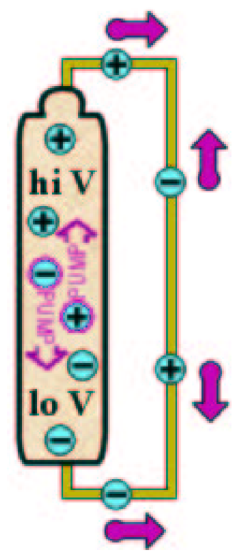
Lecture Notes #05T — *Tue 5 Feb 2002*

Currents. Batteries & Energy. $V \propto I$ & Resistance.

Batteries & Energy

Batteries provide "PUMPS" to *increase the electric potential energy of charges*. Somehow batteries manage to move positive* charge $+Q$ "uphill" through an increase in electric potential $+V$, or to move negative charges $-Q$ "uphill" through a decrease in electric potential $-V$. In either case, the PE of the charges would *increase*: by $(+Q)(+V) = QV > 0$ or $(-Q)(-V) = QV > 0$.

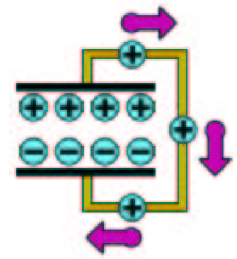


The energy to do the work to increase the PE of each charge Q is (usually) provided by exothermic chemical reactions in the battery. But in this discussion we intend "battery" to mean any device or material that raises the potential energy of electric charges. If this energy were used simply to separate + and − charges, then we'd effectively create a charged capacitor in which we've stored some energy; it would not be any more interesting than that.

To be USEFUL this energy must somehow be EXTRACTED and CONTINUOUSLY RESUPPLIED. . . .

Let us allow positive charges to *circulate* from the high-V to the low-V end of the battery through an *im*perfect conductor—a *wire*. We could alternatively use negative charges, which would circulate in the opposite direction, from low-V to high-V. In either case, the charges in question are the conducting ("free") charges of this system—some from the battery and most from the conductor. We could also imagine that the wire connects the

* Here 'V' and 'Q' denote positive quantities.



plates of an already-charged capacitor rather than the terminals of a conventional battery. The wire would still provide a path for, e.g., positive charge to flow from high-V to low-V, but this system would lack the battery's "pump" needed to continually recharge the capacitor. A charged capacitor is a "one-shot" battery.

The wire functions as a conduit. Sometimes we would also like it to function as an *absorber*, something that can remove all the electric potential energy from the charges passing through it. What we call the "wire" should be some kind of material in which charges may flow but in which the charges will deposit all the +PE given them by the battery. If the wire were an empty pipe, a noninterfering *guide* through empty space—i.e., a *perfect conductor*—then the charges' loss of PE would just turn into an increase of their kinetic energy (ΔKE $= -\Delta$PE). But a wire is a solid thing, a conducting material, comprising many small crystals of metal. Charges flowing through it COLLIDE with metallic ions in the conductor. It is through these collisions that the charges give up their KE to the wire. This is what makes normal wires *good but imperfect conductors*. If the charges suffer many collisions with the wire, when they finally leave it they re-enter the low-V terminal of the battery with no excess energy. That is, they return to their starting point with no change in their total energy: whatever PE they gained from the battery/pump they subsequently gave up to ions in the conducting wire upon colliding with them.

What happens to the wire, the metallic connector? Consult your own experiences. ... *[thinking]* ... The wire HEATS UP (emits infrared light)! If it heats up a lot it also GLOWS (also emits visible light).

This is a ***NON*****EQUILIBRIUM** situation!! There are indeed nonzero forces pushing charges around. There are nonzero electric fields in the wire pushing the charges around. Things must not have "settled down to equilibrium" yet—and *won't* as long as the battery keeps pumping on the charges, providing a potential difference and supplying them with $\Delta\text{PE} > 0$.

In summary, the battery donates some +PE to free positive charges in forcing them from low-V to high-V (high-to-low-V if they're negative).* If the + charges were allowed simply to "fall" back from high- to low-V, they would lose that PE and convert it to KE ... But any KE the charges do gain from speeding up is, we assume, lost to microscopic collisions with the wire-stuff. Battery-Energy becomes converted to Jostled-Wire-Energy, thence (usually) to ELECTROMAGNETIC RADIATION. *[What's that??]*

Electric Currents I

The charges must circulate—around a CIRCUIT—for this to work. The same charges might be pushed around again and again by the battery's potential difference V, or every once in a while replacements might be conscripted from the conductor. Wherever the charges comes from, the point is that each circulating bit of charge ΔQ gains some $\text{PE} = \Delta Q\, V$ (in the battery) and loses all of that PE (in the wire) with every trip it takes around the circuit. If it takes some time Δt, on the average, for each charge to complete each circuit, energy is transferred to the wire at the rate

$$P = \frac{\text{PE}}{\Delta t} = \frac{\Delta Q\, V}{\Delta t}\,.$$

This energy-consumption rate is the ***Power*** dissipated by this single wire. The unit of power, energy-per-time, is called the "Watt": `[W] = [J]/[s]`.

The charge serves as a medium, a kind of "working fluid," for transferring energy from the battery to the body of the wire; thereafter, the

* The flow of charge through conductors—what we normally call "electricity"—is actually performed by *electrons*, elementary particles with *negative* charge $-e$. We will see, however, that negative charges flowing in one direction are for most purposes entirely equivalent to equal positive charges flowing at the same rate in the opposite direction.

energy migrates to the outside world. The charge is typically RE-USED by the battery for this purpose. The energy is definitely *not* re-used—it is EXTRACTED and then used elsewhere—it lights up your eyes, for example. It is thrown away. The battery eventually USES UP its available, extractable store of energy.*

The rate at which the charge moves around the circuit is called the

$$\boxed{\textbf{\textit{Electric current}} \quad I \equiv \frac{\Delta Q}{\Delta t}}.$$

The unit of electric current is the "Amp" (after Ampère), charge-per-second, where $[\mathtt{A}] = [\mathtt{C}]/[\mathtt{s}]$.

If we assume that charges just keep flowing around the circuit and that *charges don't pile up anywhere*, then CONSERVATION OF CHARGE demands that $\boxed{I = \text{constant}}$. That is, the same amount of charge must keep flowing *past every point* of the circuit during any interval of time. Think of cars flowing along a highway with no traffic jams. Mark how many cars flow past an arbitrary point each minute, on a straightaway or through a toll booth, say. The number per second is the rate of car-flow, their current.

Note that simply on account of (1) *the definition of power* as energy-per-time and (2) *the definition of current* as charge-per-time, we also have

$$\boxed{P = \frac{\Delta Q}{\Delta t} V = IV}$$

for the rate at which power is dissipated by a current I that drops through a potential difference V. This statement is NOT "Ohm's Law"! It is merely

* The tendency of any source of energy to get "used up," for any concentrated supply to dissipate and become "less useful," is related to an increase of ENTROPY, the natural tendency of all isolated systems towards *dis*order.

what we mean by *Power* for a *steadily flowing current*. Also note that the units in $P = IV$ are related according to $[\mathtt{W}] = [\mathtt{A}] \times [\mathtt{V}]$.

Ω's [Ohm's] Law

An amount of charge Q that drops through a potential difference $-V$ gives up potential energy $\text{PE} = QV$. If this potential difference were altered, how would we expect the current I to change?

Well, a bigger potential drop over the same distance Δs in space must arise from stronger electric fields, on the average (recall that for a *constant* field E in 1D, $\Delta V = -E\,\Delta s$). That means a stronger force $F = QE$ is felt by each charge Q, hence a greater acceleration and more quickly moving charges. So, seeing charges fly faster past our traffic-observation point, we observe a greater current. The simplest relation between changing-V and changing-I is a *linear proportionality*, which in fact usually holds for metallic conductors under normal conditions. *This* condition is known as

$$\boxed{\textbf{\textit{Ohm's “Law”}}: \quad I \propto V}\,.$$

(Be aware, however, that there are important devices for which the dependence of current on potential difference becomes rather more complicated—such as *transistors*!) The linear proportionality between V and I means that there must also be some proportionality constant, which we call R, the ***resistance*** of the piece of material over which the potential drops:

$$\boxed{\textbf{\textit{“Ohm's Law”}}: \quad V = IR}\,.$$

This statement of Ω's "Law" is equivalent to the one above. Resistance has units $[\mathtt{V}]/[\mathtt{A}] = [\Omega]$, called "ohms." Also recall that power P is expressed in Watts: $[\mathtt{W}] = [\mathtt{J}]/[\mathtt{s}] = [\mathtt{A}] \times [\mathtt{V}]$.

The value of R for a particular piece of something depends on its length L (in the direction of current flow), its cross-sectional area A (size of the "pipe" though which the current flows), and what the something is made of as measured by the material's "resistivity" ρ: $\boxed{R = \rho L/A}$.