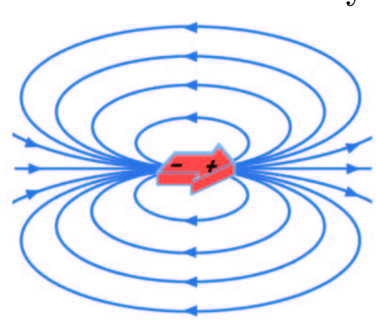
Lecture Notes #08T — *Tue 26 Feb 2002*

$\boldsymbol{B}$ *vs.* $\boldsymbol{E}$, How Currents Produce $\boldsymbol{B}$-Fields, & Magnetic Forces on Currents

How Currents Produce $\boldsymbol{B}$-Fields

Being cheap, we shall carry the concept of a ***Field*** from our electric investigations over to magnetic phenomena. The concept supposes that there are some kinds of localized magnetic "*sources*" that produce effects at all points in space around them. This information gets encoded in the pattern of the magnetic field $\boldsymbol{B}$ in space; the field determines what the $\boldsymbol{F}$orce will be anywhere on any other object capable of interacting magnetically. Since any sources that create magnetic fields also "feel" them, we have here a reciprocity reminiscent of the electric charges "Q_1Q_2" in Coulomb's Law: the sources of magnetic fields are the very things that respond to magnetic fields.

The two types of sources of magnetic fields are **(1)** pure magnetic dipoles and **(2)** electric currents. The pure dipoles are responsible for *ferromagnetism*—essentially, ordinary magnets. These ultimately derive from an intrinsic property of elementary particles called their SPIN, specifically electrons' spins. Every electron in effect carries around its own itty-bitty pointsize permanent magnet that produces a magnetic-field dipole pattern like this $\Longrightarrow$

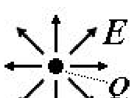


If you can convince enough of the electron spins within a material to line up, you can make a macroscopic magnet; this happens readily in iron, cobalt, gadolinium, and various other exotic stuff. Owing to the NONexistence of single magnetic charges—the NO MAGNETIC

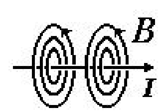
MONOPOLES law—the simplest collection of magnetic charge one can find is a +/− equal-and-opposite pair, which is just a magnetic dipole. In normal parlance, we call the poles **N/S**.

Perhaps even more remarkably, magnetic fields are produced and felt by *moving electric charges*!! One of the fundamental laws of *ElectroMagnetism*, AMPÈRE'S LAW, specifies how a given current creates its magnetic-field pattern. Another statement, the LORENTZ FORCE law, specifies the magnetic force experienced by any current that finds itself in a net magnetic field from any other source(s). Together these provide the magnetic analog of the COULOMBIC interaction amongst electrostatic source charges.

AMPÈRE'S LAW is really closer in spirit to GAUSS'S LAW than to COULOMB'S LAW itself. GAUSS'S LAW is a *global* statement, insofar as it relates a *total* electric flux distributed over an arbitrary closed surface ("Gaussian bubble") to the *total* electric charge contained within the bubble's volume. This reflects a basic property of COULOMB'S LAW, namely, that on a fundamental level electrostatic interactions occur between *point*like electric charges Q, so electric fields always radiate out from or in towards point sources. But the sources of magnetic fields, on the other hand, are currents I and every segment of current of length ℓ has a *direction*. Since magnetic field lines must always form closed loops (NO MAGNETIC MONOPOLES!),



the magnetic loops are tailored to a directional current by looping around the current's axis. Thus, in contrast to $\boldsymbol{E}$-field patterns **exploding** out of electric charges, AMPÈRE'S LAW provides $\boldsymbol{B}$-field patterns **looping** around electric currents. And instead of measuring how much electric charge interior to a bubble produces electric-field flux exploding out of the bubble, now we need to measure the magnetic field *circulating around* some closed loop-like path—magnetic field produced by currents passing *through* the loopy path.



Here is one way to state AMPÈRE'S LAW:

$$\sum_{\substack{\text{around}\\ \text{path}}} B_{\|}\Delta\ell = \mu_0 \sum_{\substack{\text{through}\\ \text{path's hole}}} I$$

The lefthand side measures the magnetic field parallel to an *arbitrary* path in space in the vicinity of each little segment of length $\Delta\ell$ comprising the path. One adds up the $\boldsymbol{B}$-parallel-to-path contributions around the entire path, weighted at each step by each segment's length, to obtain a measure of the generically looping $\boldsymbol{B}$-field.* Note that choosing a path includes specifying a sense of circulation around the path. The righthand side represents the total current passing through the hole that the path unavoidably circumscribes. If a current passes through the hole in a direction related to the path's circulation direction via a *righthand* (*lefthand*) sense, then one takes I as positive (negative); current that doesn't pass through the hole doesn't contribute to the sum. *[Cf. charge exterior to a Gaussian bubble not contributing to the bubble's net electric flux.]*

A basic though ideal case is that of an infinitely long, perfectly straight current I. The *symmetry* of this one-dimensional source is characterized by its INVARIANCE as one moves along the current's length or as one moves angularly around the current's axis. This necessitates looping magnetic-field lines that form perfect circles around the axis. At a distance r from the current axis, the magnetic field $\boldsymbol{B}(r)$ at that radius follows the circle but its strength $B(r)$ does not vary all the way around. The lefthand sum, which tracks the looping field line, then involves a constant $B(r)$ along the

* In calculus terms, this sum is actually a *path-integral* $\oint \boldsymbol{B}\cdot d\boldsymbol{\ell}$, where the symbol $\oint$ means integrating up something like $|\boldsymbol{B}|\, d\ell \cos\theta$ around a *closed* path, and θ is the angle between the field vector $\boldsymbol{B}$ and $d\boldsymbol{\ell}$ at every little directed segment $d\boldsymbol{\ell}$ of the path.

whole path's length—just its circumference $2\pi r$. The righthand, current-source-measuring sum includes the single current I, yielding

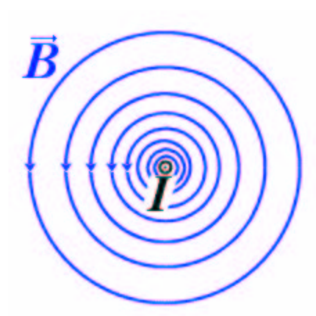
$$B(r) \times 2\pi r = \mu_0 I \quad \Longrightarrow \quad \boxed{B_{\substack{\text{long}\\\text{straight}\\\text{current}}}(r) = \frac{\mu_0 I}{2\pi r}},$$

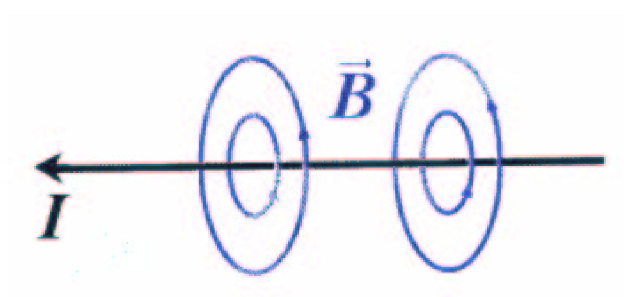
where $\boldsymbol{B}$ curls around I in the *righthand* sense—i.e.,

{`right thumb follows current`}

$\Longleftrightarrow$ {`right fingers curl around along magnetic field`}

... which from the front and the side looks like this:





How Currents React to $\boldsymbol{B}$-Fields

The flip side to currents producing $\boldsymbol{B}$-fields is the force experienced by a current segment I of length ℓ in a $\boldsymbol{B}$-field: $F_B = I\boldsymbol{\ell}\times\boldsymbol{B}$, where the vector $\boldsymbol{\ell}$ points along the direction of current flow. *[Cf. electric charges, which not only produce electric fields but experience a force when sitting in the $\boldsymbol{E}$-fields of other charges.]* More generally, any charge q moving with some nonzero velocity $\boldsymbol{v}$ in a magnetic field $\boldsymbol{B}$ feels this LORENTZ FORCE $\boxed{F_B = q\boldsymbol{v} \times \boldsymbol{B}}$. ....
Stay tuned .... **[What's "$\boldsymbol{v} \times \boldsymbol{B}$"??!]** ....
.... More on this weird force and its funny sideways nature later ....
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