Lecture Notes #01Θ — *Thu 10 Jan 2002*

Electric charge, conservation of charge, Coulomb's Law

Causes & Effects

Electric fields E and ***Magnetic fields B*** provide a description or framework for mediating the effects—forces—caused by one bit of matter and "detected" by other matter.*

Such measurable effects between things are called ***Interactions***.

For ELECTROSTATICS, objects must possess an intrinsic property called *charge* that gauges how strongly they interact with other charged objects, i.e., how strong the forces are. The symbol for an amount of charge is usually denoted Q or q.

Ultimately, there is some kind of *electric force* between charged objects that is responsible for the effects we observe in the physical world. For example, why does stuff hang together at all? To what do we ascribe the colors, smells, textures, stiffness, hardness, density, gooiness, shininess, reactivity, crystalline forms, transmutability, &c., of common things?

* Note that *vectors* such as ***E*** and ***B*** appear in **boldface** type in print, but you should signify them by writing a little arrow above the letter as usual, as in '$\vec{E}$' and '$\vec{B}$'.

Electrostatic forces are observed to have the following properties, which form the basis of our *model* of ELECTROSTATICS [“ES” for short]:

- [This one is an assumption.] The matter the world is made of—electrons and nuclei, mostly—acts the way it does because these part[icle]s have just a few “fundamental” properties: *mass* and *charge* (and another called “*spin*”).
- Pointlike objects with ZERO charge ($Q = 0$) have no ES force on them—they “feel” or “experience” no ES force—nor can they “exert” ES forces on other objects.
- There seems to be only 2 types of nonzero charge, whose values are $+$ or $-$.
- You can add some charged bits to a big object and change its charge. Adding more $+$ (or $-$) charge to a $+$'ly (or $-$'ly) charged thing gives it an even greater net $+$ (or $-$) charge.
- Adding some $+$ (or $-$) charge to a $-$'ly (or $+$'ly) charged object cancels out some or all of its net charge. Something with an equal amount of $+$ and $-$ charge is electrically *neutral* (net charge ZERO). ... For most macroscopic objects (like you and me), this is the case to a high degree of accuracy. Nevertheless, if there is even a slight *separation* of equal amounts of some $+$ and $-$ charge within an object, there can be measurable consequences—i.e., observable forces. This segregation of $+$ and $-$ charges is called *polarization*. ... And when a neutral object becomes polarized by a nearby nonneutral one, the resulting *polarization force* between them is always attractive (see `HW #01W`).

The magnitude $|\boldsymbol{F}|$ of observed electrostatic forces adheres to this behavior:

- The closer/further two charged objects are separated from one another, roughly speaking, the stronger/weaker is the strength (i.e., magnitude) of the ES force between them. In fact, the force turns out to decrease with the ***square of the objects' separation*** r (just like the gravitational force!):

$$|\boldsymbol{F}| \propto \frac{1}{r^2} \, .$$

- The single property *charge* is responsible for both "producing" an ES-force–type effect AND "feeling" ES forces. That is, the force $\boldsymbol{F}$ is ***symmetric*** between the charge Q_1 causing the force and the charge Q_2 on which the force acts. The magnitude of the force is simply proportional to the magnitudes of both charges:

$$|\boldsymbol{F}| \propto |Q_1| \, |Q_2| \, .$$

The $\underrightarrow{\text{direction}}$ of the electrostatic force vector characterizes a *central force*—the 2 charges exert their ES forces on one another directly along their "line of sight":

- The direction of the force vector on charge #2 due to the presence of charge #1 lies along the line connecting the points where #1 and #2 are located. And what happens along this line goes by the famous rule,

LIKE CHARGES REPEL:	$\Leftarrow \boxed{+} \cdots \boxed{+} \Rightarrow$	or	$\Leftarrow \boxed{-} \cdots \boxed{-} \Rightarrow$
OPPOSITE CHARGES ATTRACT:	$\Rightarrow \boxed{+} \cdots \boxed{-} \Leftarrow$	or	$\Rightarrow \boxed{-} \cdots \boxed{+} \Leftarrow$

  Label the direction from #1 to #2 by the unit-length* vector $\hat{\boldsymbol{r}}$:

  $$\boxed{\text{charge \#1}} \quad \underset{\text{unit vector } \hat{\boldsymbol{r}}}{\longrightarrow} \quad \boxed{\boxed{\text{charge \#2}}}$$

  If the 2 charges repel, the force on #2 would be parallel to $\hat{\boldsymbol{r}}$ (away from #1) while the force on #1 is in the OPPOSITE direction, parallel to $-\hat{\boldsymbol{r}}$ (away from #2). If the 2 charges attract, the situation is reversed: the force on #2 would be parallel to $-\hat{\boldsymbol{r}}$ (towards #1) while the force on #1 would be parallel to $\hat{\boldsymbol{r}}$ (towards #2).

- In any case, the forces that the 2 charges feel are ***always equal in magnitude and opposite in direction***—even if the 2 charges are different magnitudes (e.g., $\Longrightarrow \boxed{+} \cdots \boxed{--} \Longleftarrow$)! This is the hallmark of an *action-reaction pair*. (Remember Newton's 3rd Law?) Another way to put this is:

  $$\boxed{\boldsymbol{F}_{\text{due to \#2 as felt by \#1}} = -\boldsymbol{F}_{\text{due to \#1 as felt by \#2}}}$$

---

* *N.B.* A "unit-length" vector $\hat{\boldsymbol{r}}$ has $|\hat{\boldsymbol{r}}| = 1$.

Coulomb's Law

All of the above features of the ES force can be compactly summarized in ...

$$\text{COULOMB'S LAW:} \qquad \boldsymbol{F} = k\,\frac{Q_1 Q_2}{r^2}\,\hat{\boldsymbol{r}}$$

N.B. **Vectors! Units!!**

What is the absolute strength of the force? This must come from EXPERIMENT! Its value is incorporated into that factor k (or, equivalently, into $\varepsilon_0 = 1/4\pi k$):

$$k \equiv \frac{1}{4\pi\varepsilon_0} \approx 9.0 \times 10^9 \text{ N·m}^2/\text{C}^2, \qquad \varepsilon_0 = 8.855 \times 10^{-12} \text{ C}^2/\text{N·m}^2$$

The charge of 1 electron is $-e$. The *fundamental constant* e is (the magnitude of) the very small *unit of charge* out of which ALL larger amounts of charge are compounded:

$$e = 1.60 \times 10^{-19} \text{ C}$$

One could equally well say that we define our unit of charge, 1 C[oulomb], to comprise a vast number of the basic building blocks of charge e, and that that number is $\approx 6.2 \times 10^{18}$ of them. [**Why?**]

Conservation of Charge

The net change in the total charge of the Universe is ALWAYS ZERO!!!

It is indeed possible to create and destroy particles ($m \to E = mc^2$). So it is possible to create and destroy charges $+Q$ and $-Q$. But Nature guarantees they will appear or disappear as $\pm Q$ *pairs*, which have net charge ZERO, SO THE TOTAL NEVER CHANGES.

Conductors & Insulators

The stuff we're made of and the world is made of contains charged particles that are always free to move around at least a little bit.

Solid *conductors* of charge are usually metals: copper, iron, aluminum, silver, lead, ...

Solid nonconductors, called *insulators*, are usually nonmetals: glass, rubber, wood, ...

Liquid conductors, such as salt water or battery acid, are called "electrolytes" when used in batteries.

Air is an insulator—but only up to a point. When it "breaks down" charge is suddenly able to flow through it and you get a spark. A lightning bolt is a very big spark.

There are also *semiconductors*, e.g., silicon (in computer chips). They are usually insulators but can become conductors if you know how to nudge them.

"Grounding" an object occurs when you enable charge to flow from the object to some place (or to the object from some place) with a large "capacity" to absorb or expel "fleeing" charge—a reservoir of charge, in a sense. Usually one electrically "grounds" something by attaching it through a conductor deep into the ground, literally. (Here is the symbol for a grounded wire.) The circuit in your abode is likely grounded to a pipe that goes into *the* ground.