**Lecture Notes #07Θ — Thu 21 Feb 2002****Magnetostatics: Magnetic Poles, Magnetic Fields,
Magnetic Field Lines, & Ampère's Law.**

Magnetostatics compared to Electrostatics

The existence of *magnets* and their surprising relation to *electrical* phenomena compel us to recognize the similarities and differences between steady-state (“static”) magnetic effects *vs.* electrostatic effects.

Electrically charged objects interact via the Coulomb force. That the interactions among magnetic things mimic those forces leads us to invent *magnetic fields* \mathbf{B} , in analogy to electric fields \mathbf{E} . The fields serve as a device to connect the EFFECTS produced on “test” objects to their CAUSES in other “source” objects of the same type (the type being, alternatively, Electric or Magnetic in nature). So we suppose that some kind of magnetish sources produce magnetic fields—and other magnetish objects exposed to these fields react by experiencing some kind of force, which we can observe. Magnetic fields \mathbf{B} and the forces they induce in magnetic objects turn out to be SIMILAR in several respects to electric fields \mathbf{E} and the forces they induce in electric charges (see the **Table** on p. 3).

Yet the most fascinating aspects of the connection of magnetism, on the one hand, with the electric world of electric charges and their associated fields and forces, on the other, lie in the DIFFERENCES. To begin with:

**The things that produces magnetic fields and
feel magnetic fields are not static electric charges Q
... but rather *electric currents* I !**

This is totally amazing! It turns out that *moving charges* are what give rise to magnetic effects—for the most part.* We will see that in fact there exists a **very deep connection** between the electric and magnetic worlds. The apparently distinct electric and magnetic phenomena we observe actually reflect, Janus-like, two sides of a single, more complex entity:

ELECTROMAGNETISM

On the next page we compare various features of electric and magnetic fields and forces. NOTES & NOTATION:

- The vector \mathbf{r} (magnitude r , direction $\hat{\mathbf{r}}$) indicates the displacement from the (small) source to the field-observation point.
- The vector quantity $\Delta\ell$ represents a small vector displacement—a length $\Delta\ell$ of a segment of current in the direction of the current I . For a *single* charge q moving at velocity \mathbf{v} , in lieu of the string of moving charges that comprise a current I , one can replace $\{I \Delta\ell\}$ by $\{q\mathbf{v}\}$. [Check Units! $\{(\Delta Q/\Delta t) \times \Delta\mathbf{x}\} = \{\Delta Q \times (\Delta\mathbf{x}/\Delta t)\}$.]
- A “cross product” $\mathbf{A} \times \mathbf{B}$ takes two vectors \mathbf{A} and \mathbf{B} with angle $\theta < 180^\circ$ between them and forms a third vector of magnitude $|\mathbf{A}||\mathbf{B}|\sin\theta$ that points in the 3rd dimension (perpendicular to both \mathbf{A} and \mathbf{B}). E.g., $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$ means you would take $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ and produce the direction $\hat{\mathbf{z}}$ (but $\hat{\mathbf{y}} \times \hat{\mathbf{x}} = -\hat{\mathbf{z}}$). (You will learn how to find that 3rd direction with your right hand.)
- Directions are indicated in the diagrams on p. 4.

* There is a fundamental, “intrinsically magnetic” source of magnetism that does not derive from moving electric charges: the so-called *spin* of elementary particles. Every electron, proton, and neutron, for example, in effect carries around its own little permanent magnet. Electron spins are largely responsible for the magnetism of normal ferromagnets.

Electric Field (from Coulomb's Law) .. $E = k \frac{\{Q\}}{r^2} \hat{r} \quad \left[1 \frac{\text{V}}{\text{m}} = 1 \frac{\text{N}}{\text{C}} \right] \leftarrow \boxed{\text{SIM.}}$

Magnetic Field (Biot-Savart Law) $B = \frac{\mu_0}{4\pi} \frac{\{I \Delta \ell\} \times \hat{r}}{r^2} \quad \left[1 \text{ T} = 1 \frac{\text{N}}{\text{A}\cdot\text{m}} \right]$

E obeys **Gauss's Law**: Net **E**-flux over a closed *bubble* = $\frac{1}{\epsilon_0} \sum Q_{\text{in bubble}}$

B obeys **Ampère's Law**: Net **B**-path around a closed *loop* = $\mu_0 \sum I_{\text{through loop}}$

Intrinsic strength of **E** $k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$

Intrinsic strength of **B** $\frac{\mu_0}{4\pi} = 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}} = 10^{-7} \frac{\text{N}}{\text{A}^2}$

E-Source = a static charge $|\mathbf{E}| \propto \{Q\}$

Fundamental electric objects are point charges = "monopoles" (Q, q, \dots) $\leftarrow \boxed{\text{DIFF.!!}}$

B-Source = small segment steady current $|\mathbf{B}| \propto \{I \Delta \ell\}$

Magnetic monopoles DO NOT EXIST! There are "dipoles" (spins, current loops, ...)

E dies off with distance from source as .. $|\mathbf{E}| \sim \frac{1}{r^2} \quad \leftarrow \boxed{\text{SIM.??}}$

B dies off with distance from source as .. $|\mathbf{B}| \stackrel{?}{\sim} \frac{1}{r^2} \quad \dots \xrightarrow{\text{dipoles}} \sim \frac{1}{r^3}$

Direction of **E** is *radial* from source $\mathbf{E} \sim \hat{r}$ ["EXPLOSIVE" **E**] $\leftarrow \boxed{\text{DIFF.!!}}$

Direction of **B** "*crosswise*" from source .. $\mathbf{B} \sim \Delta \ell \times \hat{r}$ ["LOOPING" **B**]

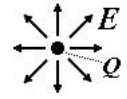
(Coulomb) Force by **E** on another $\{q\}$... $\mathbf{F} = \{q\} \mathbf{E} \quad \leftarrow \boxed{\text{SIM.}}$

(Lorentz) Force by **B** on another $\{I' \Delta \ell'\}$ $\mathbf{F} = \{I' \Delta \ell'\} \times \mathbf{B}$

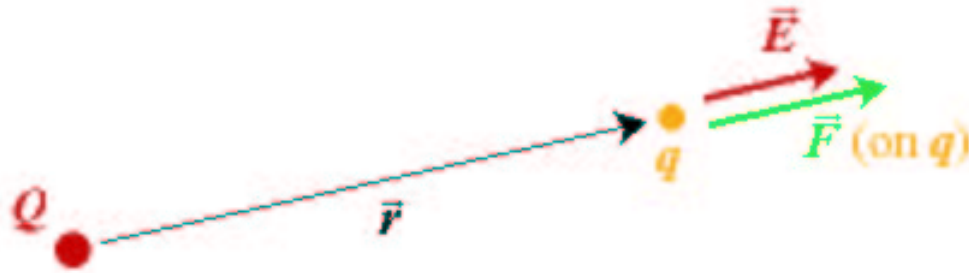
Direction of force on $\{q\}$ $\mathbf{F} \parallel \pm \mathbf{E}$ (*radial*) $\leftarrow \boxed{\text{DIFF.!!}}$

Direction of force on $\{I' \Delta \ell'\}$ $\mathbf{F} \perp \mathbf{B}$ (*crosswise*)

Electric Field & Force Directions



["EXPLOSIVE" E]



Magnetic Field & Force Directions



["LOOPING" B]

