Phy 306: Problem Set 4
(Due: February 28, 2008)

1) Consider the scalar functions $\psi(\mathbf{r})$ and $\phi(\mathbf{r})$. Write $\nabla \times (\nabla \psi)$, $\nabla \times (\nabla \phi)$, and $(\nabla \psi) \times (\nabla \phi)$ in the simplest form possible. Explain your results using geometrical arguments.

2) Work problem d in section 6.2 of Snieder. What is $\int \mathbf{v} \cdot d\mathbf{S}$ for a surface enclosing both $\mathbf{r}_+$ and $\mathbf{r}_-$? Why?

3) Work problem g in section 6.4 of Snieder.

4) Work the problems of section 6.5 of Snieder.

5) Calculate the divergence of the function

$$\mathbf{F}(x, y, z) = \hat{\mathbf{x}} f(x) + \hat{\mathbf{y}} f(y) + \hat{\mathbf{z}} f(-2z) \tag{1}$$

and evaluate it explicitly for $x = y = c\,, z = -c/2$, where c is a constant. Calculate the divergence of the function

$$\mathbf{G}(x, y, z) = \hat{\mathbf{x}} f(y, z) + \hat{\mathbf{y}} g(x, z) + \hat{\mathbf{z}} h(x, y)\,. \tag{2}$$

Calculate the curl of $F(x, y, z)$, the curl of $G(x, y, z)$, and the curl of the function

$$\mathbf{H}(x, y, z) = \hat{\mathbf{x}} yz + \hat{\mathbf{y}} xz + \hat{\mathbf{z}} xy\,. \tag{3}$$

6) Show that $\nabla \cdot (\nabla \times \vec{F}) = 0$. Give a geometrical explanation of this result.