Lecture Notes #08Θ — *Thu 28 Feb 2002*

Magnets & Ferromagnetism, Currents & Magnetic Forces, Torque & dc Motors

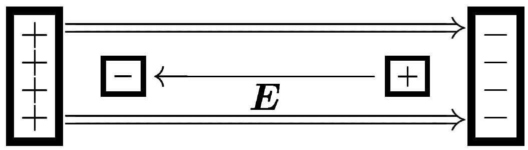
Ferromagnetism

Single magnetic charges do not exist: THERE ARE NO MAGNETIC MONOPOLES. There are, however, magnetic DIpoles. Every elementary particle has a fundamental property called "spin" and every particle that either has a net electric charge (such as electrons) or is made out of other charged particles (such as protons and neutrons, which are made of quarks) are also endowed with a *point magnetic dipole moment*.⋆ That is, the basic building blocks of matter do not leave home without their own little permanent magnet.

Let us ignore electric currents for the moment and address an important difference between magnetostatic and electrostatic effects. Magnetic fields are produced by +/− dipoles, which we label **N**-**S**, and not by individual single charges + and −, as are available for electric effects. ... So? Consider first electric consequences. If we segregate comparable groups of + and − electric charges, what happens? Well, we then have a capacitor, or something roughly like one, with an electric field streaming **between** the charges from + to − and a much smaller field "outside" the charges. If we place a polarizable material—a dielectric—between the charges, it becomes polarized with its +/− charges pulled towards the external −/+ charges,

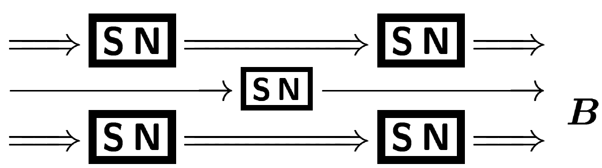
⋆ Uncharged elementary particles can have a nonzero spin yet have no magnetic moment—most importantly, *photons* = particles of light.

respectively. This is shown very schematically here:

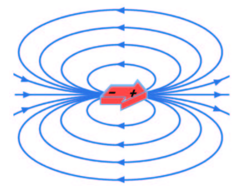


(If we place the dielectric outside the "capacitor," nothing much happens because the field strength is nothing much there.) Notice how the dielectric forms its own electric dipole "inside" the capacitor's: the dielectric's dipole lines up with the local electric field just as a compass would in the magnetic version. But the dipole's own internal field is aligned *opposite* to the very field that polarizes it and so CANCELS it, at least somewhat. The resulting net $\boldsymbol{E}$-field has been diminished.

Consider now what happens when we place a sample of soft (i.e., magnetizable) iron in the magnetic field of one or more aligned permanent magnets (hard iron). The domains in the soft iron polarize and each, like a compass, will align with the local magnetic field as described above in the electric case. This is shown, again very schematically, here:



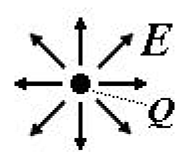
The soft-iron domain is the one sitting in the middle. There is no chance of this domain's being *between* magnetic charges—there is no way to be "inside" magnetic dipoles. This domain can only see the field "outside" the permanent magnets' **N** and **S** poles; the domain lines up with the magnets' $\boldsymbol{B}$-field there, compass-style. A soft-Fe domain's **S** pole would lie closer to some magnets' **N** poles, &c., and consequently that domain must line up from **S**-to-**N** *with the SAME orientation as the magnets*. The soft-Fe domains' own "outside" magnetic field would therefore supplement the magnets' net field, ADD to it—not oppose it, not cancel it!



If you do happen to find yourself "inside" a bar magnet, note that the magnetic field there does NOT reverse direction and go from **N**-to-**S** as does the electric field in a capacitor. **Why?** 'Cause there are no magnetic monopoles! The field doesn't really emerge from magnetic + and − charges but rather continues to loop around *from* where you'd think the **S** (−) pole is *towards* where you'd think the **N** (+) pole is. Here too, a compass or domains of a piece of soft iron would line up **S**-to-**N** in the same sense as the magnetizing magnet and once more produce a field that AUGMENTS the local magneti*zing* field, not diminish it.

The upshot of all this is that dielectrics inserted into capacitors (or into electric fields, generally) tend to reduce the local electric field and potential difference, whereas iron cores inserted into solenoids (or into magnetic fields, generally) tends to ***increase the local magnetic field.*** —Because there is no such thing as "between" magnetic charges —Because there really are no such things as individual magnetic charges.

Magnetic Forces



Nature has chosen to make the electric force rather straightforward and the magnetic force rather less straightforward. If a single point source charge Q produces an electric field $\boldsymbol{E}$, then the pattern of $\boldsymbol{E}$-vectors in space is of the "EXPLOSIVE" (or "DIVERGENT") variety: Coulomb's Law predicts $\boldsymbol{E}(r) = \hat{\boldsymbol{r}}\, kQ/r^2$, which is a purely radial pattern—it depends on r but not on angles. Once an electric field $\boldsymbol{E}$ is established at a point, by whatever means, the electric force on a charge q placed there is simply $\boldsymbol{F}_{\text{elec}} = q\boldsymbol{E}$. In other words, one point charge affects a second point charge simply by pushing the other one directly outwards away or pulling the other directly inwards towards itself.

Magnetic interactions introduce a twist—actually, two twists—to this mechanism. A magnetic field cannot arise from just one magnetic charge. There are no single magnetic charges ("NO MAGNETIC MONOPOLES"). Magnetic charges, if we insist on speaking of them, always come in pairs: + tied to −, **N**-and-**S**, *magnetic dipoles*. They generate a dipole-field pattern, in which $\boldsymbol{B}$-field *appears* to flow out of one (**N**) end of the dipole and back around into the other (**S**) end. But magnetic fields can also be generated and felt by *moving electric charges*—i.e., by an electric current $I = \Delta Q/\Delta t$. The "source" and "test"* here are not merely bits of charge ΔQ but brief segments of current of length $\Delta\ell$: $I\,\Delta\boldsymbol{\ell} = (\Delta Q/\Delta t)\Delta\boldsymbol{\ell} = \Delta Q(\Delta\boldsymbol{\ell}/\Delta t) = \Delta Q\,\boldsymbol{v}$. (The direction of the **vector** $\Delta\boldsymbol{\ell}$ indicates the direction of current flow through the little segment.) Both the charge and its motion are essential ingredients. The fact that the producers and feelers of magnetic field have *directions* in addition to their *strengths* complicates the description of magnetic effects.

The first "twist" in the SOURCE→FIELD→FORCE mechanism lies in the direction of the magnetic field $\boldsymbol{B}$ produced by any bit of current as observed at any point: $\boldsymbol{B}$ always turns out to be *perpendicular* to both the current-flow direction $\Delta\boldsymbol{\ell}$ AND the source-observer direction $\hat{\boldsymbol{r}}$. The $\boldsymbol{B}$-field receives one 90° twist relative to what you'd expect from a radial field like $\boldsymbol{E}$. Magnetic fields always turn "sideways," which compels magnetic field lines always to *loop around*, as claimed above.

The second 90° twist lies in the direction of the force felt by a second current (the "test current," if you like). Unlike the $\boldsymbol{E}$-field, which gives rise to a force parallel to $\boldsymbol{E}$, no matter what charge is acted upon, the $\boldsymbol{B}$-induced force acting on a current is also "sideways." The so-called Lorentz force on a charge q moving with velocity $\boldsymbol{v}$ in a magnetic field, $\boldsymbol{F}_{\text{mag}} = q\boldsymbol{v}\times\boldsymbol{B}$, takes charge- and field-directions and creates a force in the 3rd dimension ($\boldsymbol{v}\times\boldsymbol{B}$), perpendicular to both $\boldsymbol{v}$ and $\boldsymbol{B}$.†

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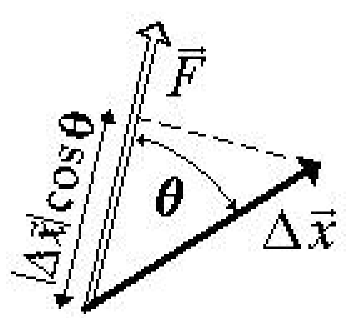
* Beware that these are only names for purposes of description and that all currents both "produce" and "feel" $\boldsymbol{B}$-fields, just as all charges q both "produce" and "feel" $\boldsymbol{E}$.

† For field-feeling currents, replace a test charge's $q\boldsymbol{v}$ by a test current's $I\Delta\boldsymbol{\ell}$.

Like electric charges repel and opposite charges attract. But the magnetic version of this has a peculiar outcome on account of the two 90° twists: parallel currents attract and antiparallel currents repel. So for the purpose of calculating forces it's not safe to blindly regard two currents as the "same current"—same I and same direction—in analogy to charges. In general, one must work out the direction of $\boldsymbol{B}$ from the whole source current's shape (all its segments $\Delta\boldsymbol{\ell}$) and from the observer position ($\hat{\boldsymbol{r}}$), and then work out the direction of $\boldsymbol{F}_{\text{mag}}$ from that $\boldsymbol{B}$ and the test charge's $\boldsymbol{v}$ (or test current's $\boldsymbol{\Delta\ell'}$). That takes two right hands.

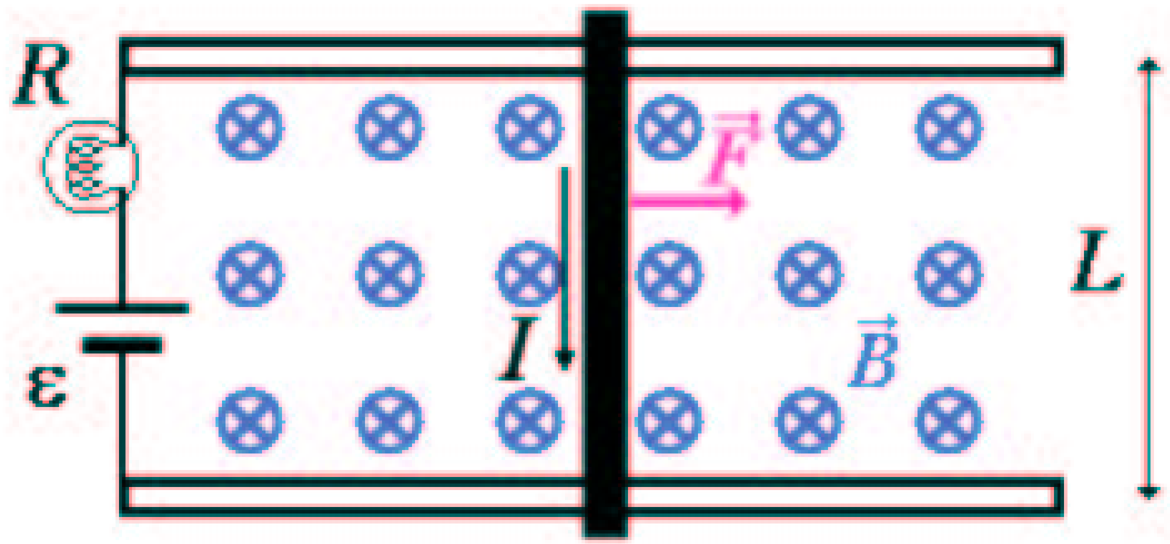
There is one other essential piece of weirdness about magnetic forces. Because by its very nature the Lorentz force is always perpendicular to the velocity ($\boldsymbol{F}_{\text{mag}} \perp \boldsymbol{v}$), it NEVER DOES ANY WORK**!!!** Recall that the work done on any object by a force $\boldsymbol{F}$ acting through a displacement $\Delta\boldsymbol{x}$ is $W = \boldsymbol{F}\cdot\Delta\boldsymbol{x} = F\Delta x\cos\theta$, where θ is the angle between $\boldsymbol{F}$ and $\Delta\boldsymbol{x}$.



Whenever $0 \le \theta < 90°$ the force has a component parallel to the displacement and you have $W > 0$: you're pushing the thing from behind, accelerating it, adding energy (kinetic energy) to it. Whenever $90° < \theta \le 180°$ the force has a component antiparallel to the displacement and you have $W < 0$: you're pushing the thing from in front, decelerating it, taking energy out of it. And if $\theta = 90°$ exactly, then you have $W = 0$: you're not changing the speed or energy of the object at all, although you are *changing its direction*, deflecting it. The Lorentz force is always sideways, always perpendicular to $\boldsymbol{v} = \Delta\boldsymbol{x}/dt$, hence always $\boldsymbol{F}_{\text{mag}} \perp \boldsymbol{v}$ and $\boldsymbol{F}_{\text{mag}} \perp \Delta\boldsymbol{x}$. And so magnetic fields, which bump things around through this force and through this force alone, can ***NEVER do work***. No matter what.

Electric Motors

So if magnetic fields never EVER do any work, how do electric motors work and how do they *do* work? Consider the Rod 'n' Rails apparatus, our prototypical electric motor:



It's pretty straightforward: the slidable rod lies across the rails, completes an electric circuit, and carries a current I. If immersed in a magnetic field perpendicular to the plane of the rails, the current flowing through the rod suffers a Lorentz force perpendicular to the rod, as shown. (Notice the three mutually perpendicular directions of current, field, and resultant force.) Its magnitude equals $|\boldsymbol{F}| = ILB$. OK, so the charges drifting through the rod experience a sideways force.

Now, how is it a MOTOR? We put electrical energy into the current (power ε^2/R) ... and expect to get MOTION out of it. We attach the source of εMF, there's a force on the rod, and so ... the rod moves. Good—we have a MOTOR.

But *wait a minute!!* The force on the rod accelerates it, we get motion where before there was none, and this change in kinetic energy has arisen because the rod was pushed by a—**magnetic force!?** The magnetic force has done work after all?! *Huh???*

There is a way out of this, everything's cool, you just have to look at it a little more closely. If there were still a current but no rod, we'd have no problem. The downward-travelling beam of charges in the diagram would be deflected towards the right (towards their left) by the force shown and presumably keep deflecting at constant speed and execute circular motion. The rod-ness of the rod does not let this happen, however. We can picture the conventional current's positive charge carriers moving downwards at the drift velocity and being deflected *within* the rod—so far no problem—but then they reach the rod's surface and CANNOT ESCAPE! There must be some "force of constraint" keeping the moving charges within the straight rod. If they need to keep deflecting, well, they'll have to carry the whole rod with them towards the right. They MAY NOT jump out of the rod; we just ain't gonna let that happen.

So there's more going on here than just the magnetic force. There are assumptions built into what it means to exert a force "on the rod" (as opposed to "on the moving charges"). Ultimately, electrostatic forces and some subtleties of quantum mechanics are responsible for gluing and confining electrons to the rod (or to any other piece of normal matter, for that matter). The magnetic force really does merely **deflect** moving charges to the side. But *these charges already have some kinetic energy*—and this is the source of the work done to accelerate the rod sideways. And that's how the electrical power source powers the motor! The sideways-deflected charges tug on the mass of the rod and *use up some of their own energy* to drag the rod along sideways with them. **The magnetic field does not provide the energy but rather provides a kind of "working fluid" to *transmit* some other energy.** ... Imagine the Road Runner running alongside Wile E. Coyote parallel to the edge of a cliff. R.R. give W.E.C. just a little poke sideways, costing R.R. no energy yet deflecting W.E.C. ...

T-183 FIG. 20-30 Torque on a current loop in a magnetic field