Broken orientational and reflection symmetries in thin film superconductors with mesoscopic magnetic dipoles

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Within Ginzburg-Landau theory, we study vortex configurations in periodic nanoscale superconducting dot arrays with ferromagnetic inclusions. the magnetic moments of the dipoles are oriented out of the plane, and no external flux is applied. We find that sufficiently strong dipoles induce the formation of vortex-antivortex pairs with the vortices usually confined in the dot regions and the antivortices forming a lattice in the interstitial areas. The vortex-antivortex pair density exhibits broad plateaus as a function of the dot dipole moment. Although in most cases, the number of antivortices per unit cell is integral, we have identified a half-integral plateau in which some antivortices are shared by two magnetic dots. Many of the plateaus correspond to vortex configurations which break lattice symmetries. Some of these configurations involve the marked deformation of vortex cores. Experimental implications of these novel results are discussed.

1. Introduction

Due to their high critical currents and fields, type II superconductors are technologically very important. However, sufficiently large applied magnetic fields or currents cause vortices to appear in these superconducting materials, leading to dissipation and thereby spoiling the perfect conductivity vital in applications involving superconductors. Hence, it is important to find ways to pin flux quanta to prevent their motion. Systematic studies of pinning have been conducted in experiments on regular nanoscale arrays in which a lattice of defects is superimposed on a thin superconducting film. Earlier experiments focused on "antidot" arrays, in which the pinning centers consist of holes or depressions in the superconducting substrate. More recently, defect arrays have been formed of mesoscopic magnetic dipoles, often deposited on top of the superconducting film [1]. Many experimental and theoretical studies have examined scenarios in which a finite net magnetic flux penetrates each unit cell. In this work, we examine magnetic dipole arrays for which the applied magnetic flux vanishes. Even in the absence of an applied magnetic field, we demonstrate rich vortex phenomena when the dot dipole moments are sufficiently strong.

We focus on the case in which the magnetic moments are oriented perpendicular to the superconducting substrate. The supercurrents induced by the dot magnets move in a clockwise direction, thereby creating a partial Meissner effect by canceling some of the field from the mesoscopic array magnets. Associated with these supercurrents is a cost in kinetic energy; if the dipole moments are sufficiently strong, it becomes energetically favorable to place a vortex near the dot, since this allows the counterclockwise vortex currents to partially cancel the induced supercurrents. The condition that zero net flux pass through the unit cell prevents vortices from forming in isolation; vortices and antivortices must appear in pairs.

A few studies have focused on isolated magnets, where the entire system has cylindrical symmetry [2,3]. For the first time, we study an infinite periodic array of dots in the framework of the

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Ginzburg-Landau theory. A few studies in the context of the London theory have examined a single cell system (with periodic boundary conditions), in which a single pair of flux quanta is present [4–6]. One study examined depinning phenomena in the Ginzburg-Landau framework, but for antidots rather than magnetic dots [7]. Since one must put vortices in by hand in the London theory, it is difficult to go beyond simple situations (typically one vortex-antivortex pair in a single unit cell.) In Ginzburg-Landau treatment, vortices and antivortices appear naturally, allowing the consideration of more complicated situations which arise when one has multiple flux quanta pairs in a unit cell. In this work, we do not assume that each unit cell has the same vortex configuration; to take into account superlattice effects, we study a large (4×4) supercell with periodic boundary conditions. In what follows, we denote by ρ_{pair} the number of vortex-antivortex pairs per unit cell. We find that, as a function of the dot dipole strength, ρ_{pair} exhibits crisply defined plateaus. As will be discussed, we find stable vortex states which break orientational and/or mirror symmetry. As the dipole strength is varied, phase transitions in the vortex configurations occur. These transitions can be abrupt or continuous. Hysteresis phenomena are found to occur in the abrupt cases, consistent with their begin first order transitions. Surprisingly, some of the first order transitions occur within a plateau. As will be seen, second order transitions mark shifts between plateaus in which ρ_{pair} either increases or decreases via the annihilation or creation of a vortex-antivortex pair. The creation or destruction of a pair is associated with a marked deformation of the vortex and antivortex cores and occurs gradually. Since London theory does not take into account variations of the Cooper pair density, and therefore cannot correctly describe vortex cores, Ginzburg-Landau theory is essential in the study of these novel states.

2. Methods and results

Operating in the Ginzburg-Landau framework, we solve the nonlinear partial differential equations which result from extremizing the Ginzburg-Landau internal energy given in Eq. 1 below.

$$E_{GL} = d\Delta\xi^3 \int \left[\begin{array}{c} \left|\psi^* \left(\vec{\nabla}/i - \vec{A}\right) \psi\right|^2 - |\psi|^2 \\ +\frac{\kappa^2}{2} |\psi|^4 + (\vec{B} - \vec{B}_{ext})^2 \end{array} \right] d^3x. \quad (1)$$

In Eq. 1, we have used dimensionless units; as a consequence, all linear dimensions are expressed in terms of the superconducting coherence length ξ. In the constant in front of the integral sign, Δ is the condensation energy per volume for a uniform bulk superconductor and d is the film thickness in units of ξ. $\kappa \equiv \lambda/\xi$, where λ is the magnetic penetration depth, is the Ginzburg parameter. Since $d \ll \xi$, we regard the superconducting substrate as a film of negligable thickness. For simplicity, the mesoscopic magnetic dipoles above the substrate are assumed to have a square cross section. Although a range of magnet thicknesses may be used, in this work we have chosen the height of the dot to be equal to its lateral size. Specifically, the array magnets are cubes 2.0ξ in a square array with a lattice constant of 6.25ξ. We specify the dot dipole moment by calculating the *positive* flux passing through each unit cell. This quantity, given in units of the fundamental flux quantum, is a convenient measure of the dipole strength.

To solve the Ginzburg-Landau equations for our system, we have replaced continuum variables with their discrete versions on a mesh fine enough to ensure convergence with respect to the discretization (to achieve this, we allow at least 5 grid points per coherence length). Our discretization scheme is a gauge theoretic formalism in which currents and vector potentials A^x_{ij} and A^y_{ij} occupy the lattice links, while the order parameter values ψ_{ij} occupy the nodes of the lattice. In our model, we impose discrete versions of the standard continuum gauge symmetries. We treat the mesoscopic array dipoles as square loops of current with a vertical thickness equal to the width of the base of the magnet; the magnetic fields and vector potential generated by the magnetic dots are then readily calculated. In our calculation, x and y (in-plane coordinates) are handled discretely, while the z direction is treated exactly, in the continuum limit.

We solve the Ginzburg-Landau equations in an iterative manner, via a conjugate-gradient technique. In this method, one begins by linearizing the Ginzburg-Landau equations about an initial guess. The resulting linear equations are then solved, and the solution is used as an input for the next iteration. In our calculations, we simulate an experiment in which dot dipole moments are varied continuously (To realize this in the laboratory, one could apply an in-plane magnetic field to tilt the moments of the dot magnets. Since the superconducting film is quite thin, the applied field would not destroy the superconductivity. However, the tilting of the dipole moments would allow one to vary their effective strength). In "rightward sweeps", we slowly increase the strength of the dot dipoles. Sweeps range from very weak magnets which do not induce the formation of flux quanta pairs to magnetic dots strong enough to destroy the superconductivity altogether. During a rightward sweep, in which dipole strengths are steadily increased, each calculation uses as its initial guess the results of the previous calculation. The Ginzburg-Landau equations are then solved for slightly stronger magnetic dots. "Leftward" sweeps are conducted in a very similar way, with each successive calculation using weaker dipoles than the previous one. The sweeps in opposite directions complement each other by making it possible to see hysteresis effects (thereby revealing which transitions are first order); and, by comparison of the results of the two sweeps, allowing the identification of the stable phase.

To illustrate the configurations which correspond to the phases shown in Fig. 2, it we display Cooper pair densities in Fig. 3 and Fig. 4. Vortices and antivortices are readily identified as regions of depleted Cooper pairs. To distinguish vortices from antivortices, it is useful to exploit the fact that there are singularities in the order parameter phase at the vortex cores; the line integral $\oint \vec{\nabla}\phi \cdot d\vec{s}$ yields $+2\pi$ (-2π) if a (anti) vortex is encompassed by the contour of integration, and is otherwise zero. In terms of currents and magnetic flux, this condition is $\oint \frac{\vec{J}}{|\psi|^2} \cdot d\vec{s} + \Phi_B = \pm 2\pi$ for a vortex or antivortex, respectively. This tool for locating flux quanta allows the precise depiction of vortex configurations, and also is a convenient way to determine ρ_{pair}.

Figure 1 depicts the Ginzburg-Landau internal energy and ρ_{pair} for both sweeps to the right and sweeps to the left (E_{GL} is given in units of $d\Delta\xi^3$). Readily evident is the discrepancy between leftward and rightward sweeps for ρ_{pair}. One can also see sharp downward jumps in E_{GL} for both rightward and leftward sweeps. The hysteresis effects in ρ_{pair} and the jumps in the Ginzburg-Landau energy are a result of metastability of some of the vortex configurations, a sign that several of the transitions are first order. One can construct a "ground state energy" by selecting the lowest energy from the leftward and rightward sweeps. The configuration corresponding to the lowest energy is then the preferred vortex state. In this manner, we have constructed a phase diagram for the system, shown in Fig 2. Figures 3 and 4 depict vortex configurations in salient cases. In the phase diagram, states are classified according to ρ_{pair}, the number of vortex-antivortex pairs. When necessary, as in the case $\rho_{pair} = 2$, phases are further classified according to the symmetry of the vortex configuration.

In discussing the phase diagram of Fig. 2, we begin from the left (weak dipole strengths) and move to the right (in the direction of greater dipole moments). For very weak dipoles, the formation of vortex-antivortex pairs is energetically unfavorable. Hence, $\rho_{pair} = 0$ for the leftmost state. The next configuration corresponds to a single vortex-antivortex pair per unit cell with vortices occupying the dot regions and antivortices residing in the centers of the interstitial areas. As the dipole moments are increased further, there is a first order transition to a vortex state for which $\rho_{pair} = 2$. As is evident in Fig. 3a, antivortices (small dark regions) are connected with the magnetic dots by lobes of depleted Cooper pair density (light areas). These molecule-like complexes orient along the unit cell diagonal, thereby breaking the $\pi/2$ orientational symmetry. The next state breaks both orientational and mirror symmetries. Not surprisingly, the transition between them is first order.

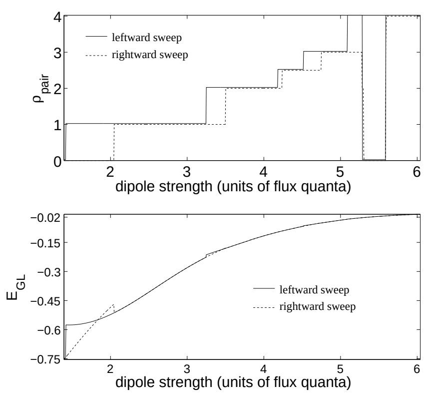

Figure 1. ρ_{pair} and E_{GL} plots for sweeps to the left and to the right. The energy is given in units of $d\xi^3\Delta$, where Δ is the volume condensation energy and d is the film thickness

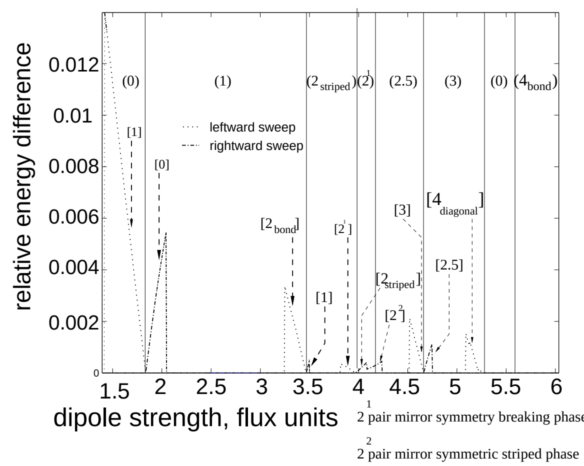

Figure 2. Phase diagram with mestastable states. The numbers in parentheses indicate stable phases. Metastable states are indicated by the arrows, and are characterized by numbers in square brackets. Energy differences are given as fractional deviations from the ground state energy.

Higher dipole strengths lead to a fractional state, for which $\rho_{pair} = 2.5$. Figs. 3b and 3c illustrate that the transition into this state occurs gradually. Ultimately, mirror symmetry is restored, as seen in Fig. 3c. The next transition, to a $\rho_{pair} = 3$ state (depicted in Fig 3d), is first order. In view of the significant dissimilarities between the two states, this is not surprising. As one continues the rightward sweep, the vortex pair density begins to exhibit nonmonotonic behavior by jumping to zero. Despite the abrupt change in ρ_{pair}, the shift to $\rho_{pair} = 0$ occurs continuously, through the sequence shown in Fig. 4. The antivortices first rearrange to form the $\rho_{pair} = 3$ configuration in Fig. 4a. Next, the antivortices are pulled inward, forming the state shown in Fig. 4b. At the same time, the vortices move outward from the dot to meet and annihilate the antivortices. The final transition is to the $\rho_{pair} = 4$ configuration shown in Fig. 4d. In each unit cell, four vortex-antivortex pairs form. The antivortices move outward toward neighboring dots, while the the vortices gravitate to the magnetic dots.

This complicated evolution is driven by the competition between intra and interdot potentials for the antivortices. Evidently, for very strong dipoles, the antivortices feel not just the effects of a single magnet, but are affected by neighboring magnets as well, and the state deforms in such a way that the vortices instead reside on nearest neighbor bonds.

3. Possibilities for experiment and conclusions

Finally, we mention some possible experimental tests for the novel vortex phenomena discussed in this work. Critical current measurements are a useful probe; different j_c^x and j_c^y values would signal broken orientational symmetry. Broken mirror symmetry implies the existence of an Ising-like order parameter, and one could look for signatures of a phase transition in the Ising universality class in the specific heat. For states such as the second $\rho_{pair} = 0$ configuration, the state

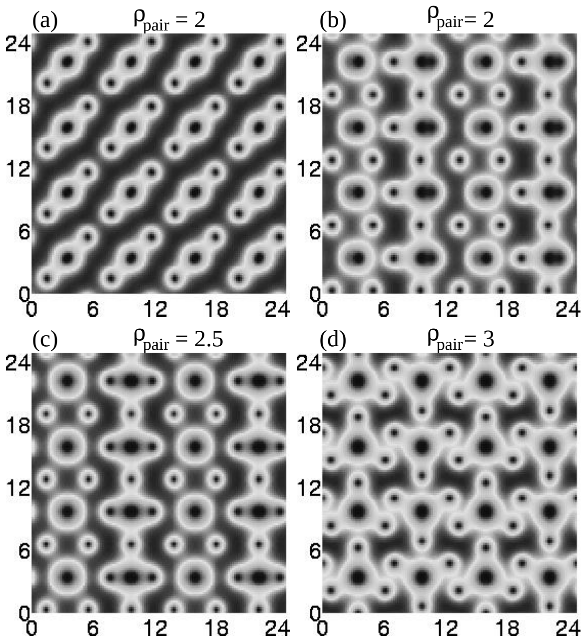

Figure 3. Cooper pair density plots of stable phases. The images shown in panels (a), (b), (c), and (d) correspond to dipole strengths equal to 3.61, 4.07, 4.33, and 4.95 fundamental flux units, respectively. The small dark spots are antivortices, while the larger dark spots are dot regions; lighter shading indicates depressed Cooper pair density.

evolves continuously, but slowly as the dot dipole moments are adjusted. This suggests the presence of a large number of quasi-degenerate states, which may be linked to a peak in the specific heat for the $\rho_{pair} = 0$ configuration.

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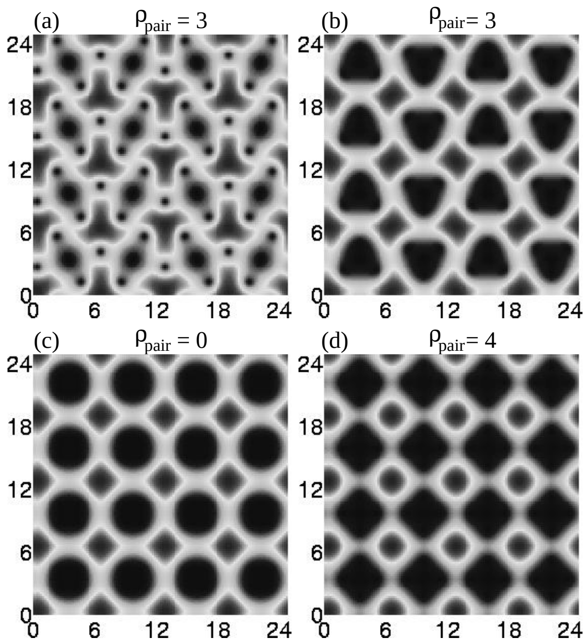

Figure 4. Cooper pair density plots of stable phases. Images shown in panels (a), (b), (c), and (d) correspond to dipole strengths equal to 5.16, 5.26, 5.42, and 6.04 fundamental flux units, respectively. Although perhaps not obvious from (c), where there are large regions of suppressed order parameter, solid evidence for $\rho_{pair} = 0$ is gleaned from the current configurations (not shown) for (c), in which there are no circulating patterns of current that would indicate vortices/antivortices.