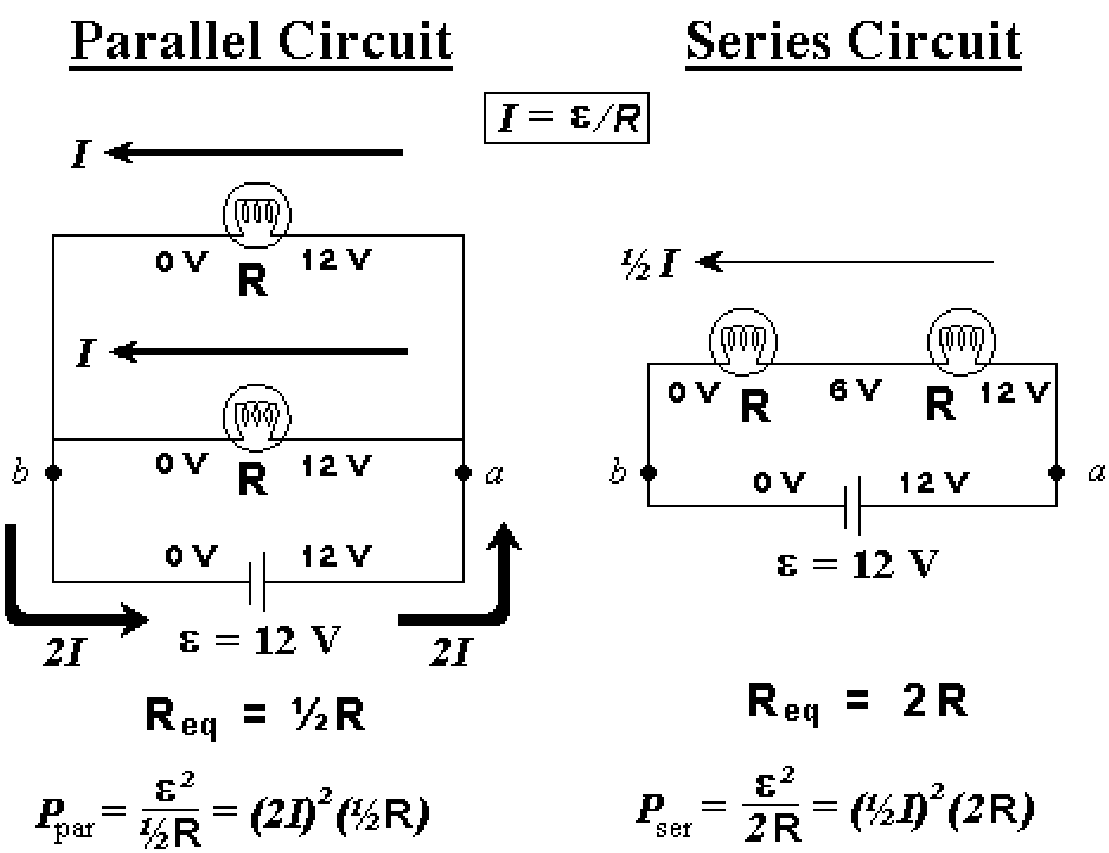
Lecture Notes #06Θ — *Thu 14 Feb 2002*

Kirchhoff's "Rules" —
Conservation of Energy & Charge in Circuits

The Fate of Electrical Energy & Charge in Parallel & Series

We need to understand the PHYSICAL PRINCIPLES behind the circuits depicted schematically here:



The symbol "$\boldsymbol{\varepsilon}$" stands for *ElectroMotive Force* — εMF for short. Although it was historically designated a "*force*," it's actually not a force at all! Like a good tune, the melody of its name has stuck even while its original mechanism has dropped out of fashion. εMF represents whatever magic causes a ***potential difference*** V to become established and maintained between two points in a circuit. The "seat of εMF" might lie within the chemical reactions of a battery, or be tied to the electromagnetic hocus-pocus of giant generators of electricity, or stand for the already-separated charges on a capacitor. As far as we're concerned, εMF's just AN ANONYMOUS SOURCE OF POTENTIAL DIFFERENCE. The work is done by somebody else.

The arrows and "I" stand for the direction and flow of electrical ***Currents***, respectively.

Resistors R, depicted as light bulbs to help us think in terms of a familiar type of power output (LIGHT!), might be light bulb filaments or imperfectly conducting wires or lousy wires or ... just any old stuff, really.

The flow of charge in a circuit implies a manifestly *NONEQUILIBRIUM* situation. TIME is involved because charge configurations are changing *as Time itself flows*. Nevertheless, if everything keeps flowing *at the same rate* there does appear to be something that doesn't change. The state of the whole circuit/system seems to be "steady": all motion of CHARGES and exchanges of ENERGY keep happening but *in the same way*. This changing-but-not-changing situation, the essence of the behavior of DC [D*irect*-C*urrent*] circuits full of just resistors and εMFs, is called a ***Steady State***.

Kirchhoff's Attitude Concerning the Conservation Laws

What happens in DC circuits? "Circuits" usually contain, among other things, some mobile electrons and fixed positive ions and other neutral junk all confined to variously connected pieces of metal. DC circuits generally have "sources of εMF," nuggets of electrical nutrition that magically give electrons the get-up-and-go they need to make their rounds. Since electrons are given potential energy, ENERGY is involved.

Recall that "potential difference" (ΔV) is synonymous with "nonzero electric field" ($|\boldsymbol{E}| \propto \Delta V \neq 0$). Charges respond to $\boldsymbol{E}$-fields by moving in such a way as to "use" or "give away" or "exchange" their potential energy—i.e., their PE turns into some other form of energy, belonging to something else. Perhaps the energy acquires a useful form (LIGHT). Since the motion of charges naturally arises, CURRENT is involved, too.

What can we say, in general, about ENERGY and CHARGE that might pertain to the steady-state situation in a DC circuit? Two things:

ENERGY IS CONSERVED!

CHARGE IS CONSERVED!

—as always.

If you follow any charge q *around one circuit*, i.e., around any closed path so that the charge comes back to where it started ... then its potential energy must not have changed: $\boxed{\Delta\text{PE}\ (\texttt{around any closed path}) = 0}$.*

* Forces that act this way are called *Conservative Forces.* Gravity and Coulombic (electrostatic) forces, the two most familiar fundamental forces of Nature (out of four), are conservative: You can always recover whatever PE you've lost, and *vice-versa*, at least in principle. *Friction* is not conservative—but neither is it "fundamental."

This is CONSERVATION OF ENERGY. Now, potential difference ΔV is *defined* to equal the change in the charge q's potential energy per amount of charge q. So for journeys around any "loop" (closed path) in a circuit, an equivalent statement of energy conservation is

$$\boxed{\Delta V\ (\texttt{around any loop}) = \frac{\Delta\text{PE}\ (\texttt{any loop})}{q} = 0}\,.$$

CONSERVATION OF ENERGY
"Kirchhoff's Loop 'Rule'"

If you follow any streaming current $I = \Delta Q/\Delta t$ through any part of a circuit, you should be able to confirm that net charge is neither created nor destroyed *at any point at any time*. CONSERVATION OF CHARGE applies everywhere everywhen. If you mark all the charge ΔQ that arrives at any place in the circuit during some brief time interval Δt, that same amount of charge must leave that place if it's a steady-state situation. The total charge entering must equal the total charge leaving. In particular, like traffic that continues to flow through a chaotic intersection of roads, as much charge must enter a junction as leaves it in any time interval Δt: $\boxed{\Delta Q_{\text{in}} = \Delta Q_{\text{out}}\ \texttt{at any junction}}$. Current I is *defined* to equal the charge ΔQ moving by during a time Δt. So at any junction (of paths/wires) in a circuit, an equivalent statement of charge conservation is

$$\boxed{I_{\text{in}}\ (\texttt{at any point}) = \frac{\Delta Q_{\text{in}}}{\Delta t} = \frac{\Delta Q_{\text{out}}}{\Delta t} = I_{\text{out}}\ (\texttt{at same point})}\,.$$

CONSERVATION OF CHARGE
"Kirchhoff's Junction 'Rule'"

The Conservation Laws in Series or Parallel

Consider any one resistor (resistance R') in a DC circuit maintaining a Steady State. Whatever conventional current I' winds up flowing through that resistor, for whatever reason, if we follow that current we fall with it through some potential difference $-\Delta V' < 0$ determined by *Ohm's Law*: $\Delta V' = -I'R'$. Follow the flow and you go *down*stream, towards *lower* potential, hence the minus sign. (The charges *lose* some of their PE, so $\Delta\text{PE} = q\,\Delta V < 0$ for every moving q.)

To say that "two or more resistors $\{R_1, R_2, \ldots\}$ are connected IN SERIES" means that they are essentially lined up end-to-end, like elephants in a circus. Any current I_1 that flows through the first must be exactly the same current I_2 that flows through the next, and the next ... and the next, and we call this common current I: $I_1 = I_2 = \cdots \equiv I$. This is just CONSERVATION OF CHARGE. The total potential drop $\Delta V < 0$ across all these resistors——i.e., the total potential energy $q\,\Delta V$ lost by a charge q passing through all these resistors—is just the sum of the potential drops through each: $\Delta V = \Delta V_1 + \Delta V_2 + \cdots$. This is just CONSERVATION OF ENERGY. Applying Ohm's Law to each resistor, $\Delta V_1 = -I_1R_1 = -IR_1$, etc., we see that

$$\Delta V = -IR_1 - IR_2 - \cdots = -I\,(R_1 + R_2 + \cdots) \equiv -IR_{\text{eq, ser.}}$$

All the resistors taken together in series, from the first to the last, are acting like *one effective* or *"equivalent" resistance*,

$$\boxed{R_{\text{eq, ser.}} = R_1 + R_2 + \cdots},$$

which is just a consequence of the two conservation laws. You could picture this as simply stringing several resistors together to make one long one. Since $R = \rho\ell/A$, it's similar to a total resistance that is proportional to *one* resistor's total length $\ell = \ell_1 + \ell_2 + \cdots$.

To say that "two or more resistors $\{R_1, R_2, \ldots\}$ are connected IN PARALLEL" means that the two ends of all the resistors are connected directly to the same pair of points **A** and **B**. "A point" here signifies one or more junctions of the circuit that lie at the same potential. The junctions are likely connected by a blob of metal of negligible resistance, the whole of which lies at some *equipotential*. *Different* currents $I_1, I_2, \ldots$ might flow through the resistors. But if you follow any one current from **A** to **B** and then any other current back from **B** to **A**, you lose some potential $\Delta V < 0$ going downhill but must regain exactly the same amount $-\Delta V > 0$ returning uphill—because that's a closed loop (!) and $(+\Delta V) + (-\Delta V) = 0$ *around any closed loop*. So all resistors in parallel share the same potential difference, which we call ΔV: $\Delta V_1 = \Delta V_2 = \cdots \equiv \Delta V$. This is just CONSERVATION OF ENERGY. The total current I entering **A** must equal the total current leaving point **B**. In between **A** and **B**, I must also equal the total current branching out to all the resistors at junction **A** and gathered together again at junction **B**: $I = I_1 + I_2 + \cdots$. This is just CONSERVATION OF CHARGE. Applying Ohm's Law to each resistor, $\Delta V = \Delta V_1 = -I_1 R_1$, etc., we see that

$$I = -\frac{\Delta V}{R_1} - \frac{\Delta V}{R_2} - \cdots = -\Delta V\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots\right) \equiv -\frac{\Delta V}{R_{\text{eq, par.}}}\,.$$

All the resistors taken together in parallel, between **A** and **B**, are acting like *one effective* or *"equivalent" resistance* determined by

$$\boxed{\frac{1}{R_{\text{eq, par.}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots}\,,$$

which is just a consequence of the two conservation laws. You could picture this as simply laying several resistors side-by-side to make one wider one. Since $1/R = A/\rho\ell$, it's similar to a total resistance whose inverse is proportional to *one* resistor's total area $A = A_1 + A_2 + \cdots$.