an oven to regulate the Rb density, a few Gauss magnetic field, laser system to polarize the Rb, and an NMR system to monitor $^3$He polarization, see Fig.4.3.1. The function of the oven is to maintain temperature inside the polarizer cell high enough to vaporize the Rb metal to density that can be optically polarized by circularly polarized laser light. The optimal temperature, at which the maximum $^3$He polarization was achieved, was 160$^\circ$C. The oven is made of high-temperature nylon and was heated by forcing hot air through it. The air temperature was controlled by a PID controller which received temperature readings from a RTD placed directly on the surface of the cell. The temperature data were communicated by fiber optics to the heater controller located outside the cave.
The oven has neutron entry and exit windows made of two 0.2 mm thick single crystal silicon wafers per window.

Optics for the Laser Light. Two identical optical boards with light tight boxes were positioned directly above and below the oven. Each board contained optical elements to collimate and polarize the laser light conducted to the optics from the diode array by a fiber. On the board the light was first expanded to the aperture of the polarizing cube, that splits the original light beam into two linearly polarized components which were then circularly polarized by a λ/4 plate each. The circularly polarized light accessed the $^3$He cell through uncoated glass windows on the top and on the bottom of the oven illuminating the entire cell. The two Coherent 30 W FAP diode laser arrays were used in the polarizer setup [ref coherent]. The spectral widths, FWHM, of FAP outputs were $\sim 2nm$. This should be compared to the Rb absorption width of $\sim 20$ times narrower, however due to partially the optical thickness of Rb, strong absorption takes place even outside the resonance region of the spectrum [47].

Measurement of $^3$He Polarization in the Cell

Cell’s Thickness. The amount of $^3He$ in the beam can be determined using equation 4.81 where neutron transmission is a ratio of the number of neutrons transmitted through the cell to the monitor when the cell is in beam and out of beam. Using the velocity dependence of the absorption cross section

$$\sigma = \sigma_0 \frac{v_0}{v},$$

(4.85)
Figure 4.9: Front view of the Polarizer. Drive coils are seen encircling the oven box.
where $\sigma_0$ is the "thermal" neutron absorption cross-section at 2200 m/s, the number thickness, $n$ can be extracted from the transmission ratio curve by fitting it to the exponent of 4.81. Several corrections have to be considered before the $n$ calculated:

1) The monitor M2 located 15 cm downstream from the oven was used to measure the neutron transmissions. Before the neutrons hit M2, they interact with different materials, such as the four Si wafers; the neutron windows of the oven, as well as several layers of thin aluminum windows of the monitor M1 and M2, and then of course, the glass of the cell itself. These interactions resulting in transmission

$$T_n^0 = \frac{I}{I_0} = e^{-\left(n\sigma + n_g \sigma_g l_g + n_{Al} \sigma_{Al} l_{Al} + n_{Si} \sigma_{Si} l_{Si}\right)},$$

(4.86)

where $n$, $n_g$, $n_{Al}$ and $n_{Si}$ stand for the number densities of $^3He$, Ge-180 glass, Al and Si, respectively, i.e. materials encountered by the beam, and $l$, $l_g$, $l_{Al}$ and $l_{Si}$ are the corresponding thicknesses. In order to extract the absolute value for the $^3He$ thickness, these contributions to the beam attenuation are divided out of the exponent in eqn. 4.86.

The two plots in Fig. 3.10 show transmissions as a function of time-of-flight through two GE-180 glass samples with thickness of 3.5 mm together with four Si wafers. The contribution of the aluminum windows was not separated since it was negligible.

One way to isolate the beam attenuation of the glass and Si is to look at the log of the transmission through the $^3He$ cell vs. TOF. The non-$^3He$ terms give the energy-independent term in the exponent:

$$T_n^0 = e^{-n \sigma(E) l - b}$$

(4.87)

Then, if one plots the logarithm of $T_n^0$ vs. TOF, the resulting straight line intercepts the
ordinate at $b$. As can be seen from the plots of Fig. (4.11) (and also as expected based on the fact that neither glass nor $Si$ possess a significant energy-dependent absorption cross-section), the TOF-dependent glass and $Si$ transmissions, as compared to the $^3He$ transmission are very small. The uncertainty in the thickness introduced by the correction for the glass and $Si$ transmissions will therefore also be very small.

Figure 4.10: Attenuation of the neutron beam by two samples of GE-180. This type of glass is used in the spin-filter cell, as a function of neutron time of flight.

2) Neutron transmission is defined by

$$T = \frac{I}{I_0},$$

(4.88)

where $I$ is the beam intensity or monitor yield with the cell in place, and $I_0$ is the corresponding intensity or monitor yield when the cell is out of beam, the ”empty” configuration.
In order for this relationship to describe accurately the energy dependent transmission, all other conditions such as beam intensity and monitor efficiencies etc. must be kept constant. Both of the intensities have to be measured by the same beam monitor but they will be performed at two different times. If the proton current incident on the spallation target fluctuates, the beam intensity will be different independently of the configuration of the polarizer between the "cell in" and "cell out" runs. Therefore in our measurements the corresponding $M_2$ signals are normalized by the signal the corresponding signal of the $M_1$.

Figure 4.11: Neutron Attenuation in four Si Wafers built into the polarizer oven.

Flight Path Length Correction  As mentioned, in order to cancel out the effects of beam fluctuations, $M_2$ signals were normalized by the corresponding $M_1$ signals on a time-bin-by-time bin basis. When performing corrections or calculating transmission ratio, the different
lengths of the flight paths has to be corrected for. The two monitors were positioned $\sim 40$ cm apart. Suppose that the distance from the moderator to $M_1$ is $L_1$, then the neutron, which arrived at $M_1$ at time $t_1$, has traveled with velocity $v_1 = \frac{L_1}{t_1}$. The monitor $M_2$ is $\Delta L$ away from $M_1$. The time it takes the neutron to cover this distance is

$$
\Delta t = \Delta L \frac{t_1}{L_1}, \quad (4.89)
$$

and time-of-flight to $M_2$ is

$$
t_2 = t_1 \left(1 + \frac{\Delta L}{L_1}\right) = t_1 \frac{L_2}{L_1}, \quad (4.90)
$$

Here, $L_2$ and $t_2$ are the distance to the second monitor from the moderator. Using this time shift in the normalization removes the unevenness in the transmission curve, near the TOF that corresponds the Al Bragg edges.

The performance of the $^3$He polarizer cell can be described by figure-of-merit (FOM), $P_n^2 T_n$, where $P_n$ is the beam polarization and $T_n$ is the transmission. In the parity violating $\gamma$-ray asymmetry measurements the experimental error is related to the FOM $1/\sigma^2 \propto P_n^2 T_n$ e.g. the FOM is related to the statistical accuracy of the experiment or the running time to achieve the statistical goal of the experiment. The FOM as a function of neutron energy in Fig. 3.12 is calculated for the $^3$He cell with the $^3$He thickness and polarization of XYZ. In Fig. 3.13 FOM is plotted as a function of the $^3He$ thickness for the $^3He$ polarization of XYZ. With these plots the optimum $^3$He cell thickness and polarization in the selected neutron energy range was selected.

$^3$He Polarization in the Cell The $^3$He polarizer system consists of a NMR system that allows a relative measurement of the $^3$He polarization in the cell. For the absolute polar-
Figure 4.12: Figure Of Merit: (a) Un-polarized spherical transmission, (b) Polarized spherical transmission
Figure 4.13: Neutron attenuation in the un-polarized $^3$He-cell, "Boo-Boo", used during the 2005 commissioning run. The transmission has been corrected for the attenuating effects in glass and Si components of the polarizer.
<table>
<thead>
<tr>
<th>Cell</th>
<th>Diameter [cm]</th>
<th>Volume [cm³]</th>
<th>Thickness [10²⁰ cm⁻²]</th>
<th>Lifetime [hrs]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Astro</td>
<td>11.3</td>
<td>640</td>
<td>1.4</td>
<td>730</td>
</tr>
<tr>
<td>Pebbles</td>
<td>11.1</td>
<td>508</td>
<td>1.1</td>
<td>350</td>
</tr>
<tr>
<td>Dino</td>
<td>10.6</td>
<td>452</td>
<td>1.2</td>
<td>700</td>
</tr>
<tr>
<td>BooBoo</td>
<td>12.6</td>
<td>587</td>
<td>1.4</td>
<td>520</td>
</tr>
<tr>
<td>Kirk</td>
<td>10.5</td>
<td>624</td>
<td>1.5</td>
<td>600</td>
</tr>
<tr>
<td>Rocky</td>
<td>13.4</td>
<td>773</td>
<td>1.2</td>
<td>100</td>
</tr>
<tr>
<td>Elroy</td>
<td>11</td>
<td>430</td>
<td>1.0</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 4.1: Polarizer Cells Created and tested at NIST. The cells Boo Boo, Dino and Pebbles were chosen as spin filters for NPDGamma experiments.

The polarization value of \( P_3 \), the relative neutron transmission measurement was used to calibrate the NMR measurement by applying Eqns. 4.81 and 4.82

\[
\frac{T_n}{T_0} = \cosh(n\sigma P_3) .
\]

In this method the thickness parameters, \( nl \), are determined by measurements with un-polarized \(^3\text{He}\) (see previous section).

Neutron Backgrounds in the Determination of the \(^3\text{He}\) Polarization When using Eqn. 4.91 to determine the \(^3\text{He}\) polarization the conditions during the measurements have to be kept constant including neutron backgrounds in the beam monitors. The monitor neutron backgrounds are produced neutrons that are scattered out from beam and then re-scattered...
to the monitor, or one of the largest monitor neutron background is the neutrons which backscatter from the collimation to the monitor that they gave already passed. If the neutron background contributions during the polarized and un-polarized runs in the monitors are changed, then the ratio $T_n/T_0$ is effected and the extracted polarization deviates from the correct polarization. To estimate the size of this effect, test measurements and simulations were performed. In the test experiments neutrons were back-scattered to $M2$ increasing the transmission signal. Having a few percent different back-scattering contributions in $M2$ “polarized” and ”un-polarized” signals compromised the fit and changed significantly the value of the fitting parameter, $P_3$. A larger $\chi^2$ was characteristic for the fittings where the neutron backgrounds were present and were not properly treated or there was present other type of noise that has to be taken care.

In the test experiments, first, the back-scattering portion of the signal was separated in a series of measurements, where the changing scattering surface area was attached to the downstream surface of $M2$. The ”pure” signal, i.e. that due to only the direct beam, was first measured with the backscatterer removed from beam, This signal was then subtracted from the total signals measured with the scattering surfaces in beam, yielding the backscattering part of the signal. The resulting waveform can then be scaled proportionally to the area of the scatterer. In the simulations back-scattering areas were picked, and the corresponding waveforms formed from the $M1$ signal by transmitting them through the same amount, $4.9 \text{ atm} \cdot \text{cm}$, of $^3\text{He}$ as is present in the real $^3\text{He}$ cell called "Boo-Boo".

In the analysis the backscattering contribution was taken care of by introducing a new fitting function. In a simple approximation of the backscattering one considers a neutron
beam incident on the $^3$He cell. According to the transmission

$$I = I_0 e^{-n\sigma t},$$

(4.92)

if the beam is un-polarized and

$$I = I_0 e^{-n\sigma t} \cosh(n\sigma l P_3)$$

(4.93)

if it is polarized. Here $I_0$ is the intensity of the incident neutron beam and $I$ is that of the transmitted beam.

If part of the transmitted beam $I$ is reflected, the intensity measured by $M2$ is for unpolarized beam

$$I_u = I_0 (1 + a_u) e^{-n\sigma t},$$

(4.94)

and for the polarized beam

$$I_p = I_0 e^{-n\sigma t} (1 + a_p) \cosh(n\sigma l P_3),$$

(4.95)

where $I_u$ and $I_p$ are the polarized and un-polarized intensities, and $a_u$ and $a_p$ are the corresponding coefficients, quantifying the reflected portion of the transmitted beam back
into the monitor. Thus the transmission ratio is

\[
\frac{T_n}{T_0} = \frac{e^{-n\sigma t} \cosh(n\sigma l P_3)(1 + a_p)}{e^{-n\sigma t}(1 + a_u)} = \frac{1 + a_p}{1 + a_u} \cosh(n\sigma l P_3).
\] (4.96)

If all other conditions between the polarized and un-polarized runs were unchanged, then the magnitudes of \(a_p\) and \(a_u\) will only depend on the properties of the backscattering surface. The term containing the reflection coefficients will cancel out, provided that the scattering surface stays the same.

The expression of Eqn. (4.96) can be reduced to

\[
\frac{T_n}{T_0} = (1 + a) \cosh(n\sigma l P_3),
\] (4.97)

where \(a = a(E_n)\), \(E_n\) being the neutron energy.

Table 4.2 compares the results of the polarization measurements corrected with backscattering part given by simulations Eqn. 4.82 to the uncorrected results of Eqn. 4.97

Figure 4.15 shows the history of the \(^3\)He polarization in the Boo-Boo cell throughout the 2005 run. An unexpected loss of polarization was observed with two decay constants; a slow loss of polarization which seems to be related to the total beam time and then a faster decay constant which seems to be related to the beam intensity. The gaps in the plot represent pauses in data acquisition due to beam interruptions and various maintenance periods. The decay of the polarization within each period immediately following return of the neutron beam has yet to be explained, as it seems to be correlated with the beam intensity. The overall decay in the polarization, however, is mainly due to the deterioration of the cell itself. Upon examination of the cell after the completion of the commissioning run, a white milky layer was found to be covering the inner surface of the glass. This would
<table>
<thead>
<tr>
<th>$P_{cosh}$</th>
<th>$\Delta P_{cosh}$</th>
<th>$P_{cosh(1+a)}$</th>
<th>$\Delta P_{cosh(1+a)}$</th>
<th>$\frac{B_U}{B_P}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.303</td>
<td>-17.98</td>
<td>5.607</td>
<td>0.387</td>
<td>4.70</td>
</tr>
<tr>
<td>0.311</td>
<td>-16.02</td>
<td>4.852</td>
<td>0.386</td>
<td>4.38</td>
</tr>
<tr>
<td>0.318</td>
<td>-14.08</td>
<td>4.104</td>
<td>0.385</td>
<td>4.05</td>
</tr>
<tr>
<td>0.325</td>
<td>-12.17</td>
<td>3.371</td>
<td>0.384</td>
<td>3.71</td>
</tr>
<tr>
<td>0.332</td>
<td>-10.29</td>
<td>2.664</td>
<td>0.382</td>
<td>3.37</td>
</tr>
<tr>
<td>0.339</td>
<td>-8.43</td>
<td>2.000</td>
<td>0.381</td>
<td>3.02</td>
</tr>
<tr>
<td>0.346</td>
<td>-6.59</td>
<td>1.394</td>
<td>0.380</td>
<td>2.68</td>
</tr>
<tr>
<td>0.352</td>
<td>-4.77</td>
<td>0.868</td>
<td>0.379</td>
<td>2.32</td>
</tr>
<tr>
<td>0.359</td>
<td>-2.97</td>
<td>0.445</td>
<td>0.377</td>
<td>1.96</td>
</tr>
<tr>
<td>0.366</td>
<td>-1.19</td>
<td>0.150</td>
<td>0.376</td>
<td>1.60</td>
</tr>
<tr>
<td>0.372</td>
<td>0.57</td>
<td>0.010</td>
<td>0.375</td>
<td>1.23</td>
</tr>
<tr>
<td>0.379</td>
<td>2.33</td>
<td>0.049</td>
<td>0.373</td>
<td>0.85</td>
</tr>
<tr>
<td>0.385</td>
<td>4.06</td>
<td>0.288</td>
<td>0.372</td>
<td>0.47</td>
</tr>
<tr>
<td>0.391</td>
<td>5.79</td>
<td>0.741</td>
<td>0.370</td>
<td>0.09</td>
</tr>
<tr>
<td>0.398</td>
<td>7.50</td>
<td>1.412</td>
<td>0.369</td>
<td>-0.31</td>
</tr>
<tr>
<td>0.404</td>
<td>9.20</td>
<td>2.294</td>
<td>0.367</td>
<td>-0.70</td>
</tr>
<tr>
<td>0.410</td>
<td>10.88</td>
<td>3.367</td>
<td>0.366</td>
<td>-1.11</td>
</tr>
<tr>
<td>0.416</td>
<td>12.56</td>
<td>4.599</td>
<td>0.364</td>
<td>-1.52</td>
</tr>
<tr>
<td>0.423</td>
<td>14.23</td>
<td>5.947</td>
<td>0.363</td>
<td>-1.93</td>
</tr>
<tr>
<td>0.429</td>
<td>15.88</td>
<td>7.365</td>
<td>0.361</td>
<td>-2.35</td>
</tr>
<tr>
<td>0.435</td>
<td>17.52</td>
<td>8.804</td>
<td>0.360</td>
<td>-2.78</td>
</tr>
</tbody>
</table>

Table 4.2: Fits using $cosh(bP_3)$ and $cosh(bP_3) \cdot (1 + a)$.  
141
Figure 4.15: $^3$He polarization in 2005. The gaps in the plot represent pauses in data acquisition due to beam interruptions and various maintenance periods.
decrease the light transmission into the cell and thus impede the optical pumping process of the $Rb$ atoms.

![Neutron Beam Polarization](image)

Figure 4.16: Neutron Beam polarization at $0.56 \, ^3He$ spin-filter polarization.

The polarization of the neutron beam depends on the neutrons energy, due to the energy dependent absorption cross section. Fig. 3.18 shows neutron beam polarization as a function of time-of-flight when the $^3He$ polarization (Boo-Boo) was 56 per cent atm cm.

Polarizer NMR.

4.3.2 Analyzer The primary purpose of the neutron beam polarization analyzer in this experiment is to measure the degree of polarization of the neutron beam. This allows a study of a beam depolarization when the beam interacts with inhomogeneous magnetic fields or targets where neutron spin-flip scattering can take place, or to monitor the efficiency of the
Figure 4.17: Diagram of the NMR system of the Spin-Filter.
RFSF as discussed in section ABC.

The use of the $^3He$ as a gauge of neutron beam polarization is based on the spin-dependent absorption cross section as discussed in section ABC. After the polarizer the neutron beam polarization is either reversed by the RFSF or left unchanged if the RFSF is off-state. After the RFSF the beam polarization is analyzed by a cell of polarized $^3He$.

The number of neutrons measured by $M3$ when the RFSF is in the on-state (off-state)

$$N_{\pm} = N_0 e^{-\frac{\sigma_0}{t_0} (n+m) \cdot t} \left( \cosh \left[ \frac{\sigma_0}{t_0} (\pm nP + mQ) \cdot t \right] \right),$$

(4.98)

where $\sigma_0$ and $t_0$ are the absorption cross-section and the time-of-flight at thermal energy, $t$ is time-of-flight of neutrons that are measured, $n$ and $m$ are the polarizer and the analyzer thicknesses in cm$^{-2}$ and $P$ and $Q$ are there respective polarizations. The beam polarization analyzing power can then expressed as

$$P_n = frac{N_+ - N_-}{N_+ + N_-} = \cosh \left[ \frac{\sigma_0}{t_0} (nP + mQ) \cdot t \right] - \cosh \left[ \frac{\sigma_0}{t_0} (-nP + mQ) \cdot t \right],$$

(4.99)

where the spin reversal has been taken care by reversing the sign of $P$ in the last term.

The optimum thickness of the analyzer cell was found by using the simulated spectrum of the neutrons coming out of the moderator Fig. ?? and propagated the neutrons through the polarizer and the analyzer, according to Eqn. 4.99. Then the normalized population difference $(N_+ - N_-)$ was plotted as a function of the $^3He$ thickness in the analyzer cell. The plot is shown in Fig. ??). In the calculation polarizer and analyzer polarizations close to those measured during the commissioning run were used ($P = 0.45$, $Q = 0.57$). The maximum was found at $\sim 4$ atm cm.
Figure 4.18: Simulated neutron spectrum used in the optimization of the analyzer thickness.
Figure 4.19: Analyzer Thickness Figure of Merit.
Figure 4.20: The Analyzer Cell (TS-12).
Measurement of the parameters of the analyzer cell

Figure ?? shows the experimental setup for the measurement of the parameters of the analyzer cell. The same setup was also used for the spin-flip efficiency measurements.

![Experimental Setup](image)

**Figure 4.21: Experimental Setup.**

The neutron transmission through the unpolarized analyzer cell was calculated by using signals of the downstream monitor (M3) when the unpolarized cell was in the beam and out of beam.

\[
T_n^0 = \frac{I}{I_0} = e^{-(n_0 \sigma l + n_g \sigma_g l_g)}, \quad (4.100)
\]

where \( n \) and \( n_g \) are the number densities of \(^3\)He gas and that of glass respectively, \( l \) and \( l_g \) - the corresponding thicknesses, \( \sigma = \sigma(E) \) is the energy-dependent absorption cross-section of neutrons in helium. The cell itself is a glass bulb with 2.6 cm in external diameter.

The glass type used in fabrication of this cell was Corning 1720 (see Table [ref] in the appendix) which contains around 5% of \( B_2O_3 \). \(^{10}\)B, which comprises 20% of natural boron has a relatively large energy dependent absorption cross-section, which accounts for about 22% of the beam attenuation and therefore needs to be taken into account. Part of the
attenuation is due to scattering, but as its cross-section is energy independent, it cancels out in transmission calculations.

\[ \sigma = \sigma(E) = \sigma_0 \sqrt{\frac{25.3 \text{meV}}{E}} \]  \hspace{1cm} (4.101)

and

\[ \sigma_B = \sigma_B(E) = \sigma_{B0} \sqrt{\frac{25.3 \text{meV}}{E}}, \]  \hspace{1cm} (4.102)

where \( \sigma_0 = 5333 \text{ barn} \) and \( \sigma_{B0} = 760 \text{ barn} \) is the “thermal” cross-section, measured at 25.3 meV.

Fitting the measured transmission curve to Eqn. (4.100) and correcting for absorption in Boron, gives the parameters which will define the thickness of helium in the analyzer cell.

Measurement of the glass thickness of the analyzer cell using $^{241}$Am $\gamma$-rays. In order to determine the glass thickness of TS-11 cell, we conducted a $\gamma$-transmission experiment using the 26.36 keV $\gamma$-rays from a $^{241}$Am source.

Aluminum and glass samples of known [25] thicknesses were used for calibration of the measurement [Table 3]. The measured thicknesses agreed with the calculated values within 5%. The thickness of the two walls, \( x_{\text{glass}} \), of TS-11 combined was calculated from,

\[ x_{\text{glass}} = \frac{1}{\mu \rho} \ln \frac{N_0}{N}, \]  \hspace{1cm} (4.103)

where \( \mu \) is the photon mass attenuation coefficient at 26.36 keV, \( \rho \) is the glass density, \( N \) and \( N_0 \) are the counts with and without the cell in the beam.
Using $\mu = 2.4 \pm 0.1 \text{ cm}^2/\text{g}$ and $\rho = 2.50 \pm 0.13 \text{ g/cm}^3$ [42] we obtained

$$x_{glass} = 1.41 \pm 0.01 \text{ mm}. \quad (4.104)$$

Substituting these values into Eqn. (4.100) the measured helium thickness was extracted from un-polarized transmission to be $6.20 \pm 0.12 \text{ atm} \cdot \text{ cm}$. 

<table>
<thead>
<tr>
<th>TS – 11</th>
<th>Recorded at Fabrication</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_2$ pressure [atm]</td>
<td>0.099</td>
<td>–</td>
</tr>
<tr>
<td>$^{3}He$ pressure [atm]</td>
<td>3.740</td>
<td>$2.51 \pm 0.054$</td>
</tr>
<tr>
<td>Attenuation in $^{3}He$</td>
<td>0.109</td>
<td>$0.079 \pm 0.002$</td>
</tr>
</tbody>
</table>

Table 4.3: $^{3}He$ Thickness of the cell TS-11.
Measurement of the $^3\text{He}$ polarization in the analyzer cell When considering neutrons observed by the downstream monitor $M3$, all materials that contribute to the attenuation of the beam through energy-independent scattering or energy-dependent absorption, have to be considered. In their path neutrons interact with such materials as aluminum, air, $^3\text{He}$ and glass which contains silicon and boron. Of these materials $^{10}\text{B}$ and $^3\text{He}$ contribute most of the energy-dependent attenuation, whereas the rest account for the energy-independent attenuation. Next we need to derive an expression that allows the analyzer polarization to be extracted from the measured transmissions when the beam is polarized.

Let $N_0 = N_0^+ + N_0^-$ be the number of neutrons incident on the polarizer. Here $N_0^+$ and $N_0^-$ are the numbers of neutrons with spins parallel and anti-parallel to the $+z$-axis which is the direction of the holding field, $B_0$. Before passing through the polarizer the beam is
Figure 4.24: Photon transmission through sample glass plate.
un-polarized and so $N_0^+ = N_0^-$. Each material which beam interacts with reduces the number of transmitted neutrons either through the scattering or absorption. In scattering which is not depending on the neutron energy, the attenuation factor is $e^{-l_i\sigma_{s_i}}$. The overall attenuation factor is then $e^{-\Sigma_i\sigma_{s_i}}$ and can be factored out in the expression for neutron beam transmission through the whole system. Therefore, we need only to consider the energy-dependent component of the attenuation. The number of neutrons with the spins “up” and “down” transmitted through the first beam monitor $M1$

$$N_1^\pm = N_0^\pm e^{-m_1l_1\sigma}, \quad (4.105)$$

where $l_1$ is the $^3He$ thickness and $m_1$ is its number density in $M1$.

After passing through the polarizer cell the number of “up” and “down” neutrons is

$$N_2^\pm = N_1^\pm e^{-nl_p\sigma(1\mp P)} = N_0^\pm e^{-m_1l_1\sigma} e^{-nl_p\sigma(1\mp P)}, \quad (4.106)$$

where $l_p$ is the $^3He$ thickness in the polarizer cell, $n$ is its number density, and $P$ is the $^3He$ polarization.

The number of ”up” and ”down” neutrons transmitted through the second monitor $M2$ is

$$N_3^\pm = N_2^\pm e^{-m_2l_2\sigma}, \quad (4.107)$$

or using the expression for $N_2^\pm$ from Eqn. (4.106)
\[ N_3^\pm = N_0^\pm e^{-m_{1\sigma}e^{-m_{2\sigma}}e^{-nl_p\sigma(1\mp P)}}. \] (4.108)

And finally, analogously to the polarizer, the number of "up" and "down" neutrons that passed the analyzer cell is

\[ N_4^\pm = N_3^\pm e^{-ml_p\sigma(1\mp Q)}, \] (4.109)

where \( l \) is the \(^3\)He thickness in the analyzer cell, \( m \) is its number density, and \( Q \) is \(^3\)He polarization. According to Eqn. (4.109) we get

\[ N_4^\pm = N_0^\pm e^{-m_{1\sigma}e^{-m_{2\sigma}}e^{-nl_p\sigma(1\mp P)}} e^{-ml_p\sigma(1\mp Q)}, \] (4.110)

after adding the time-independent factor, the total number of neutrons, \( N_4 \), transmitted through the two \(^3\)He cells is

\[ N_4 = N_0^+ e^{-\sum l_i \sigma_{s_i} e^{-m_{1\sigma} e^{-m_{2\sigma}} e^{-nl_p\sigma(1-P)}} e^{-ml_p\sigma(1-Q)} + N_0^- e^{-\sum l_i \sigma_{s_i} e^{-m_{1\sigma} e^{-m_{2\sigma}} e^{-nl_p\sigma(1+P)}} e^{-ml_p\sigma(1+Q)}}} \]

\[ \frac{N_0^+}{2} e^{-\sum l_i \sigma_{s_i} e^{-m_{1\sigma} e^{-m_{2\sigma}} [e^{-nl_p\sigma(1-P)} e^{-ml_p\sigma(1-Q)} + e^{-nl_p\sigma(1+P)} e^{-ml_p\sigma(1+Q)}]}} = \]

\[ N_0^- e^{-\sum l_i \sigma_{s_i} e^{-m_{1\sigma} e^{-m_{2\sigma}} e^{-nl_p e^{-\sigma ml}} e^{-\sigma ml}} \cosh(nl_p\sigma P + ml_p\sigma Q)} \] (4.111)

And since the transmission is

\[ T_n = \frac{N_4}{N_0} \] (4.112)

then

\[ T_n = e^{-\sum l_i \sigma_{s_i} e^{-m_{1\sigma} e^{-m_{2\sigma}} e^{-nl_p e^{-\sigma ml}} \cosh(nl_p\sigma P + ml_p\sigma Q)}} \] (4.113)
Here the monitor signals have to be averaged according to the spin sequence, i.e. distinguishing between regimes when the RFSF was on-state and off-state.

On the other hand, when both of the cells are un-polarized ($P = Q = 0$), then transmission is

$$\frac{T}{T_0} = e^{-\sum l_n\sigma_l e^{-m_1\ell_1} e^{-m_2\ell_2} e^{-ml_p} e^{-ml_0}}$$  \hspace{1cm} (4.114)

Figure 4.25: Ratio of polarized and un-polarized cylindrical transmissions.

Fitting the ratio $T_n/T_0$ to $\cosh(n\sigma P + m\sigma Q)$ and using a known value for $P$ gives for the $^3$He polarization of the analyzer cell

$$Q = 0.57 \pm 0.02.$$  

In calculation the effective $^3He$ thickness of 6.2 atm cm was used.
4.4 Curvature effect of the analyzer cell to the determination of the $^3$He polarization

In the above derivation of the $^3$He polarization it was assumed that the $^3$He thickness in the cell is effectively constant across the beam cross sectional area. This is referred to as a cylindrical approximation in this work. In reality, the shape of the analyzer cell is nearly spherical. Because of the curved walls of the analyzer cell the $^3$He thickness is not constant across the beam and in the precision determination of the $^3$He polarization the varying thickness has to be corrected.

4.4.1 Transmission through an unpolarized analyzer cell with curved walls

Next we assume that the cross sectional area of the beam is a circle with radius of $R_b$ which is smaller than the radius of the cell $R_c$. If the number density of neutrons over the beam cross-section is constant, then we can have

$$\frac{dN_0}{ds} = \frac{N_0}{\pi R_b^2}, \quad (4.115)$$

where $dN_0/\text{ds}$ is neutron number density per area unit in the beam cross section with polar coordinates $(r, \phi)$.

The origin of the polar coordinate system is placed on the axis of the beam. The number of neutrons with these coordinates after passing through a cell of geometrical thickness $l$ will be

$$\frac{dN_0}{ds} e^{-n\sigma l} = \frac{N_0}{\pi R_b^2} e^{-n\sigma l}, \quad (4.116)$$

where $l = \text{constant}$, if the cell is cylindrical and $l = l(r) = 2R_c - r^2 - 2R_c + D$ if the
Figure 4.26: Beam passing through a curved cell.
cell has a shape shown in Fig. ???. The latter expression for $l$ describes the $^3\text{He}$ thickness, where the shape of the cell is given by the combination of two spheres with radius of $R_c$ that overlap (see fig). The analyzer cell is outlined with the bold blue line. The cylindrical and spherical cells can then be regarded as extreme cases of a more general cell geometry:

$$\lim_{R_c \to \infty} l(r) = D$$

(4.117)

for the cylindrical shape and

$$\lim_{D \to 2R_c} l(r) = 2\sqrt{R_c^2 - r^2}$$

(4.118)

for the spherical shape.

In order to find out the total number of neutrons that make it through the analyzer cell, one must integrate Eqn. (4.116) over $r$ from 0 to $R_b$ and from 0 to $2\pi$ over $\phi$. If the cell is symmetrical with respect to $\phi$, then the integral looks like

$$N = \int_0^{R_b} \frac{dN_0}{ds} e^{-n\sigma l} r dr d\phi = \frac{2\pi N_0}{\pi R_b^2} \int_0^{R_b} e^{-n\sigma l(r)} r dr$$

$$= \frac{2N_0}{R_b^2} \int_0^{R_b} e^{-2n\sigma \sqrt{R_c^2 - r^2} + n\sigma (2R_c - D)} r dr$$

$$= \frac{2N_0}{R_b^2} e^{n\sigma (2R_c - D)} \int_0^{R_b} e^{-2n\sigma \sqrt{R_c^2 - r^2}} r dr$$

(4.119)

In the case of a cylindrical cell, $l = \text{const}$ and the expression for the transmission through the unpolarized $^3\text{He}$ is simply $T_0 = e^{-n\sigma l}$. Making a substitution $\xi^2 = R_c^2 - r^2$ so that $r dr = -\xi d\xi$ and taking the integral by parts, one gets

$$N = \frac{2N_0}{(2n\sigma)^2 R_b^2} e^{n\sigma (2R_c - D)} e^{-2n\sigma \sqrt{R_c^2 - r^2}} (1 + 2n\sigma \sqrt{R_c^2 - r^2}) R_b$$

(4.120)
\[ dN_2^\pm = \frac{dN_0^\pm}{ds} e^{-\sigma (1\mp P)l_P} e^{2\sigma (1\mp Q)l(r)}. \]

Then substituting \( l(r) \), we have

\[ \frac{dN_2^\pm}{ds} = \frac{dN_0^\pm}{ds} e^{-\sigma (1\mp P)l_P} e^{2\sigma (1\mp Q)l(r)} e^{-2m\sigma (1\mp Q)2\sqrt{R_c^2-r^2-2R_c-D}} = \]

and integrating over the beam cross sectional area we get

\[ N_2^\pm = \int_S \frac{dN_2^\pm}{ds} ds = \int_0^{R_b} \frac{dN_0^\pm}{ds} e^{-\sigma (1\mp P)l_P} e^{2\sigma (1\mp Q)\sqrt{R_c^2-r^2}} ds \]

\[ = 2\pi \frac{dN_0^\pm}{ds} e^{-\sigma (1\mp P)l_P} e^{2R_c-D} \int_0^{R_b} e^{2\sigma (1\mp Q)\sqrt{R_c^2-r^2}} r dr, \]

where variable \( r \) comes from the polar coordinates. Integration by parts yields

\[ \frac{dN_2^\pm}{ds} = \left. \frac{2\pi}{2m\sigma (1\mp Q)^2} \frac{dN_0^\pm}{ds} e^{-\sigma (1\mp P)l_P} e^{2R_c-D} \left( e^{2m\sigma (1\mp Q)\sqrt{R_c^2-r^2}} \left[ 1 + 2m\sigma (1\mp Q)\sqrt{R_c^2-r^2} \right] \right) \right|_0^{R_b} \]
where
\[
\frac{dN_0^\pm}{ds} = \frac{N_0}{2\pi R_0^2}
\] (4.125)
and then
\[
\frac{dN_2^\pm}{ds} = \frac{N_0}{R_b^2[(2m\sigma(1\mp Q))]^2} e^{-\nu\sigma(1\mp Q)/\mu_\nu} e^{2R_c - D} \left( e^{-2m\sigma(1\mp Q)\sqrt{R_0^2 - r^2}} \left[ 1 + 2m\sigma(1\mp Q)\sqrt{R_c^2 - r^2} \right] \right)^{R_b}.
\] (4.126)

where
\[
A_\pm = \frac{N_0 e^{-\nu\sigma(1\mp Q)/\mu_\nu} e^{2R_c - D}}{R_b^2[(2m\sigma(1\mp Q))]^2}.
\] (4.127)

This gives the final result
\[
\frac{dN_2^\pm}{ds} = A_+ \left\{ e^{-2m\sigma(1-Q)\sqrt{R_c^2 - R_b^2}} \left[ 1 + 2m\sigma(1 - Q)\sqrt{R_c^2 - R_b^2} \right] e^{-2m\sigma(1-Q)R_c} \left[ 1 + 2m\sigma(1 - Q)R_c \right] \right\}
\] + \[A_- \left\{ e^{-2m\sigma(1+Q)\sqrt{R_c^2 - R_b^2}} \left[ 1 + 2m\sigma(1 + Q)\sqrt{R_c^2 - R_b^2} \right] e^{-2m\sigma(1+Q)R_c} \left[ 1 + 2m\sigma(1 + Q)R_c \right] \right\}.
\] (4.128)

4.4.3 Analysis of the analyzer data with the new transmission functions
In order to validate the functions derived for "spherical" transmissions derived above, two sets of simulated data were produced. The first set simulates a scenario where a uniform beam passes through a cylindrical cell with parallel windows and the second data set is for a spherical cell. The
first data set was fit to the $e^{-na_l}$, and the second set to the proper "spherical" transmission function of Eqn. 4.128. The fits were performed for both the polarized and the un-polarized $^3\text{He}$. The parameters set at the time of the data generation matched those extracted from the fits, which proved that the functions used indeed describe the process.

Applying the new functions to the measured data produced the following parameters: thickness $= 6.21 \pm 0.12$ [atm-cm]. $Q = P_{\text{analyzer}} = 0.57 \pm 0.02$.

Here in order to calculate the polarization, thickness parameters used in the polarized fit were extracted from un-polarized transmission data by fitting it to the corresponding spherical transmission expression. Cell’s thickness given in $\text{atm cm}$ is calculated by multiplying the extracted $^3\text{He}$ number density by the cell’s average thickness $\bar{t}$ for comparison to the cylindrical case.

$$\bar{t} = \frac{\int_{0}^{R_b} 2\sqrt{R_c^2 - r^2} rdrd\phi}{\int_{0}^{R_b} rdrd\phi} = 2.21\text{cm} \quad (4.129)$$

**Laser Optics of the Analyzer.** The laser system used to optically pump the analyzer cells is schematically presented on Fig. ???. The 30 W diode laser is connected to the optical setup via an optical fiber. At this stage the laser beam is shaped by a focusing lens and delivered to the beam splitter cube. Here main laser beam is divided into two linearly polarized components that come out at 90° with respect to each other and enter two $\lambda/4$-plates to become circularly polarized.

One of the beams, as shown on the left-hand side of Fig. (4.28), is then projected onto the cell. The beam coming out of the second quarter-wave plate circumvents the oven
Figure 4.27: Transmission through an unpolarized (a) and a polarized (b) spherical $^3He$ cells.
Figure 4.28: Schematics of the analyzer setup.
in order to illuminate the cell from the opposite side. Given a constant laser power, this configuration results in a higher pumping efficiency - illuminating more Rb atoms - than with the use of a single laser beam. In the setup the laser beam is reflected by three gold coated mirrors shown in Fig. 4.28.

Figure 4.29: Spectrum Of the Narrowed Laser before passing through the analyzer cell.

The $Rb$ is vaporized by circulating hot air inside the oven containing the cell. The pressurized air is supplied to the 750W cartridge heater that raises the ambient temperature inside the oven to an optimum of $\sim 165^\circ C$. In order to have a high heating efficiency, the oven has to made of material and mass with small heat capacity and thermal conductivity.
In addition the hot air has to be uniformly distributed in the oven. This is accomplished by the double-layer configuration of the oven. The base of the oven is a square-shaped slab of polyetheretherketone (so called "peek") high temperature polymer. The \(^3\)He cell is placed into a cut-out in the middle of the base, to the path of the both laser beams. The heated air is contained by the double-layered transparent rectangular glass domes. The stream of hot air from the cartridge heater connects to an input channel in the peek base and spreads to pin-holes piercing the peek base and connecting the spaces between the two layers of the two glass domes. This reduces temperature gradients and hence the stress on the glass components making up the domes. The air leaves the oven through an exit channel fitted with a \(J\)-type thermocouple, which transmits the temperature reading to the PID that controls the solid state relay between the \(\leq 120\) VAC variac and the cartridge heater.

In this configuration all components, including lens and mirror holders etc., in the vicinity of the \(^3\)He cell were machined out of non-metallic materials, in order to prevent magnetic field gradients, which would cause a relaxation of the \(^3\)He polarization. The nonmetallic optics holders are shown in Fig. 4.30.

The second setup (to be added) utilizes a spectrally-narrowed external-cavity single bar diode laser Figure 4.29.

The analyzer cells were typically polarized to saturation over a \(~12\) hr period and reached polarizations on the order of \(47 - 62\%\).

**NMR system of the Analyzer** The NMR system was built to monitor the \(^3\)He polarization in the cell. CW NMR and Adiabatic Fast Passage (AFP) described in section 4.2.4, were used to measure a polarization signal within the cell. Figure (4.32) shows a typical AFP
Figure 4.30: part of the broad-spectrum optics to polarize the analyzer cell.
Figure 4.31: TS-12 pump-up curve. The negative slope is due to the negative sign in front of the signal.
signal.

Two rectangular pick-up coils were placed on two sides of the cell, both in the plane perpendicular to the plane of the Helmholtz coils and the drive coils. In order to achieve a high signal-to-noise ratio the filling factor was maximized by placing the pick-up coils very close to the cell. The relative orientation of the coils was tuned in order to minimize pick-up of the RF field created by the Drive coils. As described in section 4.2, on resonance the rotating magnetization vector traverses the pick-up coils and induces an EMF signal which is amplified by the pre-amplifier and further by the lock-in amplifier used for a phase sensitive detection. The quantization axis is defined by the static magnetic field produced by 79 cm Helmholtz coils.

In the AFP process the main magnetic field sweeps through the resonance (60 kHz) at a rate of \( \frac{dH_0}{dt} = 2.58 \text{G/s} \) in order to satisfy the adiabatic 7.37 \( s^{-1} \ll 1134 \text{ s}^{-1} \) and fast 0.002 \( s^{-1} \ll 7.37 \text{ s}^{-1} \) conditions. The drive coils maintain an RF field with RMS
Figure 4.33: Diagram of the NMR system for the analyzer. Lock-In amplifier - Stanford Research Systems, Model SR-830 DSP. Two Function Generators - SRS, Model DS 345. RF Amplifier - KROHN-HITE, Model 7500.
\[ B_1 = 0.35\, G. \] Each AFP sweep on average resulted in a loss of polarization of 0.1%. The cells used in the experiment, TS-11 and TS-12 showed polarization relaxation time constants on the order of 130 hrs.

![Figure 4.34: The full analyzer set-up with the electronics rack.](image)

### 4.4.4 Optimization the Spin Flipping Efficiency of the RFSF Using The Analyzer

During the commissioning run in 2005 the spin flipping efficiency of the RFSF was optimized using the neutron beam polarization analyzer. The amplitude of the RFSF current was optimized by maximizing the spin-up - spin-down asymmetry measured by the analyzer. Figure ?? shows the measured spin flipping efficiency as a function of the amplitude of the current of the RFSF. The data is fit to a quadratic function. Then the amplitude of the RFSF current was set to the optimum value of 750 mV and the magnitude of the static holding
field was varied to again find the maximum of the $\uparrow - \downarrow$ asymmetry. The result of this measurement is shown in Fig. ?? showing the maximum at 18.52 A.

![Amplitude Scan 1.3° On-Axis 06/25/05](image)

Figure 4.35: Spin Flipper Amplitude optimization using the Beam Polarization Analyzer.

Transportation of the analyzer cell to the experiment  The laser-optics, the NMR system, and the oven were located in a separate building $\sim$ 1 km away from the site of the experiment. After the buildup of the polarization the cell has to be delivered and installed in the flight path to the front of the monitor $M3$ with minimal losses of polarization. For the transportation a cell transporter was designed and built shown in Fig. ?? . In the figure a cylindrical analyzer cell in the middle of the Helmholtz coils can be seen. The cell transporter consists of a pair of coils in Helmholtz configuration to maintain a uniform 12 Gauss field over a radius of $\sim$ 6 cm. The 6 A current is supplied by a rechargeable battery. Typically, the polarization loss caused by transportation was on the order of
Figure 4.36: Guide Field optimization using the Beam Polarization Analyzer.

\[ \Delta P_3/P = 0.1/hr. \]
Figure 4.37: Polarized cell transporter.
5.1 Parity Violating $\gamma$-ray Asymmetry from Detector Data

Parity violation between nucleons causes a tiny change to the radiative neutron capture cross-section which leads to the $\gamma$-asymmetry - the subject of this measurement. The cross section change is observed by measuring a small change in the detector yields when $\vec{s}_n \cdot \vec{k}_\gamma > 0$ or $\vec{s}_n \cdot \vec{k}_\gamma < 0$ i.e. for the neutron spin up- and down-states. Here $\vec{s}_n$ is the neutron spin, and $\vec{k}_\gamma$ is the momentum of the photon. Quantitatively this effect results in an observed angular asymmetry, $A_{\gamma}$ which is related to the differential radiative capture cross-section by

$$\frac{d\sigma}{d\Omega} = (1 + A_{\gamma} \cos \theta), \quad (5.1)$$

where $\theta$ is the angle between the neutron spin and the $\gamma$-ray momentum and $A_{\gamma}$ is the asymmetry. This cross-section quantifies the rate of the $\gamma$-rays emitted at the given angle $\theta$.

In order to understand how the asymmetry is constructed using the signals of the detector array consisting of 48 units, let us first consider a simple case, where the measurement is performed with only 2 identical detectors, positioned directly opposite each other, so that the line connecting the two is parallel to the neutron spin. For simplicity these detectors can be infinitesimally small, which condition we can also extend to the target, in wherein
the reaction occurs. In addition, for now, we will assume that the experiment takes place under ideal conditions, so that all 100% of the neutrons in the beam are polarized in one direction, every single one of their spins is reversed by the spin-flipper when it is turned on, and none loses its orientation before being absorbed. In general, by definition the signal asymmetry $A$ between the upper and the lower detectors is the ratio

$$A = \frac{U - D}{U + D},$$

where $U$ and $D$ are the yields of the upper and lower detectors respectively. If all of the conditions listed above are satisfied, then the real, “physics” asymmetry stemming from the reaction that produces the $\gamma$-rays can be calculated according to 5.2.

In our experiment we are interested in measuring the asymmetry between the $\gamma$-yields corresponding to the to orientations of the spin $\uparrow$ and $\downarrow$. As mentioned in the section dedicated to the Radio-Frequency RFSF [?], theoretically this means measuring the signals for the two spin states, in the same detector, which however is not feasible due to drifting values of the beam current, detector gains and other parameters. The next logical step is to make use of a configuration such as described in the previous paragraph. The upper and lower detectors then in principle measure the gamma intensities corresponding to one spin state each. The asymmetry measured in this manner could therefore be said to arise purely from the reaction cross-section.

Now we have to address each of the assumptions introduced for the simplified case of the asymmetry between two detectors. We will then move towards the realistic conditions,
by relaxing the constraints on the one hand and accounting for that fact in the expression for the asymmetry, on the other.

The two detectors in question cannot be identical - even if they are positioned perfectly symmetrically about the coordinates of the capture reaction, their efficiencies will, if not by much, differ. The term “efficiency” is used in the broad sense, and signifies the efficiency of gamma detection, as well as the gains characteristic of the given detector’s circuit. Furthermore, the differences in efficiencies may not be constant, but drift with time. In the worst case scenario the circuitry of the detector, could couple to the magnetic field produced by the current driving the RFSF and thus be correlated with the spin. Since the asymmetry of the signals produced in such a way would be indiscernible from the “real” one, such coupling would result in a false measured asymmetry. This scenario is prevented by configuring all possible circuits in a fashion that avoids ground loops in the vicinity of the experiment. The uncorrelated drifts of the detector efficiencies are mitigated through the implementation of the spin sequence $\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow$, which is effective up to the second order.

The basic, ”raw” asymmetry becomes

$$A(t) = A_{\gamma,i}^{\text{raw}}(t) = \frac{U_i(t) - D_i(t) - [U_{i\uparrow}(t) - D_{i\uparrow}(t)]}{U_{i\uparrow}(t) + D_{i\uparrow}(t) + U_{i\downarrow}(t) + D_{i\downarrow}(t)}, \quad (5.3)$$

where $U_{i\uparrow}(t)$ and $D_{i\uparrow}(t)$ are the modified versions of $U$ and $D$, which we used before, with subscripts indicating the neutron spin’s direction. Since the acquired data are divided into time bins, for neutron energy resolution, the asymmetry is calculated for each time bin $t$. This information plays an important role in determining the physics asymmetry. The
superscript $i$ denotes pair to which the detectors belong - the real detector array consists of 24 such pairs positioned at different angles with respect to the direction of the holding field. The efficiency of the cancellation of drifts and other systematic effects uncorrelated with the spin sequence rest on the ability of the spin-flipper to reverse the neutron spins. If the efficiency $S(t) < 1$, the asymmetry value will be artificially reduced. Therefore $S(t)$ enters the equation as a correction factor.

If the entire detector array is centered about the target, then each detector in a pair is located symmetrically with respect to the $z$-axis of the axes, placed at the target, or at its center, if the target is of finite size. Detector pairs aligned with the direction of the holding field, as those in the simplified case, measure the Up-Down asymmetry $A_{UD}$ directly. On the other hand pairs positioned at an angle $\theta_i$ with respect to the vertical axis, are only susceptible to a projection of the asymmetry. Therefore one can see from simple geometric considerations, that in order to infer the real, physics asymmetry from signals measured in such a pair would have to weight the asymmetry by a factor of $\cos \theta_i$. From similar arguments, the measured Left-Right asymmetry, $A_{LR}$ has to be corrected by $\sin \theta_i$ in order to yield the full asymmetry. Thus, the $\cos \theta_i$ and the $\sin \theta_i$ are the “geometry factors”, that account for the dependence of the measured asymmetry on the geometry of the detector array.

The signal in the upper detector can be derived from 5.1 to be $U \propto 1 + A_\gamma \cos \theta_i$ or taking into account the contribution from the possible left-right asymmetry

$$U \propto 1 + A_{UD}^{\gamma}(s_n \cdot k_\gamma) + A_{LR}^{\gamma}(s_n \times k_\gamma),$$

(5.4)

where the cosine corresponds to the inner product of the neutron spin and the $\gamma$-ray
Figure 5.1: Detector geometry with respect to the spin direction
momentum. The sign of this term depends on the relative orientation of the $s_n$ and $k_\gamma$, so that

$$U_{\uparrow\downarrow} \propto 1 \pm A^{UD}_\gamma (s_n \cdot k_\gamma) \pm A^{LR}_\gamma (s_n \times k_\gamma) = \epsilon_U p_{\uparrow\downarrow} [1 \pm A^{UD}_\gamma \cos \theta_i \pm A^{LR}_\gamma \sin \theta_i] + \epsilon_U b_i,$$  (5.5)

where the subscript enumerates the detector pairs, $\epsilon_U$ is the efficiency of the upper (lower) detector, $p_{\uparrow\downarrow}$ is the beam intensity and $b_i$ is the signal due to backgrounds. We will assume at this point that the backgrounds, be they of electronic (pedestal) or otherwise nature, are constant for the given energy or at least uncorrelated with the neutron spin. Similarly for the lower detector

$$D_{\uparrow\downarrow} \propto 1 \pm A^{UD}_\gamma (s_n \cdot k_\gamma) \pm A^{LR}_\gamma (s_n \times k_\gamma) = \epsilon_D p_{\uparrow\downarrow} [1 \mp A^{UD}_\gamma \cos \theta_i \mp A^{LR}_\gamma \sin \theta_i] + \epsilon_D b_i.$$  (5.6)

plugging these into 5.1 we see first of all that the background terms cancel, and we get:

$$A_{\text{raw}}^{\gamma} \, i = \frac{\left( \frac{\epsilon_U - \epsilon_D}{\epsilon_U + \epsilon_D} \right) \left( \frac{p_{\uparrow\downarrow} - p_{\uparrow\downarrow}}{p_{\uparrow\downarrow} + p_{\uparrow\downarrow}} \right)}{1 + \left( \frac{\epsilon_U - \epsilon_D}{\epsilon_U + \epsilon_D} \right) \left( A^{UD}_\gamma \cos \theta_i + A^{LR}_\gamma \sin \theta \right)} \left( A^{UD}_\gamma \cos \theta_i + A^{LR}_\gamma \sin \theta \right) \frac{A_{\gamma} \cdot A_{\text{p}} + \left( A^{UD}_\gamma \cos \theta_i + A^{LR}_\gamma \sin \theta \right)}{1 + A_{\gamma} \cdot A_{\text{p}} + \left( A^{UD}_\gamma \cos \theta_i + A^{LR}_\gamma \sin \theta \right)}.$$  (5.7)

Here, the first and second multipliers in the first term by definition represent the asymmetry of the efficiencies of the top and the bottom detectors, and that of the beam intensities corresponding to the $\uparrow$ and $\downarrow$ spin states. The former, $A_{\gamma}$ has been measured to be $\sim 10^{-2}$, while the latter is on the order of $10^{-3}$ per spin sequence. Therefore the product is expected to be small $\sim 10^{-5}/\sqrt{n}$, where $n$ is the number of spin sequences. Thus this term can be neglected in the numerator. In the denominator, the same term multiplies the brackets containing the up-down and left-right $\gamma$-ray asymmetries - also small quantities. The resulting expression is
If the target-beam interface is not infinitesimally small, as per our assumptions, then its geometry affects the effective angle of the detectors. The real geometry factor $G(t)$ also has to include the effects of the finite size ($15 \times 15 \times 15 \, cm^3$) of the detectors. The geometry of the detectors is discussed in more detail in later sections. With the real geometry factors, the expression for the raw $\gamma$-ray asymmetry for the detector pair $i$ becomes

$$A_{\gamma, i}^{raw} = A_{\gamma}^{UD} G_i^{UD} + A_{\gamma}^{LR} G_i^{LR}$$

Another assumption used in the construction of the algorithm to calculate the asymmetry is that the neutron beam is completely polarized in one of the two directions along the magnetic field. In practice, the polarization of the beam is dictated by the characteristics of the spin-filter, with typical polarizations of $\sim 40 - 55\%$. The neutrons with a spin opposite to that of the spin-filter’s $^3He$ nuclei are captured, making the $\uparrow$ and $\downarrow$ populations of the neutron beam unequal. This asymmetry in the populations is by definition the beam polarization. Since the neutron capture cross-section is energy dependent, so is the resulting beam polarization

$$P_n(E) = \tanh[nlP_3\sigma(E)].$$

Therefore the low-energy part of the beam is polarized more efficiently. The typical
beam polarization is shown on fig [plot from Polarimetry]. Since the asymmetry is crucially dependent on the neutron spin orientation, neutrons with spins oriented opposite to those of the majority reduce the effect and result in a smaller value of the measured asymmetry. This introduces an energy-dependent correction factor $P(t)$ in the expression for the physics asymmetry. From similar arguments, one has to include a factor $T(t)$ that accounts for the spin-flip scattering and other processes, that lead to a partial loss of polarization within the neutron beam.

The end result of the above arguments is the expression for the physics asymmetry for a given detector pair, measured at neutron energy corresponding to time bin $t$:

$$A_i^*(t) = \frac{1}{S(t)P(t)G(t)T(t)} \left\{ \frac{U_i^*(t) - D_i^*(t) - [U_i^*(t) - D_i^*(t)]}{U_i^*(t) + D_i^*(t) + U_i^*(t) + D_i^*(t)} \right\}.$$  \hspace{1cm} (5.11)

Thus the asymmetry is calculated separately for every pair of detectors, and all times of flight divided into time bins between 10 and 32 ms. The lower limit is set by the start of each macro-pulse’s acquisition, and the upper is where the chopper begins eclipsing. The resulting number is the sequence asymmetry for the given pair-time bin. Having corrected the raw asymmetries by the factors of the beam polarization, depolarization, spin-flip efficiency and geometry factors, allows us to calculate an average asymmetry for the given detector pair. This average has to be weighted by the errors of the asymmetries for each time bin

$$A_i^* = \frac{\sum_i A_i^*(t)}{\sum_i \frac{1}{\sigma_i^2(t)}} \pm \frac{1}{\sqrt{\sum_i \frac{1}{\sigma_i^2(t)}}}.$$ \hspace{1cm} (5.12)

This produces the average physics asymmetry for the given pair of detectors, for the given sequence. The physics asymmetry for the entire array and a given sequence 5.12 is further
averaged, again weighted by the respective errors,

\[ A_{\gamma}^{\text{tot}} = \frac{\sum t \frac{A_i}{\sigma_i^2}}{\sum t \frac{1}{\sigma_i^2}} \pm \frac{1}{\sqrt{\sum \frac{1}{\sigma_i^2}}}. \] (5.13)

5.1.1 Measurement of Detector Positions by Moving the Detector Table

Since the $\gamma$-ray yield in a detector depends on its effective solid angle, then by moving the detector respect to the point-like $\gamma$-ray source the effective position of the detector can be obtained and thus a geometric factor could be calculated for each detector. The measurement of the detector position allowed also to learn the effective alignment of the detector respect to the static field $B_0$.

An attempt was made to measure the detector geometric factors by moving the detector table respect to the small diameter $\gamma$-ray source in beam. Adapting a simple approach and assuming a point-like $\gamma$-ray source and infinitesimal detectors, one could estimate the yield in a detector depending on the relative positions of the crystal and the source. Using first order approximation, we can assume that the yield in a detector is proportional to the solid angle spanned by the detector from the point of view of the source. If the detector’s coordinates are $(x, y)$ and the detector crystal is located at $(x_c, y_c)$. Then the detector signal is

\[ S(x, y) = \frac{A}{R^2} = \frac{A}{(x - x_c)^2 + (y - y_c)^2}, \]

where $A$ is a proportionality constant and $R$ is the distance between the source and the crystal.

Taking into account that
Figure 5.2: A depiction of the ideal case of a point source - point detector picture. However, in reality both, the source and the detector have finite sizes and the problem is more complicated. To estimate the "true" detector positions, Monte Carlo simulations were carried out.
\[ S_0 = S(0, 0) = \frac{A}{R_c^2}, \]

where \( R_c = x_c^2 + y_c^2 \). Then

\[ S = S_0 \frac{x_c^2 + y_c^2}{(x - x_c)^2 + (y - y_c)^2} \quad (5.14) \]

Differentiating \( \frac{\partial S}{\partial x} \) we get

\[
\begin{align*}
\frac{\partial S}{\partial x} &= 2S_0(x_c - x) \frac{x_c^2 + y_c^2}{(x - x_c)^2 + (y - y_c)^2}, \\
\frac{\partial S}{\partial y} &= 2S_0(y_c - y) \frac{x_c^2 + y_c^2}{(x - x_c)^2 + (y - y_c)^2} \quad (5.15)
\end{align*}
\]

And finally for the angle we have

\[ \frac{\partial S/\partial y}{\partial S/\partial x} = \tan \tilde{\theta} \quad (5.16) \]

Here \( \tan \tilde{\theta} \) is the angle between \( R \) and the \( x \)-axis. Our approximation however must stipulate that as the source displacements are small, so that \( \tilde{\theta} \simeq \theta \).

From similar arguments

\[ R = \frac{2S_0}{\sqrt{\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2}}} \quad (5.17) \]

there \( A \) is a proportionality constant, and \( R \) is the distance between the source and the crystal.
During the experiment we used this model to measure the effective position and effective angle of the detector array. The support structure of the NPDGamma detector array that weighs about one metric ton, was designed and built with these measurements in mind, which allows the detector array to be translated with respect to the neutron beam. For the measurements the detector array was moved through a $5 \times 5$ grid of $x - y$ coordinates around beam-center; five positions in the left-right direction and five positions in the up-down direction by stepping the array 4 mm at a time. A thin Cd piece was used as a target, since Cd possesses a very large radiative absorption cross-section and each neutron capture produces several $\gamma$-rays.

The deviations of the effective detector angles from the physical angles can be explained in terms of properties of the detector cubes. Each CsI cube is composed of two crystals which can have slightly different efficiencies, which coupled with the detector relative position may result in an effective angle different from the physical angle. On the other hand, the approximation of point-like source and point-like detector is tested as well, since the beam used for the measurements was collimated to 2.5 cm in diameter, whereas the CsI cubes are $15 \times 15 \times 15 \text{ cm}^3$. Thirdly, strong backgrounds were measured with the target removed from the beam. It is expected that this effect would influence the results in first ring, which measures all possible reaction products originating in the Al and other components of the experiment preceding the target in the neutrons' path. In addition the geometry factors for the background are different from those for the production data.

One can observe a qualitative difference between the angles reconstructed for the so-called corner detectors, i.e. those located at the $|\theta_i| = 45^\circ$ with $\pm x$-axis or the $\pm y$-axis, and the rest. In this example, detectors 25, 28, 31, and 34 are the corner detectors (see
Figure 5.3: A sample data (Ring #3) describing the effective detector angles reconstructed from motion table measurements. The effective detector angles are reconstructed using the table motion data and employing the formalism described in text. The gray squares indicate the expected, "real" positions of the detector cubes while the triangles represent the measured positions, and the line connected to them indicate the measured effective detector angles.
Fig. 5.3). Detectors 26, 27, 32, and 33 are shifted from their expected positions. However, their shift is symmetric. If the detector array is positioned in such a way that its geometric center coincides with that of the beam, then the systematic effects causing the shift in 26, 27, 32, and 33, would cancel each other out purely out of symmetry arguments. Therefore, studying the corner detectors potentially eliminates the necessity to estimate and analyze some of the systematics. Considering only the corner detectors, the average angle of the corner detectors is consistent with their expected positions. The standard deviations for the detector angles obtained from our measurements were on the order of 2-3 degrees. In reality, the uncertainty in the detector positioning is expected to be on the order of a couple of mm since the alignment was performed with the utmost care, using theodolites and optical lasers. Another factor affecting the uncertainty in the relative angle between the detector array and $B_0$ is the accuracy of the measurement of the magnetic field direction. The flux-gates used in the measurement have an intrinsic uncertainty of few percent.

5.1.2 Geometry Factors Each pair in each ring of the detector array is separately considered in the calculation of the UD asymmetry. The detector array covers $\sim 95\%$ of the solid angle viewed from the center of the target sample. However, the asymmetries measured in different pairs do not contribute equally to the final asymmetry in terms of their position relative to the origin of the $\gamma$-ray. Obviously, the two detectors in a pair closer to the vertical axis will carry more weight in the difference in the number of $\gamma$-rays measured in the UP-DOWN direction than the pair closer to the horizontal axis. As mentioned above, in the "ideal" case, where both the detectors and the target are point-like, the weights can be simply expressed based on the angle with respect to the horizontal axis: $\cos \theta$ for the
$A_{UD}^\gamma$ asymmetry, and $\sin \theta$ for $A_{LR}^\gamma$. The real detectors are in fact much larger, while the target size is large enough to cover the cross-section of the collimated neutron beam (see above). The distribution of the $\gamma$-rays emanating from the target as well as their paths will therefore differ somewhat from the simplified case studied with the table motion.

The spatial distribution of the neutrons throughout the target that produce $\gamma$-rays in radiative-capture reactions depends on the target material as well as the beam size. On the other hand the signal resulting from these $\gamma$-rays also changes with the target geometry. The amount of energy that each $\gamma$-ray deposits in the CsI crystals varies with the angle of incidence. Also, the same $\gamma$-ray may traverse more than one crystal. All these factors change the effective geometry of the detector array as opposed to the real physical geometry. The simple $\cos \theta$ and $\sin \theta$ therefore need to be averaged by taking into account the weights based on the deposited $\gamma$-ray energy. This integration was carried out through a Monte Carlo simulation using MCNP. The probability of capture along the trajectory ($\hat{z}$) decays exponentially (for each neutron energy).

\[
h(z, \delta z, t) = \int_{z-\delta z/2}^{z+\delta z/2} \frac{\lambda(t)e^{-\lambda(t)z}}{1 - e^{-\lambda(t)l}} \, dz,
\]

where $\lambda(t)$ is the decay parameter, and $l$ is the length of the target along $\hat{z}$.

The first stage of the simulation handles the transport of the neutrons through the beam line, and calculates the distribution of the neutrons within the target. This information then serves as the input of the second part of the simulation, which traces the $\gamma$-rays from the target to the CsI crystals, and calculates the corresponding relative yield in a given detector. A grid with a 5 mm step is sampled within the target volume and $\gamma$-rays generated into $4\pi$ from each of the neutron capture locations and traced through the crystal.
Each photon remains in the loop until it is ranged out of the detector or leaves more than 95% of its energy. The amount of energy deposited by the $\gamma$-rays upon a scattering event is determined recursively from the Compton scattering formula.

$$h\nu = \frac{h\nu_0}{1 + \frac{h\nu_0 (1 - \cos \theta)}{m_0 c^2}},$$  \hspace{1cm} (5.19)$$

where $h\nu$ is the energy of the scattered $\gamma$-ray and $m_0 c^2$ is the rest mass of the electron.

The weight is then for a source at point $\mathbf{r}$, of a $\gamma$-ray with an initial energy $E_\gamma^0$, emitted in the direction $(\theta, \phi)$

$$f(\mathbf{r}, \theta, \phi) = \sum_{i=1}^{\infty} \left( \frac{E_{\gamma}^{i-1} - E_{\gamma}^{i}}{E_{\gamma}^0} \right) \theta[0.95 - f(\mathbf{r}, \theta, \phi)],$$  \hspace{1cm} (5.20)$$

where $i$ is the number of scattering events. Energy after scattering is $E_{\gamma}^{i}$ and is determined by

$$E_{\gamma}^{i} = \frac{E_{\gamma}^{i-1}}{1 + \frac{E_{\gamma}^{i-1} (1 - \cos \alpha)}{m_0 c^2}},$$  \hspace{1cm} (5.21)$$

where $\alpha$ is the scattering angle [(31)].

The resulting geometry factor for the $j^{th}$ detector is then calculated as

$$G_j(\theta, \phi, E_n, \mathbf{r}) \equiv \frac{d^3 r \int_0^\pi \int_0^{2\pi} d\phi g(\theta, \phi) f(\mathbf{r}, \theta, \phi) h(z, \delta z, E_n)}{\int d^3 r \int_0^\pi \int_0^{2\pi} d\phi f(\mathbf{r}, \theta, \phi) h(z, \delta z, E_n)} \hspace{1cm} (5.22)$$

**Left-Right $\gamma$-ray Asymmetry and Cross-Correlations** If the left-right asymmetry is relatively large [(9)], then it is important to separate it from the up-down value. The position and the angle of the detector array with respect to the magnetic field are therefore important in determining the contributions from the L-R and U-D asymmetries. The geometry factors define this positional and angular dependence. In the calculations of asymmetry the
Figure 5.4: The effective detector angles calculated from the geometry factors for the case of hydrogen target.
geometry factors appear in a context of a two opposite detectors (so called detector pair). For simplicity we will denote these geometry factors as $Y$ and $X$ for up and down detectors in the pair respectively. The dependence of the asymmetry on the angles of the detector pairs is demonstrated in a measurement of the $Cl$ target, where the PV $A_{UD}$ is large and well known [[22], [5], [21]]. The $Cl$ target was therefore used during the experiment to verify the performance of the apparatus and the analysis algorithm. Using Eqn. (5.9) the raw asymmetry (or the raw asymmetry corrected by $P(t)$, $T(t)$ and $S(t)$) can be plotted as a function of the detector angle which is expected to follow the $A_{UD} \cos \vartheta + A^{LR} \sin \vartheta$ function, where $A^{UD}$ and $A^{LR}$ are the fitting parameters and $\vartheta$ is the azimuthal angle of the given detector in the ring. If the geometry factors are treated like components of a vector then $\vartheta$ can be extracted from them by writing

$$\vartheta = \tan^{-1} \frac{X}{Y}. \quad (5.23)$$

Previously we assumed that two detectors in any given pair are placed symmetrically with respect to the vertical. In principle one has to consider a case, where there is a slight misalignment between them, so that as opposed to the assumed angle $|\theta_i|$ for both, one is shifted away from the line connecting the other with the origin of the coordinate system by $\delta \theta$. The effect of an uncertainty associated with the angle of the detector pair with respect to the neutron spin direction can be quantitatively assessed by considering expressions 5.5 and 5.6. Suppose, the lower detector is shifted to a new angle $\theta + \delta \theta$. Then this time we get

$$D_{11} \propto 1 \pm A^{UD}_\gamma(s_n \cdot k_\gamma) \pm A^{LR}_\gamma(s_n \times k_\gamma) = 1 \pm A^{UD}_\gamma \cos \theta + \delta \theta \pm A^{LR}_\gamma \sin \theta + \delta \theta, \quad (5.24)$$

and the expression for the upper detector remains unchanged. Plugging this back into 5.3
and grouping like terms we get:

$$A_{\gamma}^{\text{raw}} = \cos \theta \left[ A_{\gamma}^{UD} + \frac{\delta \theta}{2} A_{\gamma}^{LR} \right] + \sin \theta \left[ A_{\gamma}^{UD} + \frac{\delta \theta}{2} A_{\gamma}^{LR} \right].$$

(5.25)

From 5.25 one can see that a misalignment of $\sim 1^\circ$ results in a shift of the UD asymmetry $\sim 1\%$ of the $A^{LR}$. This scenario is unlikely considering the robust design of the detector array and the thorough alignment. On the other hand, in principle, the entire array could be tilted with respect to the axis. The effect of such a misalignment can be estimated from considerations analogous to Eqn. (5.25). However, the table motion measurements detailed above do not show any significant deviation from the expected angles of the corner detectors. These measurements also suggest that there was no translational shift from the nominal position which would result in a changed perceived angles for those detectors.

5.1.3 Depolarization of the Neutron Beam

The neutron beam polarization is reduced by the depolarization caused by the inhomogeneities in the static magnetic field and the spin-flip scattering that occurs within the target. The static magnetic field has been measured to be homogeneous enough that no depolarization takes place but due to the incoherent scattering that occurs within the target some beam polarization is lost and the measured asymmetries need to be corrected by the energy-dependent depolarization factor $T(t)$ for the physics asymmetries. Neutron depolarization in a target, $T$, is accounted for using cross section values for the $(n, \gamma)$-reaction, spin-coherent and spin-incoherent scattering from Ref. [[40]]. A simple Monte Carlo simulation was written to propagate neutrons through the target material using these cross sections and assuming the $1/v$ dependence of the capture cross section, and upon each scattering the probability of spin-flip scattering was accounted
for by taking 2/3 of the ratio of the spin-incoherent scattering cross section to the total scattering cross section. Depolarization was determined by computing an average value for the number of spin-flip scatterings prior to capture.

The probability of absorption by the nucleus is

\[ P_a = \frac{\sigma_a}{\sigma_t}, \]  

(5.26)

where \( \sigma_a \) is the absorption cross-section and \( \sigma_t \) is the total cross-section:

\[ \sigma_t = \sigma_a + \sigma_s = \sigma_a + \sigma_i + \sigma_c, \]  

(5.27)

\( \sigma_c \) and \( \sigma_i \) being the coherent and incoherent parts of the scattering cross-section, \( \sigma_s \). In turn the probability that the neutron spin will flip upon scattering on the nucleus is proportional to \( \frac{2}{3}\sigma_i \):

\[ P_{flip} = \frac{\frac{2}{3}\sigma_i}{\sigma_c + \sigma_i}, \]  

(5.28)

the probability that it will not flip is then

\[ 1 - P_{flip} = P_{noflip} = \frac{1}{3}\sigma_i \]  

(5.29)

The neutron spin direction will remain unchanged compared to that which it had coming out of the polarizer if it absorbs right away, if it scatters without flipping before being absorbed, or if the number of spin-flip scattering events is even. Otherwise the direction will be opposite to the original.
The factor $T(t)$ depends on the neutron energy, so it has to be calculated and applied in the corresponding time bin. The values were acquired through a simple Monte Carlo simulation that propagates the neutron throughout the flight path, taking into account the factors considered above.

Figure 5.5: Neutron depolarization vs energy

Another set of simulations was carried out for beam depolarization calculations in LH$_2$ with different ortho-para concentration ratios. Figure (5.6) shows some of these results.

5.2 Systematics
Figure 5.6: Beam depolarization in LH2 at two para-ortho ratios.
5.2.1 Backgrounds from scattered neutrons and $\gamma$-rays

The $\gamma$-ray backgrounds affect the PV $\gamma$-ray asymmetry. The background itself can possess a $\gamma$-ray asymmetry which can then produce a false asymmetry in the measurement. Therefore, the $\gamma$-ray asymmetry of the backgrounds have to be measured to the sensitivity that is smaller than the sensitivity of the experiment. If the backgrounds are large, they dilute the physics asymmetry. According to Eqn. 5.3 a background yield cancels out in the numerator of $A_{raw}^\gamma$, but in the denominator the background signals add up resulting in a reduced value of the physics asymmetry.

To estimate background contributions in the experiment we need to consider which types of backgrounds are present.

1. The backgrounds can be divided into the following main categories:

   - Scattered neutrons - neutrons scattered out of the beam before they reach the target and absorbed by other parts of the apparatus, such as aluminum struts, cave walls, $^{10}\text{B}$-doped polyethylene shielding, beam stop etc. All of these materials create $\gamma$-ray(s) through neutron capture.

   - Frame-overlap neutrons - these are slow neutrons originated in earlier neutron macro-pulses as discussed in Chopper section.

   - $\gamma$-rays from the spallation source and neutron guide coming straight out of the neutron guide and mainly Compton scattering from target.

   - Electronic noise and other instrumental noise such as AC-pickup, ground loops etc.

2. The $\gamma$-ray backgrounds in this experiment are likely to originate in materials like Al
and Cu. Therefore, we studied $\gamma$-ray asymmetries on these materials in separate measurements using specific Al and Cu targets. The Al $\gamma$-ray asymmetry was measured to the accuracy of $A_\gamma \simeq 10^{-7}$. Aluminum is the prevalent material used in target containers and beam windows and hence is always exposed to the beam. In the $\gamma$-ray detector light produced by anything other than neutron capture in the target material are considered as background. This background can be subdivided into two groups:

- Background created by the target: $\gamma$-rays created when neutrons are captured outside the target, when the neutron is scattered out of the target and then absorbed.

- Target independent background; $\gamma$-ray contribution in the detector yield that is not related to the target. This contribution is measured by removing the target out of beam.

The first type is hard to deal with due to the fact that any attempt to directly measure it results in an alteration of conditions and therefore the size of background itself. Therefore this background has to be studied by modeling processes with Monte Carlo. The second kind of background however is easy to measure, since one can simply remove the target material out of the beam, leaving everything else (including the target container and support structure) in place.

If all backgrounds contributions from (1) and the last one from (2) are successfully determined with enough accuracy, then the obtained background $\gamma$-ray yields can in principle be simply subtracted from the total detector yields before the asymmetry calculations. This subtraction is allowed if the asymmetries of the pedestal - electronic noise and $\gamma$-ray back-
ground are known to be close to zero, or at least a couple of orders of magnitude smaller than the sought $\gamma$-ray asymmetry of the target. The LED and pedestal asymmetries in this experiments were measured separately and are consistent with $10^{-9}$ level.

a) Electronic pedestals, are measured between production runs without beam by closing the shutter and usually do not fluctuate significantly over a few hours. The pedestal signals are subtracted, by taking into account the TOF structure of the signals. In this way most of the electronic AC-component of the signal is removed. (See chapter Experiment, Section RFSF). The pedestal signal of the nearest run of each detector is subtracted from the corresponding detector signal with neutrons before the signal is used in the $A_{\text{raw}}$ calculations.

b) The subtraction of the background without $\gamma$-ray asymmetry, on the other hand is slightly different, since only a few runs were performed with empty $Al$ vessel, making accuracy of the background measurement limited. However, in the case of the solid targets, although different materials were contained in the vessel, the data are data are reasonably equally applicable.

Other corrections have to be considered with the $\gamma$-ray backgrounds. The backgrounds may have been measured with beam current that was different from that delivered during the production runs. Furthermore, the $^3\text{He}$ polarization of the polarizer may have been different and thus the beam flux has been different during these the production runs and background measurements. This means that the subtracted background signals must be normalized to the beam, more specifically to the magnitude of the beam flux downstream of the polarizer, which value is measured by the second neutron beam monitor, M2. Since different $^3\text{He}$ polarization also varies the shape of the signal, the normalization is done
Figure 5.7: Detector # 47 signal when the target was In, empty Aluminum can with and without the gap, and pedestal as a function of time-of-flight.

The plot above shows a typical ratio of a background signal (target out) from the detectors to the signal with the LH2 target in the beam. Prior to taking the ratio the two sets of signals were normalized to levels of monitor 2. It can be seen that the different rings experience a different amount of background relative to the LH2 signal. This is taken into account while calculating the asymmetry for each ring, by including the error associated with the knowledge of the background asymmetry in the final result.

5.2.2 Cuts Applied To The Data

5.2.3 In general, cuts to the precision asymmetry data have to be considered very carefully so that the cuts do not bias the asymmetry value. While the cuts normally do have a large
Figure 5.8: Signal to Background ratio vs TOF and Detector Number. The four layers represent the four rings of the detector array. From the top - ring 3, 2, 1 and 0.
effect on the uncertainties of the asymmetry, the central value can be sensitive to the.

A number of cuts were introduced in the analysis process as the first-order check of the validity of the data. Based on certain criteria, discussed below, each of the nominal 1250 8-step spin-sequences comprising a typical run, are assessed and passed down to the asymmetry calculation if these criteria are met.

- Spin Sequence cuts - The raw asymmetry $A_{raw}$ is calculated according to Eqn. (5.3). The $\uparrow$- and $\downarrow$-spin states for each signals for each of the terms included in the calculation are composed using all 8 steps of the spin sequence. Therefore, the data analysis is performed on one such spin-sequence at a time. The neutron macro-pulses formed in the DAQ are encoded with a number corresponding to the position within the sequence. The spin state that determines whether the current is delivered to the Spin-Flipper or to the dummy load in fact forms a 16-step sequence. Thus the numbers assigned to the neutron pulses go from 0 to 15. This allows the analysis program to verify the validity of the spin sequence. One of the requirements is that the spin sequence follows the pattern $\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow$.

- Cuts on the neutron beam intensity. This cut discards a sequence in which one or more neutron pulses were taken at a proton current below a certain threshold value. This ensures that the experimental components (detectors in particular) stay in the linearity range.

- Neutron Beam Fluctuations As an additional precaution, a spin sequence was discarded if the neutron beam fluctuated, measured by $M1$, greater than a set value of 2%. First, the $M1$ signal was integrated between time bins 10 and 50, then the
numbers were used to compute the average \( M1 \) signal for the sequence:

\[
\langle M_1 \rangle = \frac{1}{8} \sum_{i=1}^{8} M_1^i.
\]  

If the ratio \( \frac{|M_1^i - \langle M_1 \rangle|}{M_1} \geq 0.02 \), then the spin sequence was skipped.

Frame Overlap Neutrons If the neutron energy is very low compared to the energy range of interest in our experiment, it may be slow enough that its velocity is phased to the copper opening so that the neutron can reach the apparatus two frames after the neutron pulse in which it was produced, as described in the section 3.2. Before these neutrons can reach the nuclear target they have to traverse three layers of \(^3\text{He}\). Since \(^3\text{He}\) possesses a huge absorption cross-section which is inversely proportional to the velocity of the neutron, the "frame-overlap" neutrons are absorbed and they do not dilute the knowledge of neutron time of flight.

\( \beta \)-delayed Neutrons In the spallation source the prompt neutrons are created by a few hundred nano-second wide proton pulse. In addition the prompt neutrons also neutrons
are produced by $\beta$-decays. These so called $\beta$-delayed neutrons can be produced seconds up to minutes after the prompt neutrons. They therefore show up in the monitors as having a different energy, since their time of flight has been compromised by their late emission. This time independent background is estimated to be less than $10^{-5}$ - too small to have any effect on the results of this experiment.

5.2.4 False Asymmetries Parity-conserving processes like Mott-Schwinger \cite{43} scattering can produce a left-right asymmetry. If a large L-R asymmetry is expected, there has to be a mechanism for disentangling Up-Down and Left-Right asymmetries and in the first order the mixing of these orthogonal asymmetries can be prevented by careful alignment of the detector with the static magnetic field; $1^\circ$ uncertainty in the detector angles contributes 1% to the asymmetry.

The instrumental false asymmetries can be introduced by additive and multiplicative noise. The former can occur for instance when the power of the RF field of the spin-flipper, well correlated with the spin, is coupled to the detector. As we showed in the previous sections, this correlation prevents the noise contribution from being automatically canceled out in the algorithm of the raw asymmetry. The latter, also caused by coupling to the RF field, is different, in that it can affect the gains of the individual detectors. Once again, due to its correlation to the spin state, this effect can produce a non-zero false asymmetry. In order to estimate these asymmetries the VPD’s have passed separate tests with constant and alternating magnetic fields, during which the changes in the gains were measured \cite{31}. The changes in the gain were recorded at $\sim 10^{-4}/G$ and $\sim 10^{-5}G$ respectively. The effects of the magnetic fields are mitigated by the aluminum housing around the VPD’s.
The additive false asymmetries were measured in a series of pedestal runs without the neutron beam. The γ-ray asymmetry came out to be on the order of \( \sim 10^{-9} \). To measure multiplicative asymmetries the detectors have to see a measurable flux of photons and produce a signal well above noise levels. This was accomplished by mounting two Light Emitting Diodes on each detector. During the measurements the beam was off but the RFSF was turned on. The asymmetry was measured to be \( A_{LED}^{raw} = (7 \pm 6) \times 10^{-8} \) during the 2005 run and \( (3 \pm 7) \times 10^{-9} \) in 2006.

![Total LED Asymmetry](image)

Figure 5.9: A histogram of multiplicative asymmetries measured with LED and Spin-Flipper on but beam off.

Besides the instrumental asymmetries, in principle, there exist processes involving the interaction neutron with nuclei, that may produce observable asymmetries. Among these are the Mott-Schwinger scattering, capable of producing a parity-conserving left-right asym-
metry, Stern-Gerlach up-down steering of the beam in the presence of field gradients, beta
decay of the neutrons and spin rotation. All these asymmetries however have been esti-
mated to be well below the limit of our current measurement $< 10^{-10}$ [7].

5.2.5 The mean asymmetry

The final asymmetry is a single number that is extracted by
histogramming $A_{\gamma}^{\text{tot}}$ and fitting it to a Gaussian (Fig.5.11).

Using the fact that the detectors operate at Poisson statistics, i.e. that the signal in the
output of the pre-amplifiers is linearly proportional to the number of $\gamma$-rays registered by
the VPD’s, we can conclude that the relative error on the raw asymmetry $\sigma_{\text{raw}}$ is inverse
proportional to the square root of the total number of the counts. The statistical error
is extracted as the RMS width of the histogram over all spin sequences in the data set
corresponding to the given data, that passed the cut criteria (discussed below) and were
used in the calculation.

The systematic errors, corresponding to the uncertainties on the polarization, depolar-
ization, geometry factors and the spin-flip efficiency are combined in quadrature with the
final statistical error.

$$\sigma_{\text{sys}} = A_{\gamma} \sqrt{\left( \frac{\sigma_P}{P} \right)^2 + \left( \frac{\sigma_T}{T} \right)^2 + \left( \frac{\sigma_G}{G} \right)^2 + \left( \frac{\sigma_S}{s} \right)^2}.$$  
(5.31)

The error on the geometry factors is estimated to be $< 1\%$, based on the deviations of
the Monte Carlo output, due to the variations of the step [[31]]. The spin-flip efficiency and
the beam polarization have been measured with $\sim 1 - 2\%$ uncertainty [see chapter about
the Experiment]. The beam depolarization uncertainty is on the order of a few percent,
and comes from the step variation in the simulation, as well as the uncertainties on the cross-sections used. For small asymmetries, the resulting systematic error is scaled by $A_\gamma$ 5.31, hence its contribution is small.

5.3 The Geometric Mean approach

There exists an alternative approach to the calculation of the asymmetry from signals of detectors in the geometrical configuration such as in our experiment. It involves the calculation of a geometric mean of the signals of the upper and lower detectors. Consider the geometric mean of the signals $U_{1\uparrow}$ and $D_{1\uparrow}$.

$$Y_{\text{geom}} = \sqrt{\frac{U_{1\uparrow}D_{1\uparrow}}{U_{1\downarrow}D_{1\downarrow}}} = \sqrt{\frac{[1 + A_{\gamma}^{UD} \cos \theta + A_{\gamma}^{LR} \sin \theta] \cdot [1 + A_{\gamma}^{UD} \cos \theta + A_{\gamma}^{LR} \sin \theta]}{[1 - A_{\gamma}^{UD} \cos \theta - A_{\gamma}^{LR} \sin \theta] \cdot [1 - A_{\gamma}^{UD} \cos \theta - A_{\gamma}^{LR} \sin \theta]}}. \quad (5.32)$$

Then,

$$Y_{\text{geom}} - 1 = \frac{2 \left( A_{\gamma}^{UD} \cos \theta + A_{\gamma}^{LR} \sin \theta \right)}{1 - A_{\gamma}^{UD} \cos \theta - A_{\gamma}^{LR} \sin \theta} \quad (5.33)$$

and

$$Y_{\text{geom}} + 1 = \frac{2}{1 - A_{\gamma}^{UD} \cos \theta - A_{\gamma}^{LR} \sin \theta} \quad (5.34)$$

Finally,

$$\frac{Y_{\text{geom}} - 1}{Y_{\text{geom}} + 1} = A_{\gamma}^{UD} \cos \theta + A_{\gamma}^{LR} \sin \theta, \quad (5.35)$$

which is the result we obtained previously 5.8. Under ideal conditions, where the neither the efficiencies, nor the backgrounds are of importance, the this method is hence equivalent to that described above in detail. In principle however one can notice immediately the obvious differences between the two approaches - due to the construction of the “sums and differences ” expression for the raw asymmetry in 5.3, the effect of the backgrounds on
the overall asymmetry is reduced since the slowly changing backgrounds cancel out in the numerator. This is not the case in the “geometric mean” approach

\[
A^{raw}_{\text{geom}} = \frac{\sqrt{\frac{U_{\uparrow} \downarrow_{\uparrow}}{U_{\downarrow} \uparrow_{\downarrow}}} - 1}{\sqrt{\frac{U_{\uparrow} \downarrow_{\uparrow}}{U_{\downarrow} \uparrow_{\downarrow}}} + 1},
\]

(5.36)

where the background and pedestal signals cannot be simply subtracted. On the other hand this method guarantees the cancellation of the efficiencies, that multiply the yields of each detector.

Thus, in principle analyzing the data using both methods allows to get a handle on the systematic effects arising from the gain and background drifts separately.

5.4 Results

5.4.1 Hydrogen

Fig. 5.10 shows a preliminary result of the asymmetry calculation, using the measurements of the 2006 run-cycle. As evident from the plot, the minimization algorithm struggles to optimize the parameters, namely \(A_{UD}\) and \(A_{LR}\) in order to fit the data to 5.8. However the constants thus extracted do not carry enough significance to draw conclusions about the size of the asymmetry (LR or UD), since given the scale of the uncertainties with current statistics and understanding of the background, the result presents a 0 -asymmetry measurement. Nevertheless, as such the numbers agree with the expected values, and approach the accuracy of measurements by Cavignac. With an improved model of the background the uncertainty can be significantly reduced.

Ignoring the left-right asymmetry, and using the UD geometry factor to extract the physics asymmetry, the sequence asymmetries for the LH2 target were histogrammed. The resulting numbers are consistent with zero at this level of accuracy (see. Fig. 5.4.1)
Figure 5.10: Raw hydrogen asymmetry measured versus effective detector pair angle.

Para Hydrogen, $A_\gamma$

Mean $(0.61 \pm 1.3) \times 10^{-7}$
5.4.2 Chlorine

An identical procedure was implemented for the rest of the targets. The CCl₄'s large asymmetry allows to calibrate the analysis procedure as well as the apparatus. Obtaining separate values for the asymmetries for each detector pair and plotting them vs the effective detector pair angle shows that the L-R asymmetry for this target is negligible with very good precision (Fig. 5.4.2). Therefore the physics asymmetry can be alternatively extracted by histogramming the sequence physics asymmetries, obtained by using the U-D geometry factor (Fig. 5.11). The result is in good agreement with measurements by other groups.

Figure 5.11: Histogrammed sequence asymmetries for the CCl₄ target. The observed value is in good agreement with the previously reported $A_\gamma$. 

\[ A_{\gamma, \text{CCl}_4 \, \text{Target}} \]
5.4.3 Medium A Targets

The results from the rest of the targets are summarized in the table below. The analysis is identical to the ones already discussed and its description can be omitted.
<table>
<thead>
<tr>
<th>Target</th>
<th>$A_\gamma , (\times 10^{-7})$</th>
<th>$\sigma_{A_\gamma,\text{stat}} , (\times 10^{-7})$</th>
<th>$\sigma_{A_\gamma,\text{syst}} , (\times 10^{-7})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Co$</td>
<td>7.7</td>
<td>3.5</td>
<td>3.6</td>
</tr>
<tr>
<td>$Cu$</td>
<td>$-11.9$</td>
<td>5.8</td>
<td>6.0</td>
</tr>
<tr>
<td>$In$</td>
<td>6.8</td>
<td>3.0</td>
<td>3.1</td>
</tr>
<tr>
<td>$Mn$</td>
<td>$-5.3$</td>
<td>7.8</td>
<td>7.8</td>
</tr>
<tr>
<td>$Sc$</td>
<td>7.0</td>
<td>2.8</td>
<td>2.9</td>
</tr>
<tr>
<td>$Ti$</td>
<td>$-6.5$</td>
<td>3.0</td>
<td>3.1</td>
</tr>
<tr>
<td>$V$</td>
<td>1.7</td>
<td>6.3</td>
<td>6.3</td>
</tr>
</tbody>
</table>

Table 5.1: Table Caption
An experiment was conceived and implemented in order to study an important aspect of the nucleon-nucleon interaction. As opposed to the case of the interaction between the point-like quarks, the weak interaction between nucleons is not very well understood. One of the models, constructed by Desplanques, Donoghue and Holstein ascribes the interaction to the pion, the lightest meson, as the main mediator of the weak forces. The quarks of the pion, originating at the strong vertex, decay into a heavy $Z$ boson and that couples to the nucleon at the opposite, weak vertex. The process is accompanied by a violation of parity, that is exhibited in the form of a directional asymmetry of the $\gamma$-rays formed in the process of the formation of the final product of the reaction. In the simplest experimental situation that will allow an analytical solution, the neutrons are incident on hydrogen nuclei, i.e. protons, that absorb the neutrons and subsequently form a deuteron, by emitting a $\gamma$-ray in order to transition to the ground state. The asymmetry is registered as the number of the $\gamma$-rays in one direction relative to that of the neutron spin, exceeds the number in the opposite direction. This asymmetry, $A_\gamma$, is shown to be directly proportional to the coupling constant that describes the term involving the $\pi$-meson, which is the dominant exchange channel. Therefore such a measurement provides a direct quantitative assessment of the fundamental pion-nucleon coupling constant $h_\pi^1$. Provided that the systematic errors are controlled, the precision of the measurement, and hence its outcome is largely dependent on the statistics, that the experiment is able to accumulate over the scope of its operation. The
NPDGamma experiment, which has undergone its first phase at the Los Alamos Neutron Science Center, was constructed for the purpose of this measurement. During the 2004 and 2006 run cycles, the entire apparatus was subjected to a thorough systematic test in order to ensure its capability of a precision measurement. The collaboration has proposed a measurement of a quantity on the order of $\sim 5 \times 10^{-8}$, within the 10% margin of error. During the commissioning runs prior to and during 2006 the possible systematic effects including instrumental errors and possible spurious asymmetries from medium $A$ targets were measured and shown to be well below the allowable limit. The high intensity of the neutron beam at LANCSE helped to achieve the statistical error of $10^{-7}$. The experiment has been transported and is currently being re-assembled at the Spallation Neutron Source, TN, where it will undergo its second phase and reach the proposed accuracy in measuring $A_\gamma$ and $h_\pi^1$. The measurements of asymmetries in the medium $A$ targets is interesting in its own right, since coupled with the knowledge about the spectroscopic structure of the individual isotopes, they allow improve the knowledge of the weak spreading width $\Gamma_W$. The current thesis describes provides a comprehensive introduction into the theory behind the experiment, overviews the technical aspects of the apparatus and details the measurement procedures. The analysis framework is also derived and the results of the analysis described. This analysis represents the first-order approach to the NPDGamma data analysis, as a more rigorous look at the various facets of the calculations is required. In particular a more thorough Monte Carlo simulation of the background will reduce the error of the final result.


[42] PHYWE, 37070 Göttingen, Germany. PHYSWE series of publications.


APPENDICES
A. Polarizer Oven Design

Figure 6.1: The side view of the analyzer oven.

B. Powder Target Container Design

C. The diagram of the DAQ logic.
Figure 6.2: The top view of the analyzer oven.
Figure 6.3: The side view of the drive coils, the optics stand and the oven containing the cell.

Figure 6.4: The Q measurements of the RF coil.
Figure 6.5: The front plates of the target vessel support.

Two 1/8" Thick Aluminum plates

Figure 6.6: The powder target vessel design - side and top views.
Figure 6.7: The Ti and Al targets in a powder and solid forms, respectively. The powder target material is loaded into an aluminum can and positioned in neutron beam inside the γ-ray detector. The solid targets were in a form of thin sheets loaded onto the holding rack.
<table>
<thead>
<tr>
<th>Targets</th>
<th>A, g/part</th>
<th>Weight [g]</th>
<th>Density [g/cc]</th>
<th>h_top [cm]</th>
<th>H-h_top [cm]</th>
<th>V [cm^3]</th>
<th>n [atoms/ce]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cl</td>
<td>35.45</td>
<td>0.00</td>
<td></td>
<td>12.50</td>
<td>3.06</td>
<td>320.74</td>
<td>S.44E+19</td>
</tr>
<tr>
<td>In</td>
<td>114.82</td>
<td>7.31</td>
<td></td>
<td>5.90</td>
<td>9.66</td>
<td>1013.08</td>
<td>6.53E+21</td>
</tr>
<tr>
<td>Cu</td>
<td>63.58</td>
<td>8.60</td>
<td></td>
<td>6.70</td>
<td>8.86</td>
<td>929.16</td>
<td>6.80E+21</td>
</tr>
<tr>
<td>Mn</td>
<td>54.94</td>
<td>762.00</td>
<td>2.38</td>
<td>3.06</td>
<td>1128.47</td>
<td>2.27E+22</td>
<td></td>
</tr>
<tr>
<td>Ti</td>
<td>47.87</td>
<td>2036.00</td>
<td>1.80</td>
<td>10.76</td>
<td>1128.47</td>
<td>2.27E+22</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>50.94</td>
<td>2304.20</td>
<td>2.27</td>
<td>9.66</td>
<td>1013.08</td>
<td>2.27E+22</td>
<td></td>
</tr>
<tr>
<td>Sc2O3</td>
<td>137.87</td>
<td>751.40</td>
<td>0.81</td>
<td>6.70</td>
<td>8.86</td>
<td>929.16</td>
<td>6.80E+21</td>
</tr>
<tr>
<td>Co3O4</td>
<td>240.75</td>
<td>787.50</td>
<td>2.72</td>
<td>12.90</td>
<td>2.66</td>
<td>278.77</td>
<td>6.80E+21</td>
</tr>
<tr>
<td>Sc</td>
<td>44.95</td>
<td>2.99</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.06E+21</td>
</tr>
<tr>
<td>Co</td>
<td>58.93</td>
<td>8.90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.12E+22</td>
</tr>
<tr>
<td>O</td>
<td>15.99</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Diagrams:**

**YME 1**

**YME 2**

**YME 3**