Phy 306: Problem Set 3
(Due: February 19, 2008)

1) Consider three vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$, which share a common origin. Show that the area of the parallelogram associated with two of the vectors, $\mathbf{a}$ and $\mathbf{b}$, say, is given by $|\mathbf{a} \times \mathbf{b}|$. Show that the volume of the parallelepiped associated with the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ is given by $|\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})|$. Show that $|\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})|$ is equivalent to $\det(\mathbf{a}, \mathbf{b}, \mathbf{c})$. Given this, how are $\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$, $\mathbf{c} \cdot (\mathbf{b} \times \mathbf{a})$, $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$, $\mathbf{a} \cdot (\mathbf{c} \times \mathbf{b})$, $\mathbf{b} \cdot (\mathbf{a} \times \mathbf{c})$, and $\mathbf{b} \cdot (\mathbf{c} \times \mathbf{a})$ all related?

2) Work problems e, f, and g in section 5.2 of Snieder.

3) Work all the problems in section 5.4 of Snieder.

4) Work problem h in section 5.6 of Snieder.

5) Verify that

$$\int_C \mathbf{F} \cdot d\mathbf{r} = 0 \,, \tag{1}$$

where $\mathbf{F} = \nabla f$ and $f = xyz$, choosing for the curve C:

i) the square in the xy-plane with vertices at $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$.

ii) the triangle in the yz-plane with vertices at $(0,0)$, $(1,0)$, and $(0,1)$.

iii) the circle of unit radius centered at the origin and lying in the xz-plane.

6) a) Consider a vector function $\mathbf{F}$ in cylindrical coordinates such that

$$\mathbf{F}(\rho, \varphi, z) = \frac{1}{\rho}\hat{\varphi} \,. \tag{2}$$

Consider a unit circle in the xy plane and integrate

$$\int_C \mathbf{F} \cdot d\mathbf{r} \,, \tag{3}$$

for two different choices of C. In specific, consider a curve C_R which follows our unit circle from $x = z = 0$, $y = -1$ to $x = z = 0$, $y = +1$ on the right-hand side of the circle, as well as a curve C_L which follows our unit circle from $x = z = 0$, $y = -1$ to $x = z = 0$, $y = +1$ on the left-hand side of the circle. Why does the fact that the two paths give different results imply that there is no scalar function f such that $\mathbf{F} = \nabla f$?

b) A force $\mathbf{F}$ is *conservative* if its line integral from points A to B, that is,

$$\int_{C'} \mathbf{F} \cdot d\mathbf{r} \,, \tag{4}$$

is independent of any path C' from A to B. Could either the function $\mathbf{F}$ of problem # 5 or that of problem #6 a) represent a conservative force? Why or why not? If $\mathbf{F}$ is a conservative force, is there always a scalar function f for which $\mathbf{F} = \nabla f$? Why or why not?