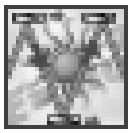


Lecture Notes #09T — *Tue 5 March 2002*

ElectroMagnetic Induction: Changing Magnetic Flux in the Scheme of Things

The Fourth Fundamental Law of ElectroMagnetism: Faraday's Law

Thus far, our understanding of electric and magnetic effects has hinged on a two-step paradigm:

> (⋆) SOURCES PRODUCE FIELDS ...
>
> (⋆) ... WHICH PRODUCE FORCES ON OTHER SOURCES.

The "sources" here can be (1) electric charges for electric fields and forces and/or (2) moving electric charges (or magnets) for magnetic fields and forces. "Effects" means *observable forces*, on which a physical theory such as ElectroMagnetism bases its claims.

Our investigations into electrostatics and magnetostatics have touched upon three of the ***four fundamental laws of ElectroMagnetism***: (1) Coulomb's Law of electrostatics for electric charges; (2) a Coulomb's Law for magnetostatics but *without* single magnetic charges; and (3) Ampère's Law of magnetostatics, though only in part. The remaining item, (4) Faraday's Law of *Electromagnetic Induction*, is a law of DYNAMICS (not statics), as is the rest of Ampère's Law.

We have invented the constructs "Electric field $\boldsymbol{E}$" and "Magnetic field $\boldsymbol{B}$" as convenient mechanisms or connections for describing how forces arise among static or moving charges. But we haven't proven that $\boldsymbol{E}$ and $\boldsymbol{B}$

are actually necessary or even "real." We have illustrated how to use them, how their field-line patterns have certain features, why they might be convenient, and perhaps why their patterns might be construed as interesting.

Faraday's Law is fundamentally different from the ElectroMagnetic laws of statics, in that it involves quantities that *vary in time*. It is different, moreover, in that it is *purely a relation between electric and magnetic fields!* The existence of effects in the *absence of sources*—i.e., fields without stationary or moving electric charges—implies that ***E*- and *B*-fields have a reality of their own**. It implies that these "fields" are indeed not merely constructs. This discovery comes laced with a gloss of MAGIC, for it opens the way to your understanding the nature of LIGHT.

The Four Fundamental Laws of ElectroMagnetism

Here we summarize (descriptively rather than precisely) the four fundamental laws of ElectroMagnetism, whose mathematical incarnations are collectively known as ***Maxwell's Equations***. We must not forget to supplement these statements with the two powerful constraints on what the "Laws" happen to be allowed to allow happen:

- CONSERVATION OF ENERGY
- CONSERVATION OF CHARGE

(1) **Coulomb's Law**

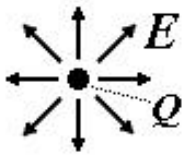
Electric forces occur between (all) pairs of objects possessing net electric "charge."

Each point charge Q_1 produces an "electric field" $\boldsymbol{E}_1 = \dfrac{kQ_1}{r^2}\,\hat{\boldsymbol{r}}$ at any other point a distance r from Q_1, directed radially away $(\hat{\boldsymbol{r}})$ from Q_1.

The force on another point charge Q_2 located at the "other point" is

$\boldsymbol{F}_{\text{on } Q_2} = Q_2\boldsymbol{E}$, where $\boldsymbol{E}$ is the TOTAL electric field at Q_2's location. The force between 2 point charges $\therefore$ has magnitude $|\boldsymbol{F}| = k\dfrac{|Q_1 Q_2|}{r^2}$.

Electric fields and forces emanating from **1** fundamental, point charge Q display an "EXPLODING" (for $Q > 0$) or "IMPLODING" (for $Q < 0$) field-line pattern around the point—indicating the existence of a single point "source" of electric effects.



Gauss's Law measures the amount of $\boldsymbol{E}$ exploding through a closed surface ("bubble") due to the net electric charge contained inside.

(2) ***No Magnetic Charges* exist for "Magnetic Coulomb's Law"**

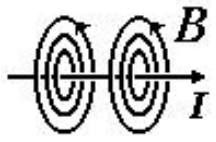
Isolated magnetic charges—i.e., magnetic monopoles '**N**' & '**S**' (or '+' and '−') DO NOT EXIST.

So there are NEVER patterns of EX-/IM-PLODING magnetic field $\boldsymbol{B}$—because there are no single magnetic source charges.

(3) $\frac{1}{2}$ **Ampère's Law** *[Biot-Savart Law; Lorentz Force]*

Magnetic fields $\boldsymbol{B}$ can be generated by indivisible pairs of apparent magnetic charge ('**N**' & '**S**')—magnetic dipoles. These occur either as objects possessing a net magnetic "spin" (elementary charged particles, ferromagnets) OR by electric current loops—i.e., by *moving electric charges*! Moving charges also experience a "Lorentz" force when subject to a field $\boldsymbol{B}$: it's $\boldsymbol{F} = I\Delta\boldsymbol{\ell} \times \boldsymbol{B}$ for a length $\Delta\boldsymbol{\ell}$ of current I or, equivalently, $\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$ for a point charge q with velocity $\boldsymbol{v}$.

Magnetic fields produced by moving electric charges (currents) always form "LOOPING" ("*non*EXPLODING") field-line patterns.



Ampère's Law, by having us trace a closed path ("loop"), measures the amount of looping-$\boldsymbol{B}$ generated by the net electric current piercing the "hole" inside the loop. [*The other* $\frac{1}{2}$ *of Ampère's Law*? Well, there is

actually one more way to produce magnetic fields ...]

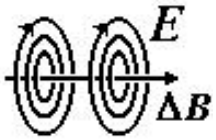
(4) **Faraday's Law**

The existence of electrically charged objects is NOT THE ONLY WAY to make electric fields!!

DYNAMICS!!

Changing magnetic fields produce a new kind of electric field pattern—"LOOPING," sourceless, *non*EXPLODING electric fields!

More precisely: $\boxed{\varepsilon = -\dfrac{\Delta\Phi_{\text{B}}}{\Delta t}}$.



The net $\boldsymbol{E}$ around a closed path ("loop"), weighted by the path length, equals a net εMF ε (a change-of-Voltage) around the loop.♠

The εMF is generated or "INDUCED" by *changing "magnetic flux"* Φ_{B} through the area of the loop, where $\Phi_{\text{B}} = \{\boldsymbol{B}\texttt{-fields}\} \times \{\texttt{Area}\}$.

The direction of the εMF around the loop and *the current* I_{induced} *it induces* in a real wire loop (resistance $R \Rightarrow I_{\text{induced}} = \varepsilon/R$) is such that all consequences OPPOSE THE CHANGE IN FLUX ($-\Delta\Phi_{\text{B}}$). This feature of Faraday's Law has been dubbed "Lenz's Law."

There are 3–6 ostensibly distinct ways to realize a nonzero change in flux, $\Delta\Phi_{\text{B}} \neq 0$: **(1)** change the *loop position* in the $\boldsymbol{B}$-field, **(2)** change the *loop orientation* with respect to the $\boldsymbol{B}$-field, **(3)** change the *loop area*, **(4)** *re-orient the* $\boldsymbol{B}$*-field* through a fixed loop, **(5)** *move the source of* $\boldsymbol{B}$*-field* (e.g., a magnet) with respect to a fixed loop, & **(6)** don't move anything but *change* $\boldsymbol{B}(t)$ *in time*. ★★★★★★★★★★★★★★★ The MAGIC one is **(6)**.

♠ The "loop" may be an imaginary string of points in space or may coincide with a real wire loop. Note that positive direction *around* the loop correlates with positive direction *through* the loop via a right-hand mnemonic. This εMF circulates in the *negative* sense.