Phy 306: Problem Set 2
(Due: February 5, 2008)

1) Prove the Euler formula, $e^{ix} = \cos x + i \sin x$. You may take x to be real, though the result holds for complex x as well.

2) Work the problems of Sec. 3.2 in Snieder.

3) Noting $(e^{i\theta})^n = e^{in\theta}$ for any n and the Euler formula prove de Moivre's theorem:

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i \sin n\theta \tag{1}$$

Use this with $n = 4$ to show that

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1 \tag{2}$$

and deduce that

$$\cos\frac{\pi}{8} = \left(\frac{2+\sqrt{2}}{4}\right)^{1/2} \tag{3}$$

4) Two complex numbers z and w are given by $z = 3 + 4i$ and $w = 2 - i$. What are the modulus ($|z|$) and argument ($\arg z$) of z? Of w? On an Argand diagram, plot the following: a) $z + w$, b) $z - w$, c) wz, d) w/z, e) $z^*w + w^*z$, f) w^2, and g) $(1 + z + w)^{1/2}$.

Hint: for part g), if we write $z = re^{i\theta}$, where r and θ are real, then you can use de Moivre's theorem to calculate its n^{th} root.

5) Work problems a, g, and h in section 4.1 of Snieder.

6) Work problems d and e in section 4.3 of Snieder.

Extra Credit: Work problem d of section 4.2 of Snieder.