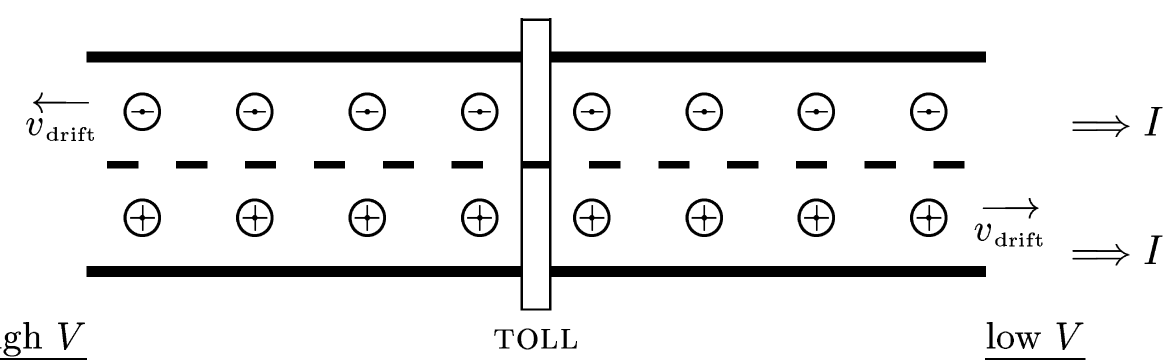
Lecture Notes #05Θ — *Thu 7 Feb 2002*

Ohm's Law, *R*esistance, & *ρ*esistivity.
Electrical Power.

Current flow

Current is like *traffic flow*—it has a direction—except that the vehicles in "charge-traffic" also have a sign. "Conventional" current $I = \Delta Q/\Delta t$ is *defined* to flow in the direction of positive charge. Negative charge flowing in the *opposite* direction at the same rate is assigned a positive current $+I$ (not $-I$!), equivalent to positive charge flowing in the original direction.

This convention springs from CONSERVATION OF CHARGE: If there were *equal* densities of positive and negative charge flowing together to the right, say, their combination would constitute a flow of ZERO net charge—no current at all, $I = 0$. That is, the sum of the positive charges' current $+I$ plus the negative charges' current must vanish, implying that negative charges flowing rightwards must be assigned a negative current, $-I < 0$, as must positive charges flowing leftwards. **Negative charges moving in one direction can therefore be regarded as a *positive current flowing in the opposite direction*.** In particular, a "current" of electrons flows, by convention, in the direction *opposite* to their velocity.



This is illustrated above, where positive charges pass the TOLL booth driving towards the right at some steady *drift speed* $v_{d[\text{rift}]}$, which constitutes a current I flowing to the right. Negative charges driving past the TOLL booth towards the left at the same speed v_d and with the same density (same spacing) *also* constitute a positive current I (not $-I$), i.e., flowing towards the *right* here. You *could* equally well say that the negative charges form a current moving towards the left, but it would have a *negative* value, $-I < 0$, because they are, after all, *negative* charges. For most purposes, you couldn't tell whether the charged traffic involves + charges going one way or − charges going the other way or even some mixture of these (unless you exploit the *Hall effect*); their physical effects would be identical.

Consider that both cases entail the same behavior of the system with respect to ENERGY: you've got either positive charges moving through a decrease in electric potential, from + to −, with $\boxed{(+Q)(-\Delta V) < 0}$, or negative charges moving through an increase in potential, from − to +, with $\boxed{(-Q)(+\Delta V) < 0}$. In both cases the charges' PE *de*creases, and this lost energy is passed on to the "outside world" via the charges' interactions with the "highway" (wires, &c.). Note that if we were to "retrace our steps" by going "upstream" against an electric current the potential energy would be increasing and we'd feel like we're climbing back up rather than falling.

Microscopic Model of Conduction

Charges drift along at an AVERAGE *drift speed* v_d because they are exposed to a *non*zero electric field and a *non*constant potential in the conducting "pipe" through which they flow. It is a ***non*equilibrium** situation. That is, there is an electric field E inside the conductor pushing them along. (We'll assume $E \approx$ constant.) Thus, there must exist a potential difference V between the ends of a piece of conductor—i.e., between the ends of the pipe. *The charges flow in the direction that decreases their electric potential energy* (ΔPE < 0)—towards a lower potential for positive charges and towards a higher potential for negative charges.

If the charges flow a distance L parallel to the electric field E, the magnitude of their change in potential over this distance adds up to $\boxed{V = EL}$. Recall that this is just the definition of WORK per acted-upon charge:

$$\{\texttt{Work/charge}\} = \{\texttt{Force/charge}\} \times \{\texttt{distance}\}\,.$$

Now believe it or not, we can derive Ohm's Law $\boxed{I = V/R = EL/R}$ and an expression for the *resistance* $R = \rho L/A$ AND the *resistivity* ρ from a simple 1-dimensional MODEL of how drifting charges lose energy in the conductor.

Let's suppose that positive carriers of charge e and mass m are accelerated in the field E by the force $F = eE$. They acquire an acceleration a:

$$F = eE = ma \quad\Longrightarrow\quad a = \frac{eE}{m}\,, \qquad \underset{(\text{charge } +e)}{\oplus} \quad \underset{[\text{vectors}\ \ \ldots\ \text{effects}\ \ldots]}{\Longrightarrow\quad \boldsymbol{E}, \boldsymbol{F}, \boldsymbol{a}}$$

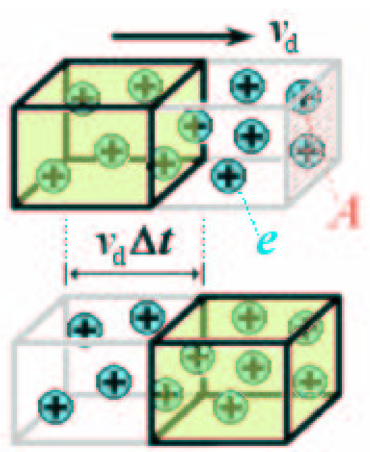
and keep speeding up, converting their loss of PE into a gain of KE. Here is our MAIN ASSUMPTION: Each charge *collides* with something relatively massive in the conducting material—an ion, say—after accelerating for some time τ, and in the collision suddenly LOSES ALL ITS KINETIC ENERGY. When a charge collides and comes to rest, the conductor gains that KE, jiggles, heats up a bit. This is our mechanism for tranferring the charges' PE—and ultimately the energy originally supplied by some kind of "battery"—to the conductor and, by extension, to the rest of the world.

.... *Here's the plan.* .... *We first figure out how much the charges are sped up by* $E = V/L$, *on the average, between collisions. We also note how densely the drifting charges are packed in the conductor. Then we can say how much charge passes by in a given time interval. This* *is just the current I, and we'll see how it depends on the applied V and known characteristics of the conductor.* ....

ON THE AVERAGE this collision time has some value τ ("tau") that depends on the particular material, which determines how easy (long τ) or difficult (short τ) it is for charges to flow in it. Then the average maximum speed a charge attains just prior to a collision is $v_{\max} = 0 + a\tau$, since it came to rest after the previous collision and then accelerates at rate a for time τ. The *average speed* of all charges is called the *drift speed*:

$$v_d = \frac{1}{2}(0 + v_{\max}) = \frac{1}{2}a\tau = \frac{1}{2}\frac{F}{m}\tau = \frac{e\tau}{2m}E = \boxed{\frac{e\tau}{2mL}V = v_d}.$$

Not surprisingly, the bigger the $\boldsymbol{E}$-field in the conducting pipe—or the potential difference V between the ends of the pipe—the greater is the drift (average) speed v_d.



The rate at which the charges drift along clearly has something to do with the *current* I, which is, after all, just the rate of charge flow. We want to know how much charge ΔQ flows past a point in the conductor ("TOLL booth") in a time interval Δt when the drift speed is v_d. Picture all that flowing charge as a volume of water moving down a pipe of cross-sectional area A. In a time Δt the length of water (i.e., charge) that flows at speed v_d past any point equals $v_d\Delta t$ (see Fig. 18–23 in GIANCOLI/5). How many charge carriers are there in a section of water this long with cross-sectional area A? That is, how many are to be found in the *volume* $(v_d\Delta t)A$? That depends on the *density* "n" of charge carriers—the number per volume—in the water, or in a particular conducting material. The total number N of charge carriers in the volume would be the density (number per volume) times the volume: $N = n \times (v_d\Delta tA)$.

Each carrier has a charge e ... so the total amount of charge ΔQ that passes by any point in time Δt is $\Delta Q = Ne = nev_d\,\Delta t\,A$.

Finally, the amount of charge drifting by *per time* IS the current:

$$I = \frac{\Delta Q}{\Delta t} = \frac{Ne}{\Delta t} = \frac{nev_d\,\Delta t\,A}{\Delta t} = \boxed{neAv_d = I}\,.$$

Using our 1st expression for v_d in terms of the potential difference V yields

$$\boxed{I = neA\left(\frac{e\tau}{2mL}\,V\right) = \underbrace{\underbrace{\frac{ne^2\tau}{2m}}_{\equiv 1/\rho}\,\frac{A}{L}}_{\equiv 1/R}\,V} \quad \ldots$$

Not only have we derived ***Ohm's Law*** (the proportionality $I \propto V$) from this simple model—we have also obtained an expression for the *resistivity* ρ in terms of properties of the conductor (n and τ) and its charge carriers (e and m), as well as the dependence of the *resistance* on the material's dimensions and resistivity:

$$\Longrightarrow \quad \boxed{R \equiv \frac{V}{I} = \rho\,\frac{L}{A} \quad \text{where} \quad \rho \equiv \frac{2m}{ne^2\tau}}\,.$$

Notice how a smaller density n of charge carriers or smaller charge e corresponds to a bigger resistivity ρ—i.e., less current in response to a given V. Also larger m (more sluggish masses) implies a greater ρ. That a small τ implies greater resistivity follows because a small collision time means that the charges never get a chance to speed up very much; they move more slowly on the average, and the current is smaller (for given V or E).

Beware that this is only a rough model, proposed by P. Drude in 1900, when the then-recently discovered electron was all the rage. Much more sophisticated theories now exist to predict both the density n and the collision time τ (typically, $n \sim 10^{23}$ cm^{-3} and $\tau \sim 10^{-14}$ s). One might say that all the "physics" of conduction in the Ohm's-Law regime is hidden in the single dynamical parameter τ.

N.B. The "drift speed" v_d is NOT the "speed of electricity." If you calculate v_d from $I = neAv_d$ for typical values of I, n, and A, you'll discover that the charges crawl along *very slowly*. [Go on, calculate it!]

Yet ... your lights or any other electronic devices still seem to come on essentially instananeously when you flick on a light switch. *That* lightning speed depends not on how quickly the charges move but rather on how rapidly the imposed electric field $\boldsymbol{E}$ (or potential difference V) gets established in the circuit's conductors. And *that* effect propagates at the *speed of light*. We'll learn about LIGHT, light's speed, what it does when it travels that fast, how it relates to electric (and magnetic) fields, what "it" is, &c.
.... later ....