**Lecture Notes #02T — *Tue 15 Jan 2002***

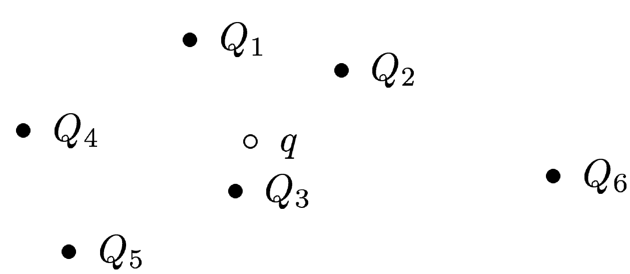
# $\vec{F}$orces, $\vec{E}$lectric Fields, & Field Lines

## Electric Fields

Let us use a little *positive charge q to test the electrostatic* ["ES"] *neighborhood* of an arbitrary arrangement of $N$ point charges which we list as $\{Q_1, Q_2, Q_3, \ldots, Q_N\} = \{Q_i,\ i = 1, 2, \ldots, N\}$. Here $N$ is just the number of charges in the neighborhood of $q$: $N = 1$ or 2 or 3 or whatever it is. Let us suppose that the positions of these charges are *fixed*—they are glued down, somehow.

We'll have to assume that $q$ is not placed right on top of any of these charges. [**Why?**]

Put $q$ somewhere, anywhere. For example, here's a charge labeled "$q$" sitting among a bunch of 6 other charges, labeled "$Q_1$" … "$Q_6$":




We can describe the location of $q$ with respect to each of the fixed charges $Q_i$. Suppose that $q$ is a known distance $r_i$ from $Q_i$ (charge #$i$). And suppose that the direction from charge #$i$ to the test charge $q$ is parallel to some unit vector $\hat{\boldsymbol{r}}_i$. Then $q$ feels a ***Coulombic force from each charge***: it feels a force $\boldsymbol{F}_1$ from $Q_1$, a force $\boldsymbol{F}_2$ from $Q_2$, &c.

The force that $q$ feels due to any one of the charges $Q_i$ equals the vector

$$\boldsymbol{F}_i = k\,\frac{Q_i q}{r_i^2}\,\hat{\boldsymbol{r}}_i\ .$$

This is COULOMB'S LAW. By the PRINCIPLE OF SUPERPOSITION, the ***total ES force*** on $q$—due to ALL the charges $\{Q_i, i = 1, \ldots, N\}$—is just the vector sum of all the forces acting on $q$:

$$\boldsymbol{F}_{\rm net} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 + \cdots + \boldsymbol{F}_N = k\,\frac{Q_1 q}{r_1^2}\,\hat{\boldsymbol{r}}_1 + \cdots + k\,\frac{Q_N q}{r_N^2}\,\hat{\boldsymbol{r}}_N \equiv \sum_{i=1}^{N} k\,\frac{Q_i q}{r_i^2}\,\hat{\boldsymbol{r}}_i .$$

Notice that this net-force idea is the same as for any other situation with forces—the total force on a given object is ... well, the sum of all the forces on it! Although 2 forces might happen to be equal and opposite and cancel, there is no way for one source to "prevent" another source's force from acting on the object. They're all there, for better or worse, and all we can do is note what the force vectors are and add them up to find the net force on our object. ***Fixed source charges don't "shield" one another in any way.*** They don't prevent one another from acting.

CAUTION: "Net force" always refers to the sum of all forces on *one object*. You must first identify an object of interest, then identify all the forces on it, then add up all those forces. We *never* add the forces on distinct objects to obtain a "net force on one of those objects."

*Reminder:*

$$\text{NET FORCE ON } q = \boldsymbol{F}_{\rm net} = \sum_{i=1}^{N} k\,\frac{Q_i q}{r_i^2}\,\hat{\boldsymbol{r}}_i$$

If we move $q$ around to some other position, the charges $\{Q_i\}$ and their positions don't change but the distance $r_i$ and direction $\hat{\boldsymbol{r}}_i$ from charge $\#i$ to charge $q$ will generally alter *for every* $i = 1, \ldots, N$. That means that all of the forces $\{\boldsymbol{F}_i\}$ will vary as we move $q$ around, on account of the forces' dependences on the distance $r_i$ and direction $\hat{\boldsymbol{r}}_i$ from each $Q_i$ to $q$. (That dependence is $\boldsymbol{F}_i \propto {r_i}^{-2}\hat{\boldsymbol{r}}_i$.) So $\boldsymbol{F}_{\rm net}$ ***depends on $q$'s position.***

EXAMPLE: Put $q$ *extremely close* to one of the charges, say #3, so that $r_3 \approx 0$ and $1/{r_3}^2$ is enormous. Then $r_3 \ll r_j$ compared to all the other charges ($j = 1, 2, 4, 5, 6, \ldots, N$) and $1/{r_3}^2 \gg 1/{r_j}^2$. In this case the force $\boldsymbol{F}_3$ will swamp all the other forces (i.e., $|\boldsymbol{F}_3| \gg |\boldsymbol{F}_j|$ for all $j \neq 3$). It will appear to $q$ that $Q_3$ is the only charge around, more or less, because $q$'s $\boldsymbol{F}_{\text{net}}$ is dominated by that one huge $|\boldsymbol{F}_3|$. Therefore, ***As you approach close enough to any single point charge, the total force on your test charge appears to be a Coulombic force from that one charge alone.***

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But something simple emerges no matter where $q$ is: $\boldsymbol{F}_{\text{net}}$ is ALWAYS proportional to the charge $q$—the one that "feels" the net force from all the other charges. In other words

$$\{ \text{ Net } \boldsymbol{F}\text{orce on } q \} = \{ \text{ "testing" charge value } q \} \times \{ \text{ something (a vector) that depends on } q\text{'s position and all the other charges } Q_i \text{ and their locations } \}$$

Since the { something ... } depends on the locations and values of all the *source charges*—the ones glued all over the neighborhood—but NOT on the value of the test charge $q$, we can interpret that "something" as a property of the collection of sources but not of any other charges. And "it" varies with the particular position we happen to be "testing"! It's called the *Electric Field (vector)* $\boldsymbol{E}$ at a particular position; i.e., by the definition of $\boldsymbol{E}$ we have $\boxed{\boldsymbol{F}_{\text{net}} = q\boldsymbol{E}}$. The strength and direction of $\boldsymbol{E}$ (and therefore of $\boldsymbol{F}_{\text{net}}$) depends on the location of our "test" point in space.

So the electric field $\boldsymbol{E}$ at a point in space simply equals the total force that would be felt at that position by ANY nonzero charge $q$, but divided by the value of $q$. The division removes any dependence of the force on the test charge's value, leaving only a dependence on a *position* in space. Using

Coulomb's Law, the collection of charges $\{Q_i\}$ produces an electric field at the test charge's location equal to

$$\boxed{\boldsymbol{E}\ (\text{at } q\text{'s position}) = \frac{\boldsymbol{F}_{\text{net}}\ (\text{at } q\text{'s position})}{q} = \sum_{i=1}^{N} k\,\frac{Q_i}{r_i^2}\,\hat{\boldsymbol{r}}_i}$$

(notice how the factor $q$ has been divided out of every $\boldsymbol{F}_i$).

If at any location you were to replace $q$ with any other charge $q''$, *the electric field there would have the same value* $\boldsymbol{E}$! The net force $\boldsymbol{F}''_{\text{net}}$ on that new charge $q''$ would change only because the value of the charge has changed. We would obtain the new force $\boldsymbol{F}''_{\text{net}}$ merely by multiplying the same $\boldsymbol{E}$ by the new charge $q''$ instead of the old value $q$: $\boxed{\boldsymbol{F}''_{\text{net}} = q''\boldsymbol{E}}$. For example, if the arrangement of source charges $\{Q_i\}$ is fixed, their effect on a test charge twice as strong as $q$ is a net force twice as great as $\boldsymbol{F}_{\text{net}}$—stronger but still in the same direction as before: $\boldsymbol{F}''_{\text{net}} \| \boldsymbol{E}$!

IMPORTANT BUT TRICKY: Recall that the $\boldsymbol{E}$-field at the location of a charge $q$ depends only on the fixed charges $\{Q_i\}$ influencing $q$ and not on the value of $q$ itself. If the sign of the test charge used were *negative* ($q'' = -q$, say), then the force $q''\boldsymbol{E} = -q\boldsymbol{E}$ would simply reverse ($\boldsymbol{F}''_{\text{net}} = -\boldsymbol{F}_{\text{net}}$) to the direction OPPOSITE the electric field: $\boldsymbol{F}''_{\text{net}} \| -\boldsymbol{E}$. That's what you'd get from Coulomb's Law for a collection of charges anyway if you were to switch the sign of just the *one* charge named $q$: you'd turn each repulsion $\boldsymbol{F}_i$ acting on $q$ into an attraction $-\boldsymbol{F}_i$ acting on $q$ (or *vice-versa*).

The $\boldsymbol{E}$-field therefore has the same sign as the force on a *positive* charge $q$, which turns out to be convenient for us. In mapping the $\boldsymbol{E}$-field in a region of space we can regard it as a representation of "the force on a positive test charge": patterns of the electric field look like patterns of the electric force.

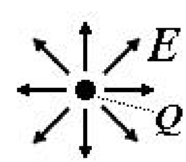
## Electric-Field (or ES-Force) Patterns

The electric field $\boldsymbol{E} = \hat{\boldsymbol{r}}kQ/r^2$ around a single positive point charge $Q$ is directed radially outwards from the charge. At small distances $r$ from $Q$ the field $|\boldsymbol{E}|$ is large and at large distances $|\boldsymbol{E}|$ is small (recall $|\boldsymbol{E}| \propto r^{-2}$)—but the *direction always points away from the center when the source charge is positive. ...* Because when a *positive test charge* $q$ is placed at any point, the force on it there equals $\boldsymbol{E}$ multiplied by $q > 0$, which correctly gives an *outward* repulsive force $\boldsymbol{F} = q\boldsymbol{E}$ parallel to $\boldsymbol{E}$.

If the source charge were negative ($Q < 0$), on the other hand, then the $\boldsymbol{E}$-field would point radially inwards everywhere. A *positive test charge* $q$, in this $\boldsymbol{E}$-field would feel an attractive radial force inwards—the force $\boldsymbol{F} = q\boldsymbol{E}$ on $q < 0$ would be *anti*parallel to $\boldsymbol{E}$ in this case. The *reversal of the force* when the test charge changes sign has *nothing to do with the electric field*—it comes about simply because the "$q$" in $\boldsymbol{F} = q\boldsymbol{E}$ changes sign.

***The electric field is a property of the source charge(s),* NOT *of any test charges!*** *

The whole pattern of vectors $\boldsymbol{E} = \boldsymbol{F}/q$ around one point source charge $Q > 0$ has an outwards, **explosive** pattern as shown here. The field around an equal but opposite source charge $Q < 0$ would produce the same situation but with all vectors reversed, an **implosive** pattern.




***One could say that all charges, in their role as sources of electric effects, broadcast* messages*—Electric Fields E—which, when* detected *by any other charge* $q$*, produce a force* $\boldsymbol{F} = q\boldsymbol{E}$ *on* $q$.**

* Bear in mind that the designations "source" charge and "test" charge depend on context and are really arbitrary. Coulomb's Law is fundamentally SYMMETRIC between the two interacting charges. All charges function as sources of electric field and any charge could in principle be used to test the presence of others.

## Electric Field Lines

*Field lines* are constructs drawn over or in place of electric-field patterns to help illustrate the "flow" of field (or force) vectors produced by one or more source charges. At each point on a field line the electric field is *tangent* to the field line along the direction of the field line's arrow. Since all such "flows" of electric fields ultimately begin as repulsion (of positive test charges) from positive point charges and ultimately end as attraction (of positive test charges) towards negative point charges, we can formulate this "rule" for ***E***-field lines:

***Electric field lines always begin on positive charges and end on negative charges!***

In particular,

***Electric field lines never form closed loops (in electrostatics).***

[Can you see **Why?**] See, e.g., Fig. 16–29 on p. 491 in GIANCOLI/5.

(*N.B.* We'll see later that *non*static, time-dependent magnetic fields *can* indeed produce loopy electric fields. But that's not from electric charges and that ain't Electro*statics* so don't worry about it for now. ***E***-fields from single point charges are of the ex-/im-plosive variety—PERIOD.)

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YOU SHOULD PLAY WITH THE COMPUTER PROGRAM `EMField6` TO GET A VISUAL FEEL FOR FORCES, ELECTRIC-FIELD PATTERNS, AND FIELD-LINE PATTERNS PRODUCED BY ONE OR MORE CHARGES. It is available on all the PCs in the Chem.-Phys. MicroLab (Rm. CP 148; phone 257–4325).